Optimization Problems Motivated by Hamiltonian Structure in Power System Dynamics

Wisconsin Institute for Discovery Optimization Seminar Series (WID-DOW), November 19, 2012

Professor Chris DeMarco

Electrical & Computer Engineering

University of Wisconsin-Madison

THE UNIVERSITY of WISCONSIN MADISON

Background & Motivation

- In normal operation, North America's three major synchronous power grids are extremely stable.
- But when large, wide-area blackouts occur (e.g., eastern U.S. Aug 2003, WECC July/Aug 1996), dynamic instability is often the ultimate culprit.
- Industry state-of-the art today: highly developed time domain simulation tools. Able to predict physical system response with high fidelity on milliseconds to minutes time scales ***(caveat: … for a precisely specified scenario, given detailed model data.)***

Background & Motivation

- Given extremely high cost of wide area blackouts, avoidance of instability phenomena is among the key drivers justifying major grid investments, under broad umbrella of "enhancing grid reliability."
- Wisconsin example – Rockdale/West Middleton 345 kV power line being constructed along Madison's Beltline. Justification for this line appears in 2009 testimony by American Transmission Company (ATC), before WI Public Service Commission.
- Interpreting ATC's justification will require some translation from the power-system-ese…

Background & Motivation

Feb. 2009 Testimony to WI PSC, by ATC's Manager for Transmission Planning, Mr. Jamal Khudai, docket 137-CE-147, under "Need for the Project":

- "The transmission system in Dane Co. … is marginally adequate under normal operating conditions. Low voltage conditions are projected to occur in the 2015-2020 period with the system intact… If not addressed, these issues lead to non-convergence of the power flow model (the problem doesn't 'solve'), which indicates voltage collapse conditions. No significant thermal violations are projected in this period under system intact or system normal conditions."
  http://psc.wi.gov/apps35/ERF_view/viewdoc.aspx?docid=107754

Background & Motivation

Translation from power engineering terminology:

- The existing, “intact” system (without the new transmission line) was judged adequate for steady state operation through 2020 (“no significant thermal violations are projected in this period under system intact or system normal conditions.”)

- This new transmission line, 32 miles at a cost of $220 million, was justified largely to protect against dynamic instability phenomena (“voltage collapse”).

Power system specialists will recognize that grid standards require reliability be maintained under failure of a major transmission line; “N-1 contingency analysis.” Mr. Khudai’s testimony goes on to describe some thermal overloads, and more severe voltage collapse, under contingencies.

Background & Motivation

- Abstracting, significant analytic challenge implicit here.
- Capital investment decisions for transmission system have very long time horizon (30+ year life). Yet they're driven in part by their impact on the stability of a very high-dimension, nonlinear dynamical system, ***on very short time scales*** (milliseconds to minutes).
- Current state of the art, that of scenario-by-scenario time domain simulation, is woefully inadequate. Critical need for tools to relate qualitative stability properties to network structure, and incorporate such tools into infrastructure optimization.

What are Grid's Dynamics, and why Hamiltonian Structure?

- Power grid is primarily a very large electric circuit, with some mechanical dynamics at select nodes.

- A standard means to derive O.D.E. or D.A.E. state equations is common throughout EE – start from constitutive relations of individual components (e.g., $v = L\,di/dt$), then algebraically couple components via topological network constraints (KVL, KCL).

- Alternative viewpoint, much less common in EE: Euler-Lagrange formalism focuses on stored energies as fundamental quantities, then derives dynamics via variational operations on these.

What are Grid's Dynamics, and why Hamiltonian Structure?

- Full Euler-Lagrange beyond scope of this brief talk. But as simple special case, recall lossless Hamiltonian system, as studied in classical mechanics.

- Classic mechanical Hamiltonian system has an even dimension state space (with "symplectic structure"). In naïve view, defined by a vector of positions, say **z**, a like-dimensioned momenta vector, **p**, and a scalar function H(**z**, **p**) {H is often potential + kinetic energy}.

What are Grid's Dynamics, and why Hamiltonian Structure?

- ODE's describing Hamiltonian dynamics are then:

$$\dot{\mathbf{z}} = \nabla_{\mathbf{p}}\mathbf{H}(\mathbf{z},\, \mathbf{p}) \; ; \; \dot{\mathbf{p}} = -\nabla_{\mathbf{z}}\mathbf{H}(\mathbf{z},\, \mathbf{p})$$

or more compactly,

$$\begin{bmatrix} \dot{\mathbf{z}} \\ \dot{\mathbf{p}} \end{bmatrix} = \mathbf{A}\nabla_{\mathbf{z},\mathbf{p}}\mathbf{H}(\mathbf{z},\, \mathbf{p})$$

with $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix}$ (negative semidefinite, full rank)

Nearly Hamiltonian Structure in Power Grid Stability Studies

- This suggests a simple generalization, which proves common to wide class of power system models:

$$\dot{\mathbf{x}} = \mathbf{A}\nabla_{\mathbf{x}}\Phi(\mathbf{x})$$

with $\mathbf{A}$ negative semidefinite, full rank.

- There exists a long literature devoted to construction of energy/Lyapunov function for power systems, with objective of estimating basins of attraction. In this author's experience, ALL of these prove to have form above, with $\Phi(\mathbf{x})$ serving as Lyapunov function.

Nearly Hamiltonian Structure in Power Grid Stability Studies

- Suggests this scalar valued, “energy-like” Lyapunov function $\Phi(\mathbf{x})$ captures considerable information regarding dynamic grid behavior.

- Q2: How to relate $\Phi(\mathbf{x})$ to practical stability problems, such as voltage collapse phenomena, and then to grid topology, to inform optimal network expansion?

- Q1: As simpler first step, how to relate $\Phi(\mathbf{x})$ to standard, familiar computations in grid studies, such as the power flow equations?

Structure in the Power Flow Problem

- Perhaps the most ubiquitous of computations in grid studies is that of the power flow equations.

- These characterize the active and reactive power absorbed by the network at each bus(node), as a function of the magnitudes and phase angles of the sinusoidal voltages at each bus.

- Commonly exploited feature in power flow equations is fact that their Jacobian matrix is very nearly symmetric (and can be made exactly symmetric with approximations that “move” line losses from branches to nodes).

Power Flow Problem Structure

- But recall basic fact of vector calculus:

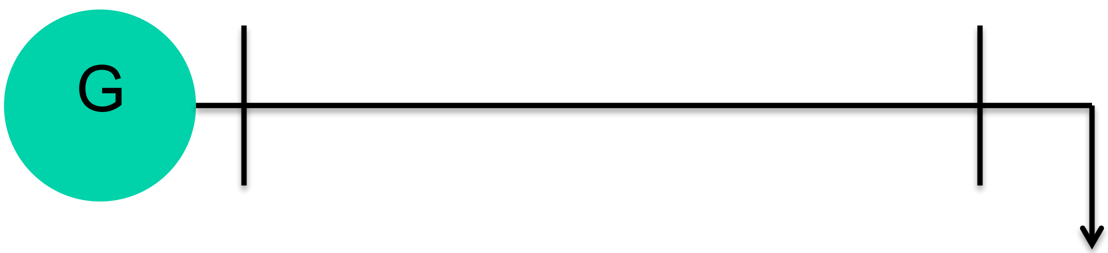
  Consider a function $\mathbf{f} : \Re^n \rightarrow \Re^n$

- Condition for such a function to be exactly integrable is that its Jacobian be structurally symmetric.

- Hence (with mild modeling approximation), power flow equations are exactly integrable! Their integral is the dominant ***network-related*** term in $\Phi(\mathbf{x})$
  for power system stability analysis (other terms relate to load/generation, and are local to each node).

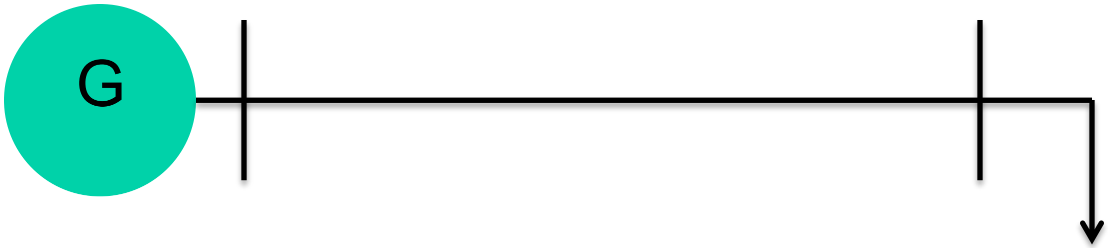
Power Flow Structure and Voltage Collapse Problem : Example

- Consider simple one-line power system, with one load, active power demand, P, reactive power demand Q. Generator holds its voltage magnitude constant, and adjusts its power output to meet load.
- Then only two variables: voltage magnitude at load bus (denote V), and relative phase angle (denote δ).



Load Demand:
Active Power P
Reactive Power Q

Power Flow Structure and Voltage Collapse Problem : Example

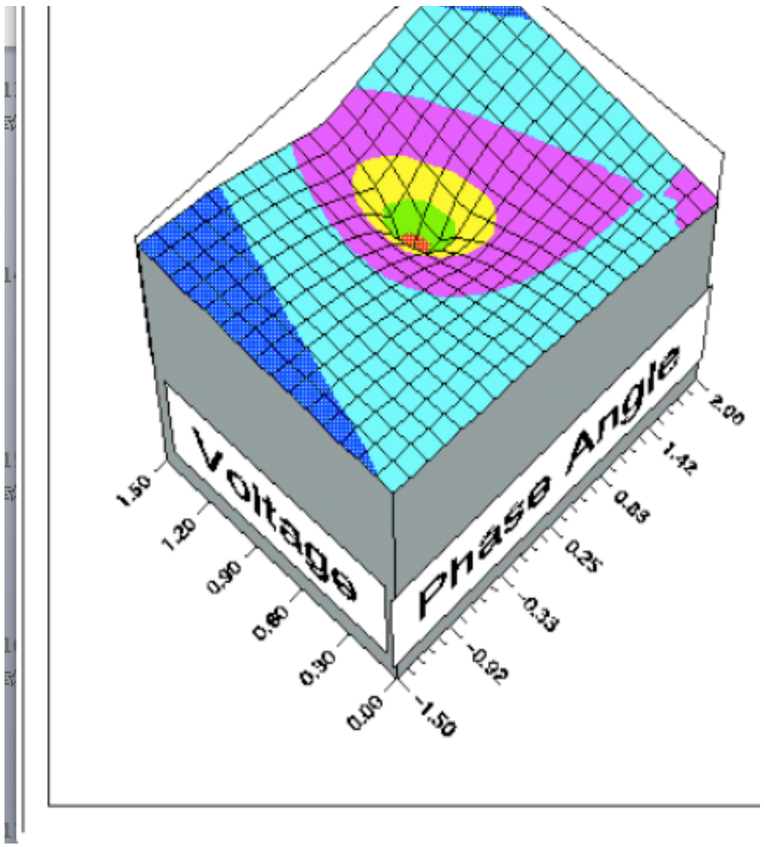


Load Demand:
Active Power P
Reactive Power Q

- Power flow solution is then a (δ, V) pair that defines an equilibrium operating point for system. But if power flow equations are $\nabla\Phi(\mathbf{x})$, such a solution must be a stationary point of $\Phi(\mathbf{x})$ (and not coincidentally, a ***stable*** point must be a local minimum of $\Phi(\mathbf{x})$).

Power Flow Structure and Voltage Collapse Problem : Example

- Viewed in $\mathbf{x}$ = (δ, V) plane, color contour plot of $\Phi(\mathbf{x})$ for this example appears as below.

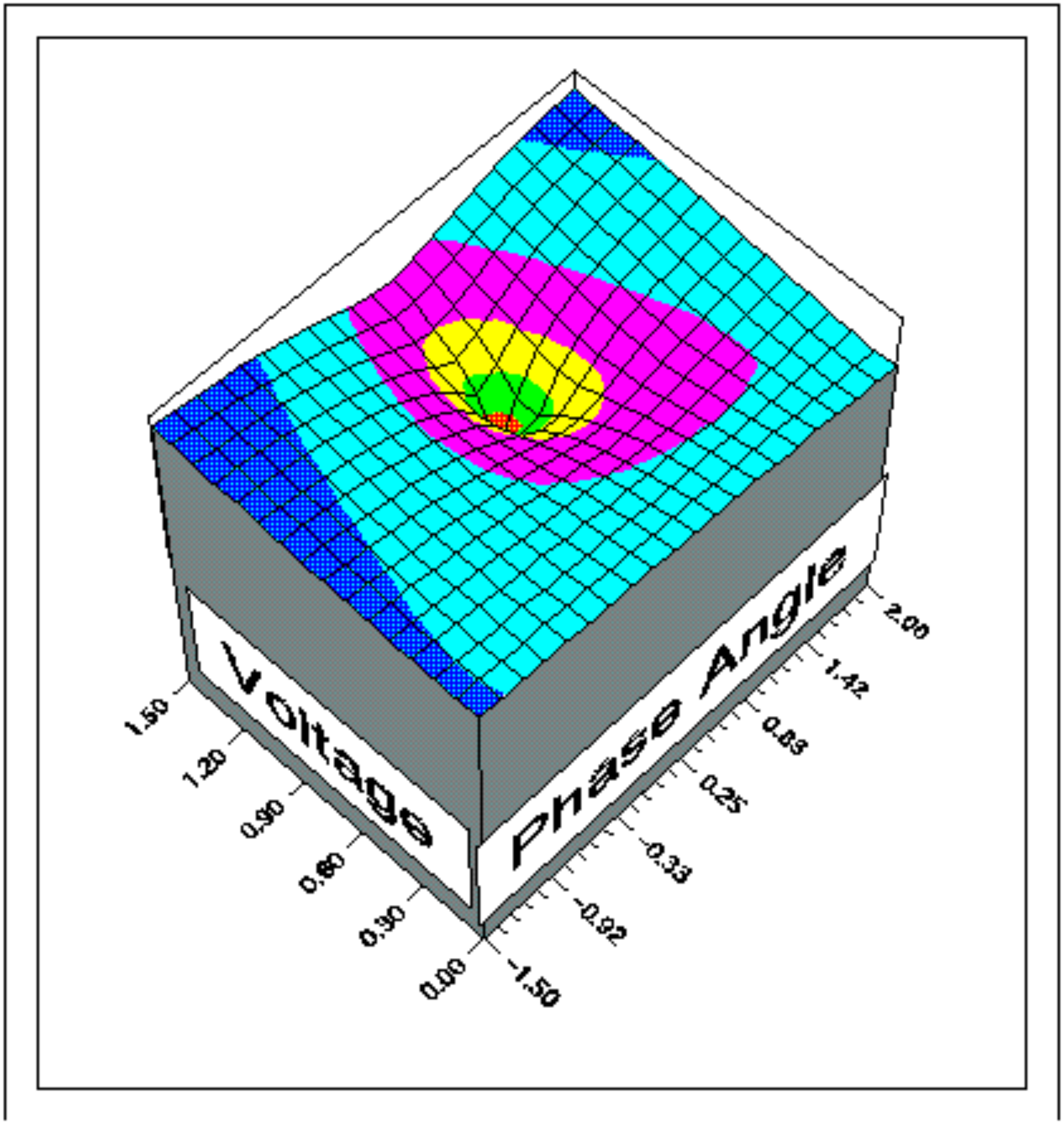


Power Flow Structure and Voltage Collapse Problem : Example

- More interesting is change in these contours of $\Phi(\mathbf{x})$ as load level slowly changes (detail: we impose a one-degree-of-freedom increase in load, with reactive load increase more pronounced).

- In animation on following slide (embedded movie), watch as the two stationary points of $\Phi(\mathbf{x})$, initially well-separated, coalesce under the influence of increasing load.

- As this (reverse) bifurcation occurs, the potential well about the operating point "opens up," and stability is lost.



Voltage
Phase Angle
1.50
1.20
0.90
0.60
0.30
0.00
-1.50
-0.92
-0.33
0.25
0.83
1.42
2.00

Power Flow Structure and Voltage Collapse Problem

- Recall American Transmission Co.'s testimony on issues justifying Dane County's new $220 million line: "these issues lead to non-convergence of the power flow model (the problem doesn't 'solve'), which indicates voltage collapse conditions."

- Note that if gradient of $\Phi(\mathbf{x})$ yields the power flow equations, the Hessian of $\Phi(\mathbf{x})$ (its curvature) must correspond to power flow Jacobian.

- Opening of potential well implies power flow Jacobian becomes singular – precisely the conditions under which standard Newton-Raphson "doesn't solve."

And where are the optimization problems in all this… ?

Short answer:

- scale computations from this n=2 dimensional example, to n=80,000 that describes bulk transmission system for Eastern Interconnect of U.S. power grid.

And where are the optimization problems in all this… ?

More substantive answer – deterministic:

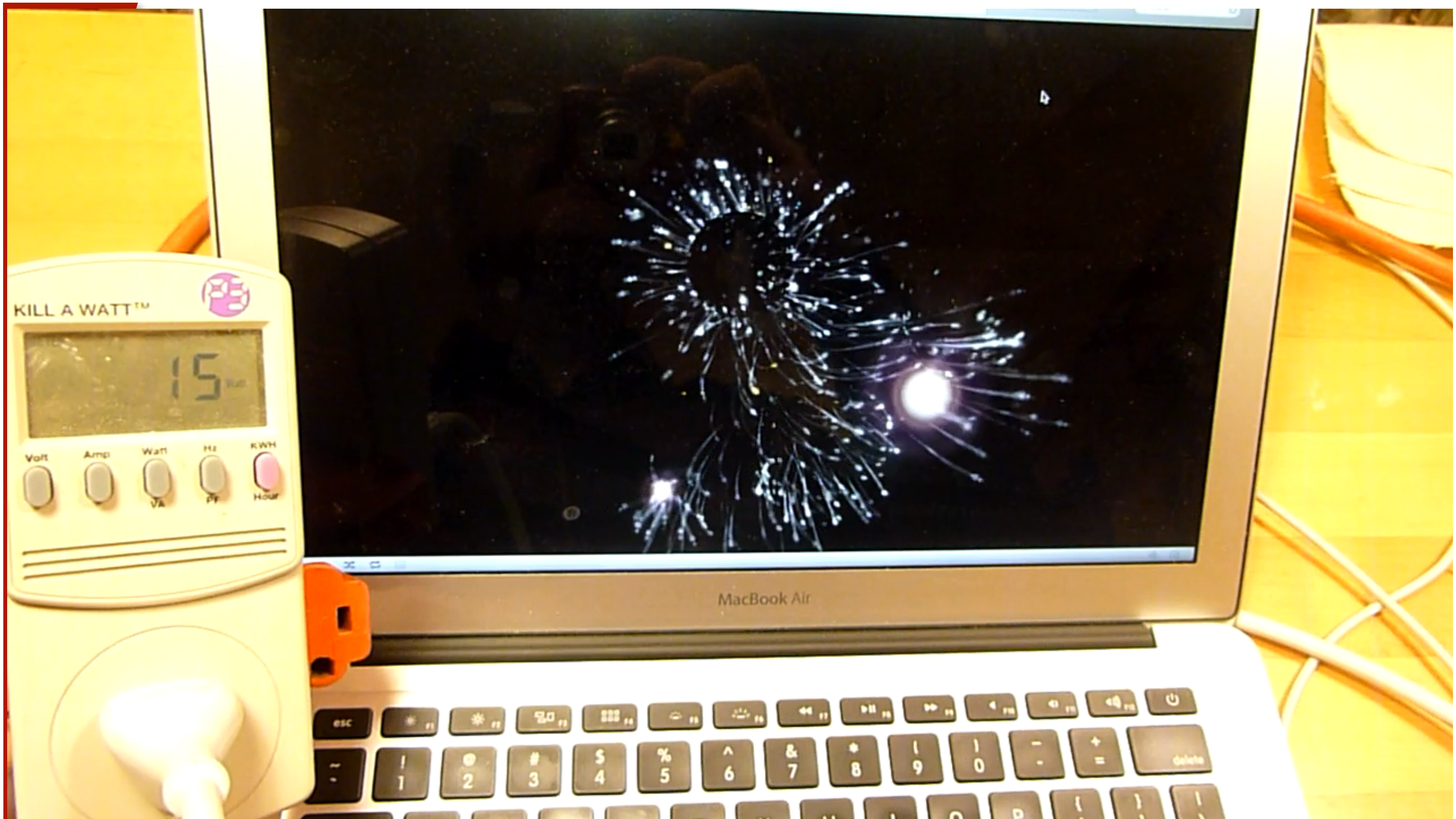
- Suppose one didn't know *a priori* the relation between the vector field defining the power grid's dynamics, and the potential function $\Phi(\mathbf{x})$.
- Recognize that load demands (and wind/solar power generation) also have fast variation, $\Delta\mathbf{P}(t)$, $\Delta\mathbf{Q}(t)$, on timescale commensurate with dynamics.
- Stability-related optimization problem: What is the "size" of the smallest disturbance ($\Delta\mathbf{P}(t)$, $\Delta\mathbf{Q}(t)$) that drives system state out of its basin of attraction?

And where are the optimization problems in all this… ?

- Moreover, these fast time scale variations in power demands $\Delta\mathbf{P}(t)$, $\Delta\mathbf{Q}(t)$ are inevitably stochastic.

- Load demand at a major distribution substation bus is the aggregation of 100's of thousands of individual customer devices. These switch on/off and vary demand under influence of individual human decisions, and very local automatic control systems.

- A laptop power supply provides microcosm of load behavior on grid. A predictable, slow moving average power demand. About the average, a random jump process, order ±15%, second-by-second.



- Experimental validation of random jump behavior in load demand (lab instrumentation courtesy of Ace Hardware)

And where are the optimization problems in all this… ?

- With $\Delta\mathbf{P}(t)$, $\Delta\mathbf{Q}(t)$ stochastic, more sophisticated problem formulations could include stochastic stability measures, treating state as a diffusion, e.g.

  - Expected exit time from basin;

  -Relative probability of different paths of exit.

- Assuming certain structural features, answers to several of these problems reduce to a deterministic counterpart in the asymptotic limit of "small noise."

- More general cases depend on statistics of noise – these become ***extremely*** challenging.

Optimization Problems in Power System Stability Characterization

- With **u**(t)=(Δ**P**(t), Δ**Q**(t)) treated as deterministic "control" input, it enters power model linearly.

- Measuring size of this control in standard integral squared fashion, one might first like to know minimum cost of control to steer system state from the stable equilibrium to any other point **x**.

- With some added assumptions (weights with which these inputs enter), associated cost of control is solvable in close form for traditional power system model, and is precisely our $\Phi(\mathbf{x})$.

Optimization Problems in Power System Stability Characterization

- But even with closed form cost of control, challenging problem remains: finding easiest path of escape out of potential well surrounding a stable equilibirum, $\mathbf{x}^{\mathrm{s}}$…

  … more precisely, wish to find lowest energy saddle point on boundary of potential well.

- One simplistic approach, that does not scale well: from neighborhood of stable equilibrium, expand constant contours of $\Phi(\mathbf{x})$. On any given constant contour, seek $\mathbf{x}$ that minimizes $\left\|\nabla\Phi(\mathbf{x})\right\|$. Terminate when contour and associated $\mathbf{x}$ at which gradient goes to zero is located – this is lowest saddle point, $\mathbf{x}^{\mathrm{u}}$.

Optimization Problems in Power System Stability Characterization

- How might one use this to characterize stability impact of network expansion (i.e., addition of new transmission lines)?
- Both the energy function itself, AND its associated stable equilibrium and lowest saddle exit would need to be parameterized in candidate line additions.
- Seek line addition(s) to maximize $\Phi(\mathbf{x}^u) - \Phi(\mathbf{x}^s)$.

Optimization Problems in Power System Stability Characterization

- As new classes of control technologies added to transmission grid to enhance its performance (e.g., new types of High Voltage DC transmission, other power electronics), dynamic behavior farther from basic physics, more determined by controllers.

- Analytic impact: accurate models yield vector fields that do not lend themselves to closed form identification of an associated $\Phi(\mathbf{x})$. Direct fix would seek numerical solution to Hamilton-Jacobi-Bellman eqns – but probably not too promising for n=80,000.

Optimization Problems in Power System Stability Characterization

- Might consider approach akin to recent SDP methods for "automatic" Lyapunov function generation.
- Given model $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, seek fit of full rank negative semidefinite $\mathbf{A}$, and (from suitable family of functions) a potential $\Phi(\mathbf{x})$, to minimize over some volume:

$$\left\| \mathbf{f}(\mathbf{x}) - \mathbf{A}\nabla\Phi(\mathbf{x}) \right\|$$

Conclusions

- U.S. appears poised for period of grid infrastructure expansion in coming decades, with potentially 100's of billion $'s of investments at stake.
- Many pieces in this expansion will be decided by role of new transmission lines in maintaining grid stability.
- Industry's state of the art for examining stability impact today is rather ad hoc, with time domain simulation performed over a ***very*** modest number of future scenarios, selected based on engineering judgment.

Conclusions

- Suggests opportunity for modern decision and optimization tools to greatly improve the transmission expansion process.
- However, even where first steps in this direction are being taken in research literature, few offer means for optimizing on contribution of transmission reinforcement to grid stability.
- Work here intended to suggest a path forward. Treat the stability enhancement objective in a computationally tractable fashion, by exploiting the nearly Hamiltonian structure in grid dynamics.