Inferring Structure from Optimization Model Solutions

John Birge, University of Chicago
Ali Hortaçsu, University of Chicago
Michael Pavlin, Wilfrid Laurier University

November 11, 2012

Themes

- Solutions of optimization models are often observed while the relevant parameters are not.
- Common examples include problems from network industries such aa electricity.
- When the data include constraint coefficients, the inverse optimization to discover the parameters is only partially identified.
- Identification can be possible through norm minimization.

A Motivating Problem

Economics of Two-Stage Electricity Markets?
(Veit et al 2006; Sioshansi, Oren, O'Neill 2010; Botterud et al 2011)
Market Power In Electricity Markets?
(Cardell, Hitt, Hogan 1996; Jiang, Baldick 2005; Hogan 2012)
Results rely assumptions r.e. participant objectives:
⇒Case Study on Wind Producer Objectives in Midwest ISO

- Two-stages: Day Ahead (DA), Real Time (RT) markets
- Forecast: Wind producers submit a DA production commitment
- Stochastic production: shortfall or surplus made up via RT prices

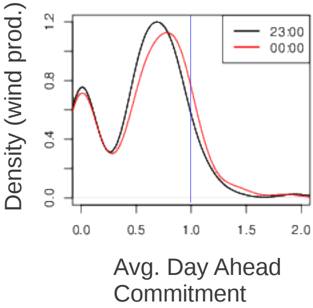
Wind DA Revenues:

$$Rev(Q_{DA}) = Q_{DA} \times \mathbf{P_{DA}} + (\mathbf{Q_{RT}} - Q_{DA}) \times \mathbf{P_{RT}}$$

Economic value of intermittent generation depends on forecast quality
(Gowrisankaran, Reynolds, Samano 2011; Skea, Anderson 2008 ...)

Quick Look at the Data

Midwest ISO 2010 Data:
Forward Premium
$E(P_{DA} - P_{RT}) > \$2.00$
But Under-commitment!



Why?

- Bad forecasting?
- **Risk aversion?**
- **Exercise of Market Power?**

Prerequisite for answer:
estimate $E(\partial P_{DA}/\partial Q_{DA})$ and $E(\partial P_{RT}/\partial Q_{DA})$
Hard with standard econometrics due to endogeneity

Research Problem:

- Ideal solution to endogeneity question:
  An accurate model of price determination process
  - Zonal prices
    uniform price multi-unit auction
    (c.f. Reguant 2012; Hortaçsu, Puller 2008; Wolak 2004)
- However, Locational Marginal Pricing dominate North American Markets
  (e.g. PJM, Midwest ISO, CAISO, ERCOT)
  - Prices depend on entire network structure
  - Network structure not directly observable
    Critical Infrastructure Information Act (2002)
- However, Available information:
  - Midwest ISO: prices, quantities, bids, active transmission constraints

Can a useful model of the network be inferred from this information?
(Useful for market researchers, participants, *designers of CIIA*)

Roadmap

- Locational Marginal Price (LMP) Market: *Linear model*
- The Estimation Problem... *via Inverse Optimization*
- An Algorithm: *A sufficient explanation*
- Application to Data: *Midwest ISO*

Electricity Dispatch Model

Relaxation of the unit commitment problem:

Market Participant $i \in \{1..N\}$

- Produces x_i MWh at an announced cost of c_i/MWh

Lossless network with links (i, j)

- Transmission between i and j of $y_{(i,j)}$ MWh
- Topology defined by matrices E, A, and D
  - Network Flow Constraints $Ey = x$
  - R Physical Constraints $Ay = 0$
  - L Transmission Constraints $Dy \leq d$

$$\begin{aligned} \min\ & c^T x \\ \text{s.t.}\ & Ey = x \\ & Ay = 0 \\ & Dy \leq d \\ & u \geq x \geq l \\ & y \geq 0 \end{aligned}$$

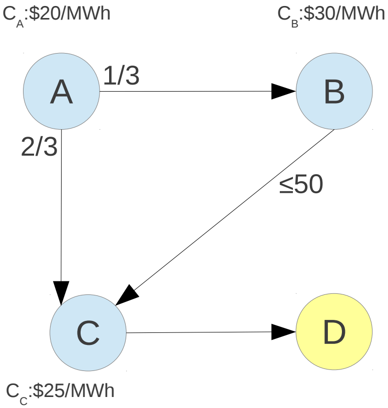
Locational Marginal Prices

- Definition:
  1. The Locational Marginal Price (LMP) is the immediate cost of supplying one additional MW of power at a particular node.
  2. The LMP is the shadow price of the flow constraint

$$
\begin{array}{rlr}
\min & c^T x & \text{dual variables} \\
\text{s.t.} & Ey = x & \pi \\
& Ay = 0 & \sigma \\
& Dy \leq d & \rho \\
& u \geq x \geq l & \alpha, \gamma \\
& y \geq 0 &
\end{array}
$$

LMP Example

(Louie, Strunz 2008)



Corresponding LP:

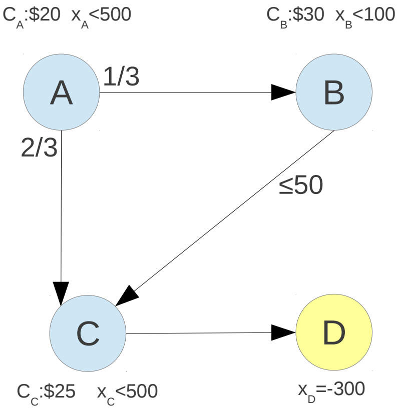
$$\min 20x_A + 20x_B + 20x_C$$

$$\text{s.t. } Ey = x$$

$$2y_{AB} - y_{AC} = 0 \qquad \text{A constraints}$$

$$y_{BC} \leq 50 \qquad \text{D constraints}$$

LMP Example



Corresponding LP:

$$\min 20x_A + 20x_B + 20x_C$$

$$\text{s.t.} \quad Ey = x$$

$$2y_{AB} - y_{AC} = 0 \qquad \text{A constraints}$$

$$y_{BC} \leq 50 \qquad \text{D constraints}$$

$$0 \leq x_A \leq 500$$

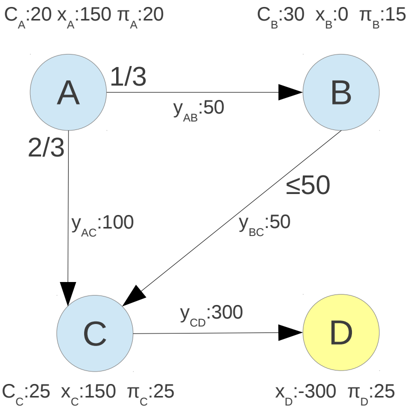
$$0 \leq x_B \leq 100$$

$$0 \leq x_C \leq 500$$

$$-300 \leq x_D \leq -300$$

$$y \geq 0$$

LMP Example



Solution:

$$x_A = 150 \qquad \pi_A = 20$$
$$x_B = 0 \qquad \pi_B = 15$$
$$x_C = 150 \qquad \pi_C = 25$$
$$x_D = -300 \qquad \pi_D = 25$$
$$y_{AB} = 50$$
$$y_{AC} = 100$$
$$y_{BC} = 50 \qquad \rho_{BC} = -15$$
$$y_{CD} = 300$$

Estimation Problem (Single Sample)

	dual variables
$\min c^T x$	
s.t. $Ey = x$	π
$Ay = 0$	σ
$Dy \leq d$	ρ
$u \geq x \geq l$	α, γ
$y \geq 0$	

- Given data:
  - c, x, u, l, π, ρ
- Generate a network model: $\bar{E}$, $\bar{A}$, $\bar{D}$, $\bar{d}$
- Explaining shadow prices π and ρ

General Inverse Optimization

Given:

- a partial specification of an optimization model
- a (partial) specification of an optimal solution

Infer missing model parameters such that:

- Consistency: *Known Parameters consistent with optimality*
- Simplicity: *Missing parameters minimize a norm*

$\min c^T x$		dual variables
s.t.	$Ey = x$	π
	$Ay = 0$	σ
	$Dy \leq d$	ρ

Standard form Zhang, Liu 1996,1999 (linear); Ahuja, Orlin 2001 (general):

- Feasible set known
- Opt. solution known
- Cost parameters unknown
- Minimize L1 or L∞ norm
  Extensions: Yang, Zhang 2007; Ahmed, Guan 2005; Iyengar, Kang 2005; Wang 2009; Ahuja, Orlin 1998,2002; Cui, Hochbaum 2010

Our Inverse Optimization Problem

Standard form:

- Feasible set known
- Opt. solution known
- Cost parameters unknown
- Minimize L1 norm

Electricity Market Problem:

- Feasible set **unknown**
- Opt. solution **partially known**
- Cost parameters **known**
- Minimize **L1 norm**

	dual variables
$\min c^T x$	
s.t. $Ey = x$	π
$Ay = 0$	σ
$Dy \leq d$	ρ
$u \geq x \geq l$	α, γ
$y \geq 0$	

	dual variables
$\min c^T x$	
s.t. $Ey = x$	π
$Ay = 0$	σ
$Dy \leq d$	ρ
$u \geq x \geq l$	α, γ
$y \geq 0$	

Our Inverse Optimization Problem

- Find a "simplest" $\bar{A}$,$\bar{D}$,$\bar{d}$,$\bar{E}$ satisfying optimality conditions
- Minimize 1-Norm
- Regularize $\bar{A}$ s.t. $\sigma_r = 1$

Resulting Optimization Problem:

$$\min \ ||\bar{A}||_1 + ||\bar{D}||_1 + ||\bar{d}||_1$$

$$\sum_{r<R} \bar{A}_{r(i,j)} + \sum_{\ell<L} \bar{D}_{\ell(i,j)} \rho_{(i,j)} = \pi_j - \pi_i \qquad \forall (i,j) \ \bar{y}_{ij} > 0$$

$$\bar{E}\bar{y} = x$$

$$\bar{A}\bar{y} = 0$$

$$\bar{D}\bar{y} \leq \bar{d}$$

2-Step Algorithm:

1. Find $\bar{E}$ and $\bar{y}$:
2. Find $\bar{A}$, $\bar{D}$, $\bar{d}$:

Step 1: Determine $\bar{E}$ and $\bar{y}$

- Assume no loss
  $\bar{E}_{k(ij)} \in \{-1, 0, 1\}$
- limit to flows between sources and sinks
  $\bar{E}_{i(ij)} = 1$ only if $x_i > 0$,
  $\bar{E}_{j(ij)} = -1$ only if $x_j < 0$,
  $\bar{E}_{k(ij)} = 0$ otherwise
- Minimize requirements on $\bar{A}$ and $\bar{D}$ to satisfy

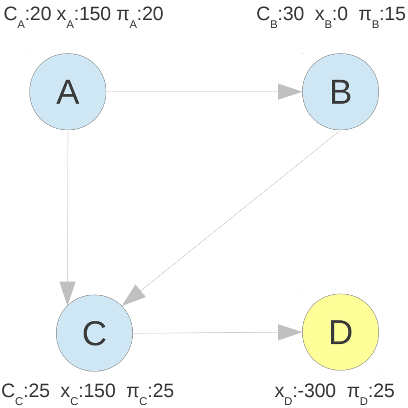
$$\sum_r \bar{A}_{r(i,j)} + \sum_\ell \bar{D}_{\ell(i,j)} \rho_{(i,j)} = \pi_j - \pi_i \quad \forall \bar{y}_{ij} > 0$$

- By solving:

$$\min \sum_{ij} \bar{y}_{ij} \cdot \max\{\pi_i - \pi_j, 0\}$$

$$\text{s.t. } \bar{E}\bar{y} = x$$

$$\bar{y}_{ij} \geq 0$$



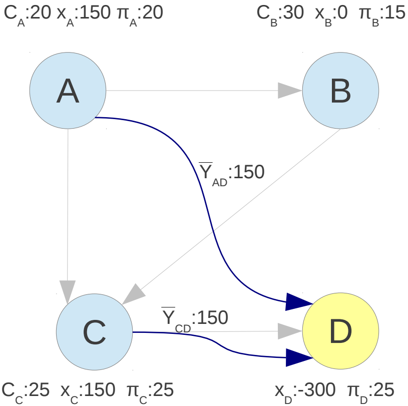
Step 1: Determine $\bar{E}$ and $\bar{y}$

- Assume no loss
  $\bar{E}_{k(ij)} \in \{-1, 0, 1\}$
- limit to paths between sources and sinks
  $\bar{E}_{i(ij)} = 1$ only if $x_i > 0$,
  $\bar{E}_{j(ij)} = -1$ only if $x_j < 0$,
  $\bar{E}_{k(ij)} = 0$ otherwise
- Minimize requirements on $\bar{A}$ and $\bar{D}$ to satisfy

$$\sum_r \bar{A}_{r(i,j)} + \sum_\ell \bar{D}_{\ell(i,j)}\rho_{(i,j)} = \pi_j - \pi_i \quad \forall \bar{y}_{ij} > 0$$

- By solving:

$$\begin{aligned} \min y_{ij} \cdot \max\{\pi_i - \pi_j, 0\} \\ \text{s.t. } \bar{E}\bar{y} = x \\ \bar{y}_{ij} \geq 0 \end{aligned}$$



Step 2: Determine $\bar{A}$, $\bar{D}$, $\bar{d}$

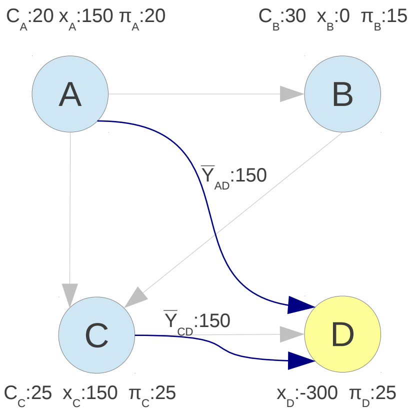
Minimize sum of 1-norms subject to optimality constraints
Solving:

$$\min \sum_{r,(i,j)} |\bar{A}_{r(ij)}| + \sum_{r,(i,j)} |\bar{D}_{\ell(ij)}| + \sum_{\ell} \bar{d}_{\ell}$$

$$\text{s.t.} \;\; \bar{A}\bar{y} = 0$$

$$\sum_{r} \bar{A}_{r(i,j)} + \sum_{\ell} \bar{D}_{\ell(i,j)} \rho_{(i,j)} = \pi_j - \pi_i \quad \forall \bar{y}_{ij} > 0$$

$$d_{\ell} \geq 0$$



A and D Constraints:
$\frac{1}{3}\bar{y}_{AD} \leq 50$

Multiple Samples

For set of samples 1..S

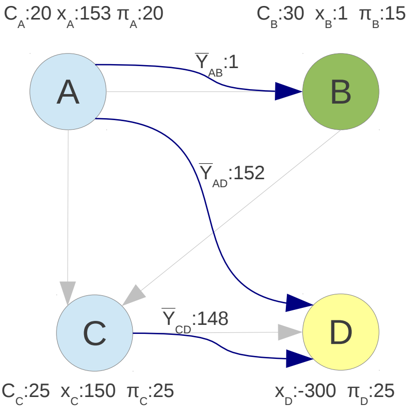
1. Calculate $\bar{y}^s$ independently
2. Add constraints for each sample

$$\min \sum_{r,(i,j)} |\bar{A}_{r(ij)}| + \sum_{r,(i,j)} |\bar{D}_{\ell(ij)}| + \sum_{\ell} \bar{d}_\ell$$

$$\text{s.t. } \bar{A}\bar{y}^s = 0 \qquad \forall s$$

$$\sum_r \bar{A}_{r(i,j)} + \sum_\ell \bar{D}_{\ell(i,j)}\rho^s_{(i,j)} = \pi^s_j - \pi^s_i \qquad \forall \bar{y}^s_{ij} > 0$$

$$\bar{d}_\ell \geq 0$$



A and D Constraints:
$\frac{1}{3}(\bar{y}_{AD} - 2\bar{y}_{AB}) \leq 50$

General Implementation

- Observations for each day and hour
  - x, π, ρ, c vary
  - E, D, A, d constant (approximately)
- Transmission and generation outages may change d
- Transmission losses not included
- Original "optimization" may be adjusted for other reasons (e.g., frequency, reliability)

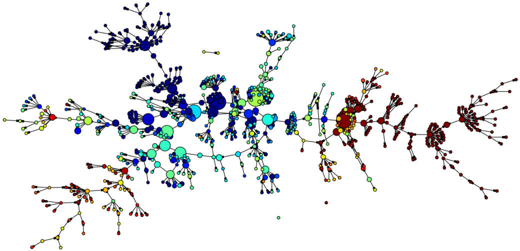
Algorithm Observations

- Extension with multiple observations
  - Step 1 performed independently
  - Step 2 single optimization adding all constraints
- Algorithm feasible if rows in A greater number of samples
- Polynomial approximation
  - Step 2: LP standard transformation
  - Each step presents $O(n^2)$ variables

Results: Application to Midwest ISO

2010/01/01 00:00:00

- 1403 Nodes
- 768 Aggregated Nodes
- 772 Active Links
- Transmission bounds per hour
- Imperfect data
- Naive implementation 20 ARows $\rightarrow$ 40min



Power Flow in Midwest ISO Network

Additional Examples

- Macro-economic models
  - Prices observed in different regions
  - Purchase quantities observed
  - Equilibrium model set up as potential optimization
  - Unknown transportation routes and costs to discover
- Supply chain interactions
  - Prices and quantities observed
  - Unobserved relationships between suppliers and customers
  - Discover relationships and transactions

Future Directions

- Discussed modelling price determination process in LMP based Electricity Markets
  - Inverse optimization based formulation/algorithm consistent with dual interpretation of LMPs

Next Steps

- Predicting market characteristics: *price response, congestion costs...*
- Structural estimation: *model participant decision making*
- Solution quality: *solution robustness to data imperfections*
- Econometrics of general competitive markets: *extend to linear market models*

Conclusions

- Inverse optimization to discover constraints
  - Needed to determine objective of market participants
  - Many markets include price and quantity observations but not constraints
  - Difficulty from bilinear form with constraint and unknown variable values
- Solution method
  - Two-step process
  - Determine consistent primal variables first
  - Choose constraint coefficients with minimum 1-norm
- Results
  - Possible to discover simple network configurations
  - Reasonable results with multiple data observations
  - Possible inconsistencies from unknown parameter changes