Redshift space distortions
and
The growth of cosmic structure

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with

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Outline

- Introduction
    - Why, what, where, …
- The simplest model.
    - Supercluster infall: the Kaiser factor.
    - Legendre expansion.
- Beyond the simplest model.
    - What about configuration space?
    - Difficulties in modeling RSD.
    - Insights from N-body.
    - Some new ideas.
- Conclusions.

RSD: Why

- What you observe in a redshift survey is the density field in redshift space!
  - A combination of density and velocity fields.
- Tests GI.
  - Structure growth driven by motion of matter and inhibited by expansion.
- Constrains GR.
  - Knowing a(t) and ρ_i, GR provides prediction for growth rate.
  - In combination with lensing measures Φ and Ψ.
- Measures "interesting" numbers.
  - Constrains H(z), DE, m_ν, etc.
- Surveys like BOSS can make percent level measurements – would like to have theory to compare to!
- Fun problem!

RSD: What

- When making a 3D map of the Universe the 3rd dimension (radial distance) is usually obtained from a redshift using Hubble's law or its generalization.
  - Focus here on spectroscopic measurements.
  - If photometric redshift uses a break or line, then it will be similarly contaminated. If it uses magnitudes it won't be.
- Redshift measures a combination of "Hubble recession" and "peculiar velocity".

$$v_{\rm obs} = Hr + v_{\rm pec} \qquad \Rightarrow \qquad \chi_{\rm obs} = \chi_{\rm true} + \frac{v_{\rm pec}}{aH}$$

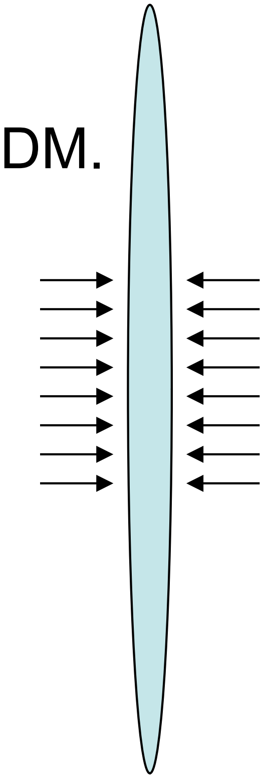
- Galaxies expected to be (almost) unbiased tracers of the cosmic velocity field (but not the density field).

Focus on galaxy surveys

- It is, of course, possible to observe RSD in things other than galaxy surveys.
- A topical example is Lyα forest.
  - Non-linearity of the map from density to observable makes life more interesting.
  - Existence of multiple regimes (line dominated vs. FGPA) increases the complexity of the problem.
  - Kaiser *form* works well, but determining β is “interesting”.
- I won’t discuss this here, but see:
  - Seljak (2012; JCAP, 3, 4).

Growth of structure

- A key test of dark energy vs. modified gravity models is the growth of structure.
  - Also helps break some DE degeneracies …
- For fixed expansion history/contents, GR makes a unique prediction for the growth of structure (and velocities).
  - Growth predicted to ~1% for a BOSS-like survey for ΛCDM.
- We can measure the growth of structure using redshift space distortions.
  - $z_{obs} = Hr + v_{pec}$.
  - $v_{pec} \sim a\, t \sim (\nabla\Psi)\, t \sim (\nabla\nabla^{-2}\rho)\, t$
  - Distortion correlated with density field.
- Constrain dD/dln(a)~$f\sigma_8$.

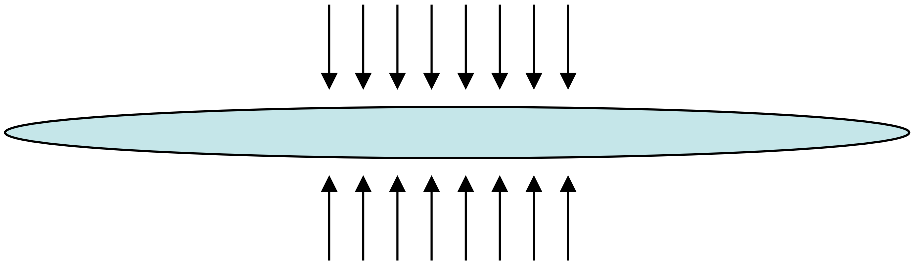


Two regimes

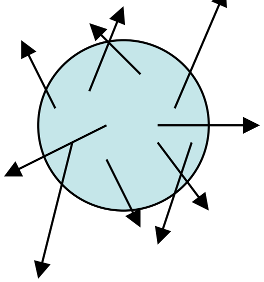
The distortions depend on non-linear density and velocity fields, which are correlated.

Velocities enhance power on large scales and suppress power on small scales.

The transition from enhancement to suppression occurs on “interesting” scales.



Coherent/supercluster infall



Random (thermal) motion

(fingers-of-god)

Interest rekindled

- There has been a lot of theoretical activity pointing out the promise of redshift space distortions recently.
- Rekindled interest in measuring RSD
    - 2dFGRS: Peacock++01, Hawkins++03, Percival++04
    - SDSS: Zehavi++05, Tegmark++06, Cabre++08, Okumura++08, Sanchez++09, …
    - VVDS: LeFevre++05, Garilli++08
    - 2SLAQ: daAngela++08
    - WiggleZ: Blake++11.
    - BOSS: Reid++12.

(See these papers for parameter/model constraints, degeneracy breaking, other references, etc. This talk will focus on the theory …)

RSD: What not

- Throughout I will be making the “distant” observer, and plane-parallel approximations.
- It is possible to drop this approximation and use spherical coordinates with *r* rather than Cartesian coordinates with *z*.
- References:
  - Fisher et al. (1994).
  - Heavens & Taylor (1995).
  - Papai & Szapudi (2008).
- Natural basis is tri-polar spherical harmonics.
- Correlation function depends on full triangle, not just on separation and angle to line-of-sight.

Spherical expansion

(Heavens & Taylor 1995)

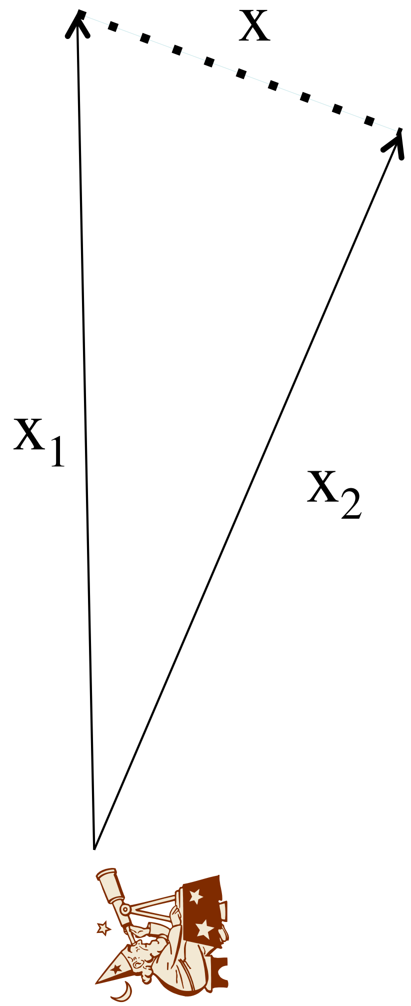
- For wide fields it makes sense to use spherical coordinates
  - note we've now "broken" translational symmetry.
- Do a "spherical" FT to get coefficients:

$$\rho_{\ell mn}(s) = c_{\ell n} \int d^3s \; \rho(\vec{s}) j_\ell(k_{\ell n} s) Y^*_{\ell m}(\theta, \phi)$$

- and expand the Bessel functions and use recurrence relations to get "mode coupling" relations between real- and redshift-space coefficients.

$$j_\ell\left(k_{\ell n} s\right) \simeq j_\ell\left(k_{\ell n} r\right) + v(r) \frac{d}{dr} j_\ell\left(k_{\ell n} r\right) + \cdots$$

Tripolar harmonics

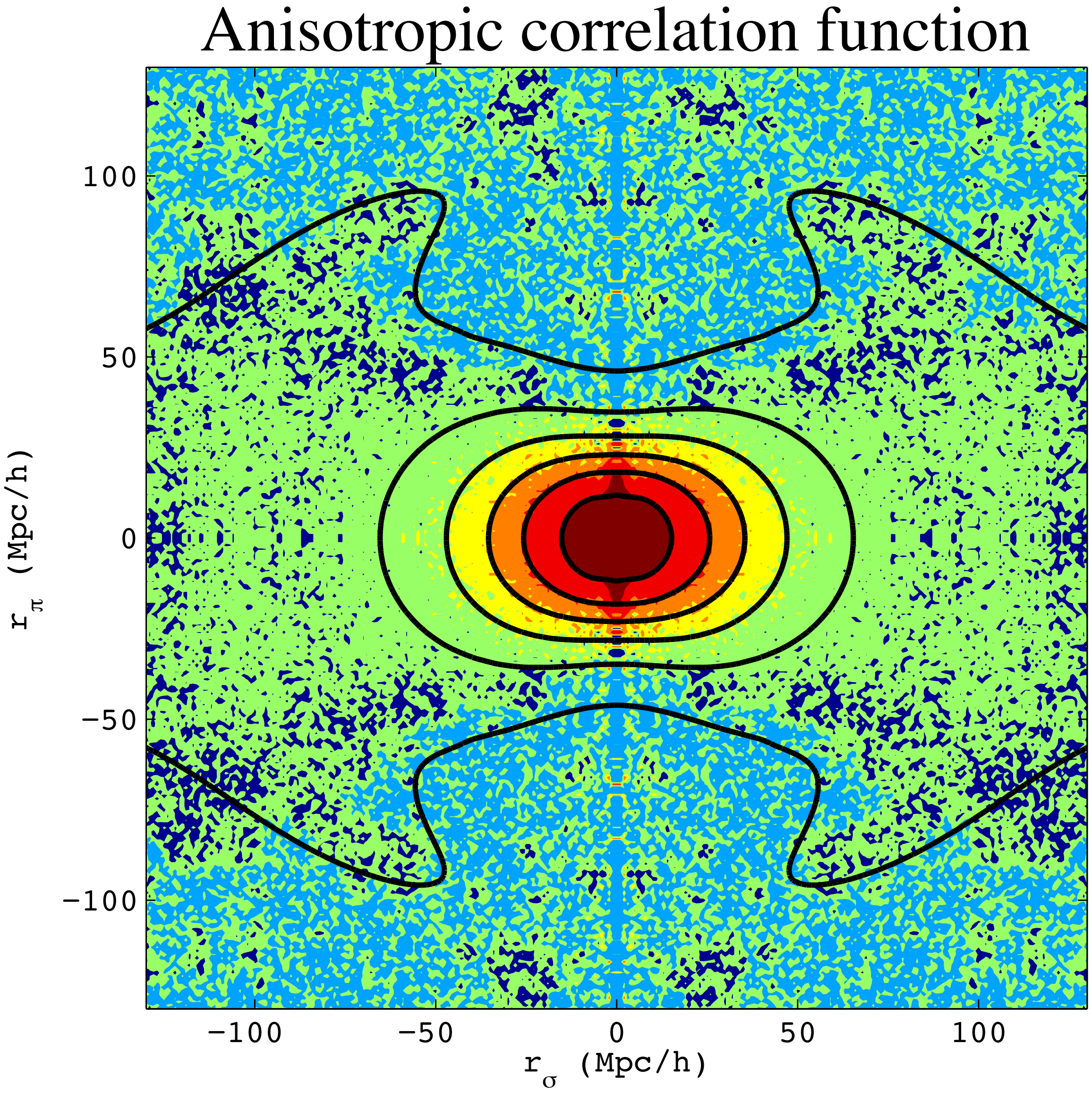


- Natural basis is tripolar harmonics:
- $\xi(x_1,x_2,x)=B^{abc}\,S_{abc}(x_1,x_2,x)$
- Expressions for B^{abc} can be derived in linear theory.
- Generally find corrections are quite small.

$$S_{\ell_1\ell_2\ell}(\widehat{x}_1,\widehat{x}_2,\widehat{x}) = \sum_{m_1,m_2,m} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & m \end{pmatrix} \cdots Y_{\ell_1 m_1}(\widehat{x}_1) Y_{\ell_2 m_2}(\widehat{x}_2) Y_{\ell m}(\widehat{x})$$

Back to small-angle …

Redshift space distortions



BOSS: Reid et al. (2012)

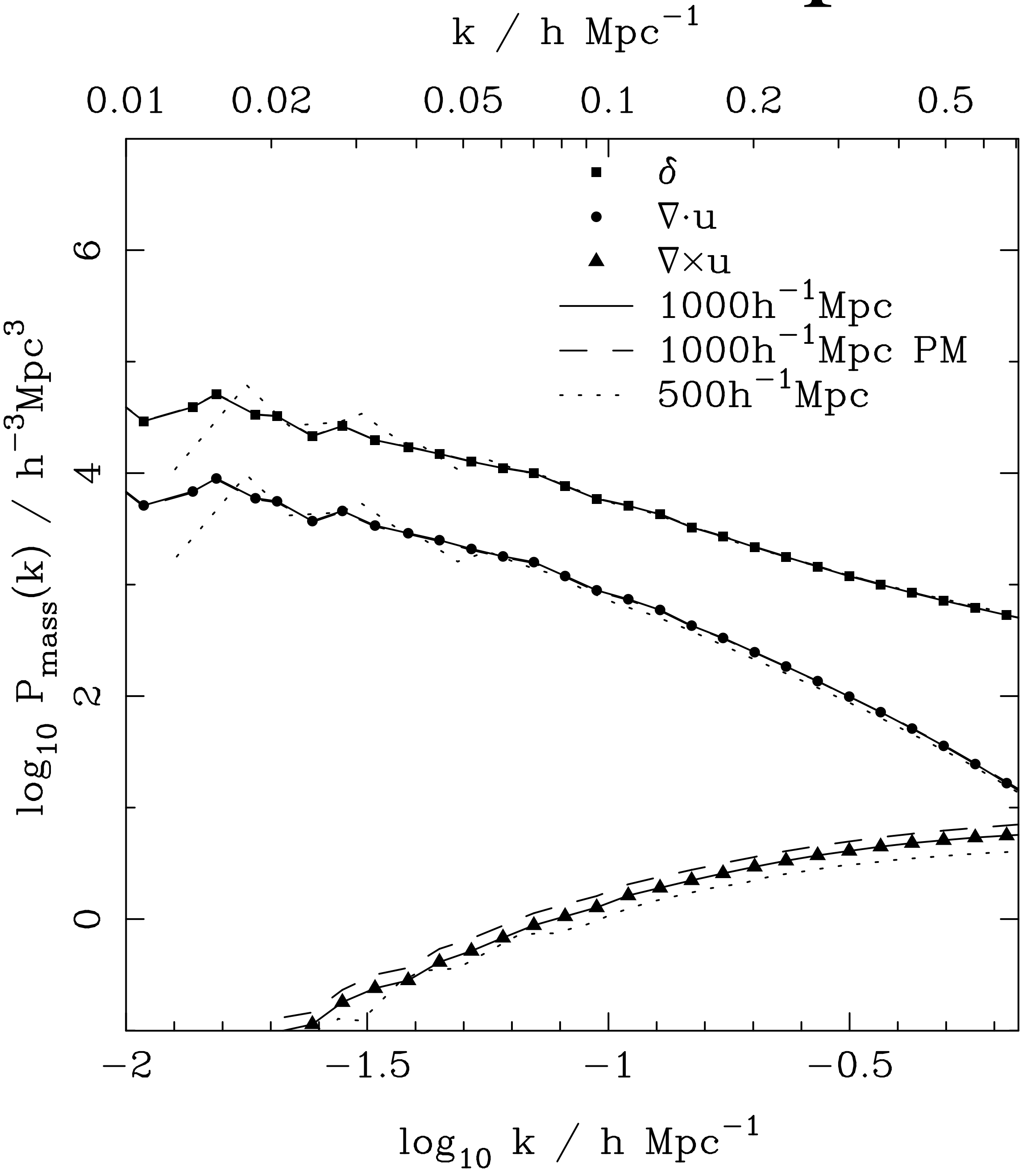
Line-of-sight selects out a special direction and breaks rotational symmetry of underlying correlations.

We observe anisotropic clustering.

Amount of anisotropy is related to rate of growth of structure.

Velocities are ≈ potential flow

Percival & White (2009)



Assume that *v* comes from a potential flow (self-consistent; curl[v] ~a^{-1} at linear order) then it is totally specified by its divergence, θ.

Continuity equation

- Relates velocities to growth of structure.
- Can be easily derived by stress-energy conservation, but physically:
  - Densities are enhanced by converging flows (and reduced by the stretching of space).
- To lowest order

$$\dot{\delta} = -a^{-1}\nabla \cdot v$$

$$\delta \, \frac{d\ln\delta}{d\ln a} \, H = -a^{-1}\nabla \cdot v$$

$$f\delta = \theta \equiv -\frac{\nabla \cdot v}{aH}$$

Kaiser formula

(Kaiser, 1987, MNRAS, 227, 1)

- Mass conservation $$(1+\delta^r)\,d^3r = (1+\delta^s)\,d^3s$$
- Jacobian $$\frac{d^3s}{d^3r} = \left(1+\frac{v}{z}\right)^2\left(1+\frac{dv}{dz}\right)$$
- Distant observer $$1+\delta^s = (1+\delta^r)\left(1+\frac{dv}{dz}\right)^{-1}$$
- Potential flow $$\frac{dv}{dz} = -\frac{d^2}{dz^2}\nabla^{-2}\theta$$
- Proportionality $$\delta^s(\mathbf{k}) = \delta^r(\mathbf{k}) + \mu_k^2\theta(\mathbf{k}) \simeq \left(1+f\mu_k^2\right)\delta^r(\mathbf{k})$$

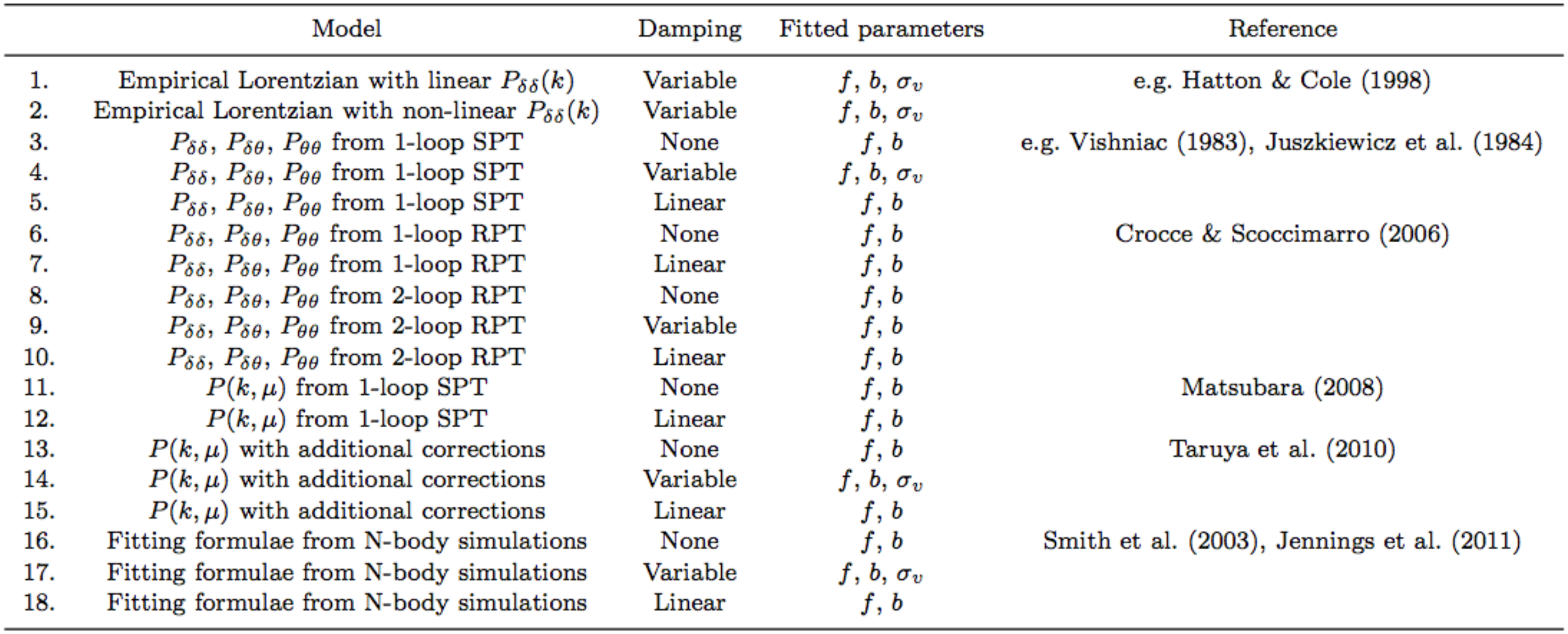
Power spectrum

- If we square the density perturbation we obtain the power spectrum:
  - $P^s(k,\mu)=[1+f\mu^2]^2\ P^r(k)$
- For biased tracers (e.g. galaxies/halos) we can assume $\delta_{obj}=b\delta_{mass}$ and $\theta_{obj}=\theta_{mass}$.
  - $P^s(k,\mu) = [b+f\mu^2]^2\ P^r(k) = b^2[1+\beta\mu^2]^2\ P^r(k)$
- Note: the two types of terms depend on $b\sigma_8$ and $f\sigma_8$, since $P\sim\sigma_8{}^2$.

Fingers-of-god

- So far we have neglected the motion of particles/galaxies inside “virialized” dark matter halos.
- These give rise to fingers-of-god which suppress power at high *k*.
- Peacock (1992) 1st modeled this as Gaussian “noise” so that
  - $P^s(k, \mu)= P^r(k)\ [b+f\mu^2]^2\ \mathrm{Exp}[-k^2\mu^2\sigma^2]$
- Sometimes see this written as $P_{\delta\delta}+P_{\delta\theta}+P_{\theta\theta}$ times Gaussians or Lorentzians.
  - Beware: no more general than linear theory!

Widely used



	Model	Damping	Fitted parameters	Reference
1.	Empirical Lorentzian with linear $P_{\delta\delta}(k)$	Variable	f, b, σ_v	e.g. Hatton & Cole (1998)
2.	Empirical Lorentzian with non-linear $P_{\delta\delta}(k)$	Variable	f, b, σ_v	
3.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 1-loop SPT	None	f, b	e.g. Vishniac (1983), Juszkiewicz et al. (1984)
4.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 1-loop SPT	Variable	f, b, σ_v	
5.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 1-loop SPT	Linear	f, b	
6.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 1-loop RPT	None	f, b	Crocce & Scoccimarro (2006)
7.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 1-loop RPT	Linear	f, b	
8.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 2-loop RPT	None	f, b	
9.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 2-loop RPT	Variable	f, b	
10.	$P_{\delta\delta}$, $P_{\delta\theta}$, $P_{\theta\theta}$ from 2-loop RPT	Linear	f, b	
11.	$P(k, \mu)$ from 1-loop SPT	None	f, b	Matsubara (2008)
12.	$P(k, \mu)$ from 1-loop SPT	Linear	f, b	
13.	$P(k, \mu)$ with additional corrections	None	f, b	Taruya et al. (2010)
14.	$P(k, \mu)$ with additional corrections	Variable	f, b, σ_v	
15.	$P(k, \mu)$ with additional corrections	Linear	f, b	
16.	Fitting formulae from N-body simulations	None	f, b	Smith et al. (2003), Jennings et al. (2011)
17.	Fitting formulae from N-body simulations	Variable	f, b, σ_v	
18.	Fitting formulae from N-body simulations	Linear	f, b	

(Blake et al. 2012; WiggleZ RSD fitting)

Measuring parameters

- Redshift space distortions can be well measured by redshift surveys and “new” surveys have impressive constraining power.

$$F_{ij} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial \ln P}{\partial p_i}\right)\left(\frac{\partial \ln P}{\partial p_j}\right)V_{\rm eff}(\mathbf{k})$$

$$\left(\frac{\partial \ln P}{\partial b}\right) = \frac{2}{b+f\mu^2} \quad , \quad \left(\frac{\partial \ln P}{\partial f}\right) = \frac{2\mu^2}{b+f\mu^2}$$

- The use of multiple tracers with different biases all tracing the same velocity field helps constrain parameters (but non-linearities cause issues).

White, Song & Percival (2009)

Legendre expansion

Rather than deal with a 2D function we frequently expand the angular dependence in a series of Legendre polynomials. The Rayleigh expansion of the plane-wave related the moments in *k*-space and *r*-space:

$$\begin{aligned}\Delta^2(k,\hat{k}\cdot\hat{z}) &\equiv \frac{k^3 P(k,\mu)}{2\pi^2} = \sum_\ell \Delta^2_\ell(k) L_\ell(\mu) \\ \xi(r,\hat{r}\cdot\hat{z}) &\equiv \sum_\ell \xi_\ell(r) L_\ell(\hat{r}\cdot\hat{z}) \quad , \quad \xi_\ell(r) = i^\ell \int \frac{dk}{k}\ \Delta^2_\ell(k) j_\ell(kr)\end{aligned}$$

If we use recurrence relations between j_l we can write ξ_l in terms of integrals of ξ times powers of *r*. e.g.

$$\int \frac{dk}{k}\Delta^2_2(k) j_2(kr) = \frac{3}{s^2}\int_0^s s^2 ds\ \xi(s) - \xi(s) = \bar{\xi}(<s) - \xi(s)$$

Legendre expansion

Note that the ratios of the moments is independent of *k* but not of *r.*

The Kaiser formula involved only terms up to μ^4, so on large scales ($k\sigma<<1$) this series truncates quite quickly.

$$\begin{pmatrix} \Delta_0^2(k) \\ \Delta_2^2(k) \\ \Delta_4^2(k) \end{pmatrix} = \Delta^2(k) \begin{pmatrix} b^2 + \frac{2}{3}bf + \frac{1}{5}f^2 \\ \frac{4}{3}bf + \frac{4}{7}f^2 \\ \frac{8}{35}f^2 \end{pmatrix}$$

Typically only measure (well) ***l***=0, 2.

Quadrupole-to-Monopole ratio

For many years it was believed that the ratio of the quadrupole to monopole components of the power spectrum could be used to measure β and (if *b* was known) Ω_m.

In the power spectrum the ratio is *k*-independent within the Kaiser approximation:

$$\frac{Q}{M} = \frac{(4/3)\beta + (4/7)\beta^2}{1 + (2/3)\beta + (1/5)\beta^2}$$

Ultimately this proved less fruitful than hoped.

See Berlind, Narayanan & Weinberg (2001; ApJ 549, 688) for a summary of past work and a discussion of issues with this approach ...

Projected statistics

- Naively projecting along the redshift direction would erase "redshift" space distortions.
  - But if the projection is not infinite some remain.
- Theory of projected statistics is nice.
  - Fisher et al. (1994).

$$1+\delta_2(\widehat{n}) = \int d\chi\ \phi(s)\left[1+\delta_3(\chi\widehat{n})\right]$$

  - Expand ϕ for u<<width of ϕ.
  - Use Rayliegh expansion:

$$C_\ell = 4\pi \int \frac{dk}{k}\Delta^2(k)W_\ell(k)$$

$$\begin{aligned} W_\ell(k) &= \int d\chi\,\phi(\chi)j_\ell(k\chi) + \beta\int d\chi\,\phi(\chi)\left[\frac{2\ell^2+2\ell-1}{(2\ell+3)(2\ell-1)}j_\ell \right. \\ &- \left. \frac{\ell(\ell-1)}{(2\ell+1)(2\ell-1)}j_{\ell-2} - \frac{(\ell+2)(\ell+1)}{(2\ell+1)(2\ell+3)}j_{\ell+2}\right] \end{aligned}$$

β terms small when ϕ wide.

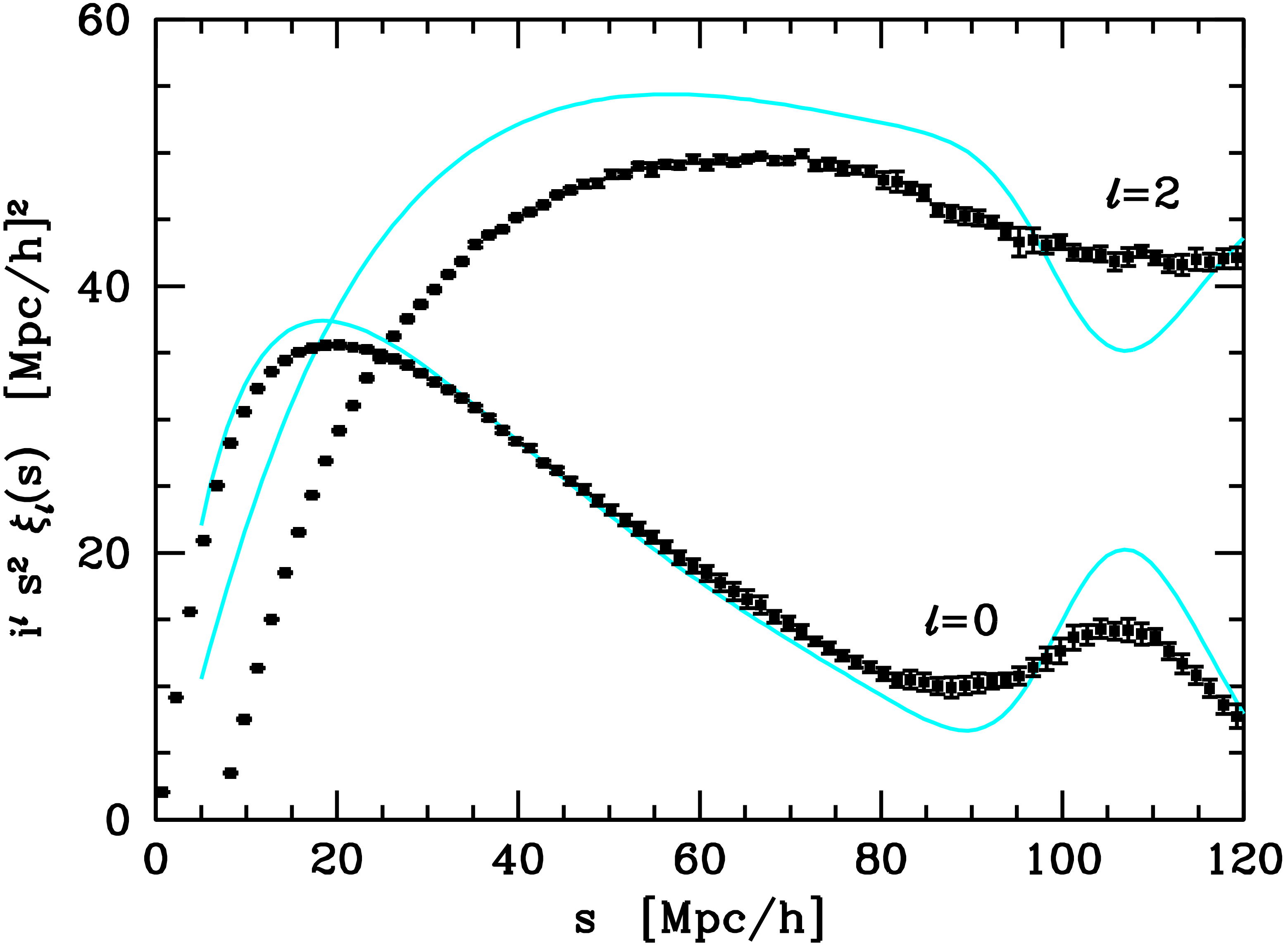
RSD from photometric surveys

- You need to be careful to include those β terms if you are working on scales "comparable" to the scale of variation of ϕ.
- When l>>1 the terms are small … but not zero.

- There has even been work suggesting one can measure RSD from upcoming photometric surveys.
  - Ross, Percival, Crocce, Cabre, Gaztanaga (2011; MNRAS, 415, 2193).

So are we done?

Kaiser is not particularly accurate



Beyond Kaiser?

- It is certainly feasible to simulate RSD
  - Study the impact of different halo populating schemes, centrals at rest vs. in motion, velocity bias, etc.
  - Naturally deals with higher-order effects and mixing of small and large scales by fingers-of-god etc.
  - Good for numbers, less so for understanding!
- Can we improve our understanding of the issues via analytic techniques?

In configuration space

- There are valuable insights to be gained by working in configuration, rather than Fourier, space.
- We begin to see why this is a hard problem …

$$1+\xi^s(R,Z)=\left\langle \int dy\ (1+\delta_1)(1+\delta_2)\delta^{(D)}(Z-y-v_{12}) \right\rangle$$

$$1+\xi^s(R,Z)=\left\langle \int dy\ (1+\delta_1)(1+\delta_2)\int \frac{d\kappa}{2\pi}e^{i\kappa(Z-y-v_{12})} \right\rangle$$

- Note all powers of the velocity field enter.

Gaussian limit

(Fisher, 1995, ApJ 448, 494)

- If δ and v are Gaussian can do all of the expectation values.

$$1+\xi^s(R,Z)=\int \frac{dy}{\sqrt{2\pi\sigma_{12}^2(y)}}\exp\left[-\frac{(Z-y)^2}{2\sigma_{12}^2(y)}\right]\times$$

$$\left[1+\xi^r(r)+\frac{y}{r}\frac{(Z-y)v_{12}(r)}{\sigma_{12}^2(y)}-\frac{1}{4}\frac{y^2}{r^2}\frac{v_{12}^2(r)}{\sigma_{12}^2(y)}\left(1-\frac{(Z-y)^2}{\sigma_{12}^2(y)}\right)\right]$$

Expanding around y=Z:

$$\xi^s(R,Z)=\xi^r(s)-\frac{d}{dy}\left[v_{12}(r)\frac{y}{r}\right]\Bigg|_{y=Z}+\frac{1}{2}\frac{d^2}{dy^2}\left[\sigma_{12}^2(y)\right]\Bigg|_{y=Z}$$

Linear theory: configuration space

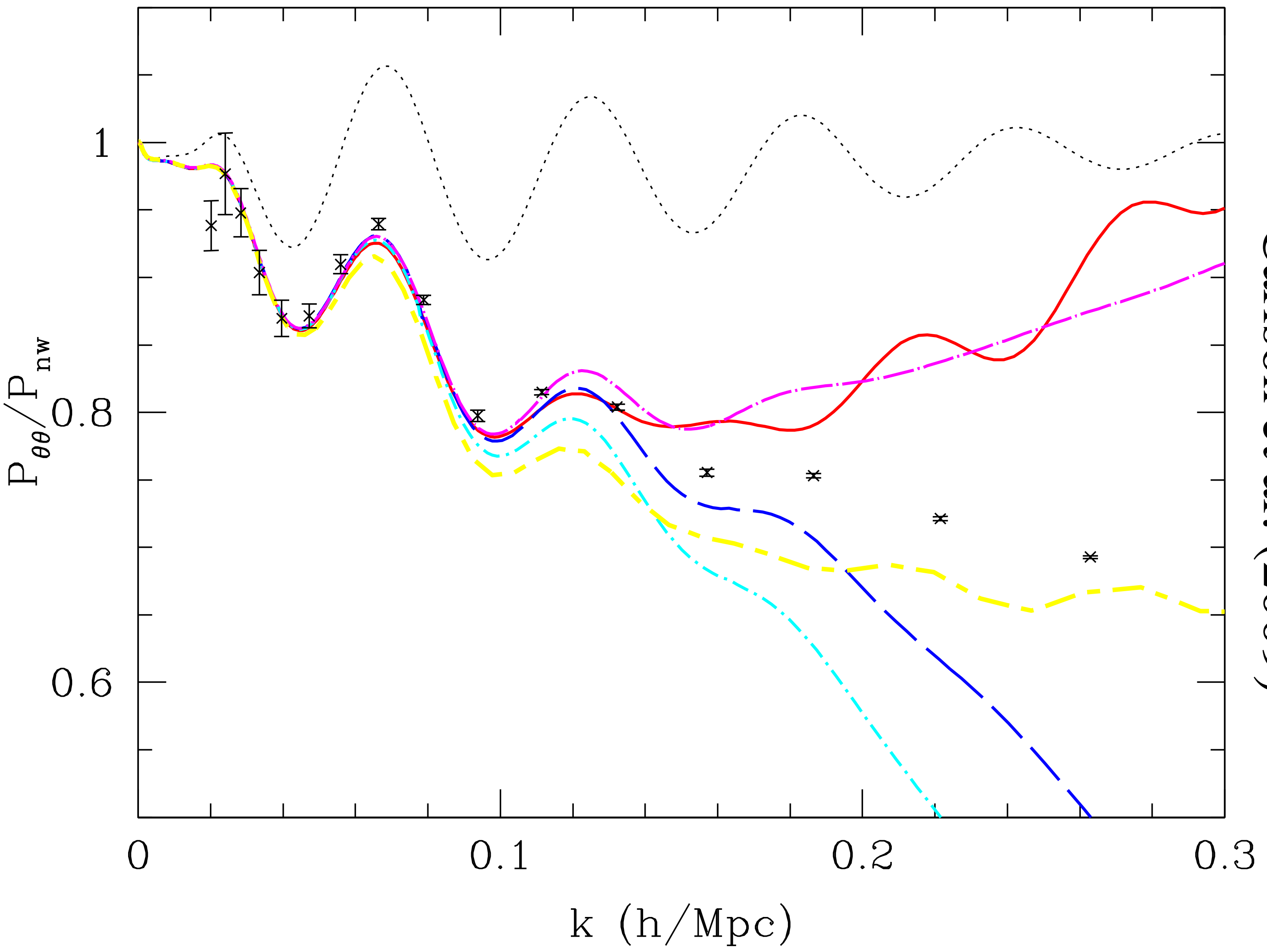
(Fisher, 1995, ApJ 448, 494)

- One can show that this expansion agrees with the Kaiser formula.
- Two important points come out of this way of looking at the problem:
  - Correlation between δ and v leads to v_{12}.
  - LOS velocity dispersion is scale- and orientation-dependent.
- By Taylor expanding about r=s we see that ξ^s depends on the 1st and 2nd derivative of velocity statistics.

Two forms of non-linearity

- Part of the difficulty is that we are dealing with two forms of non-linearity.
  - The velocity field is non-linear.
  - The mapping from real- to redshift-space is non-linear.
- These two forms of non-linearity interact, and can partially cancel.
- They also depend on parameters differently.
- This can lead to a lot of confusion …

Velocity field is nonlinear



Carlson et al. (2009)

Non-linear mapping?

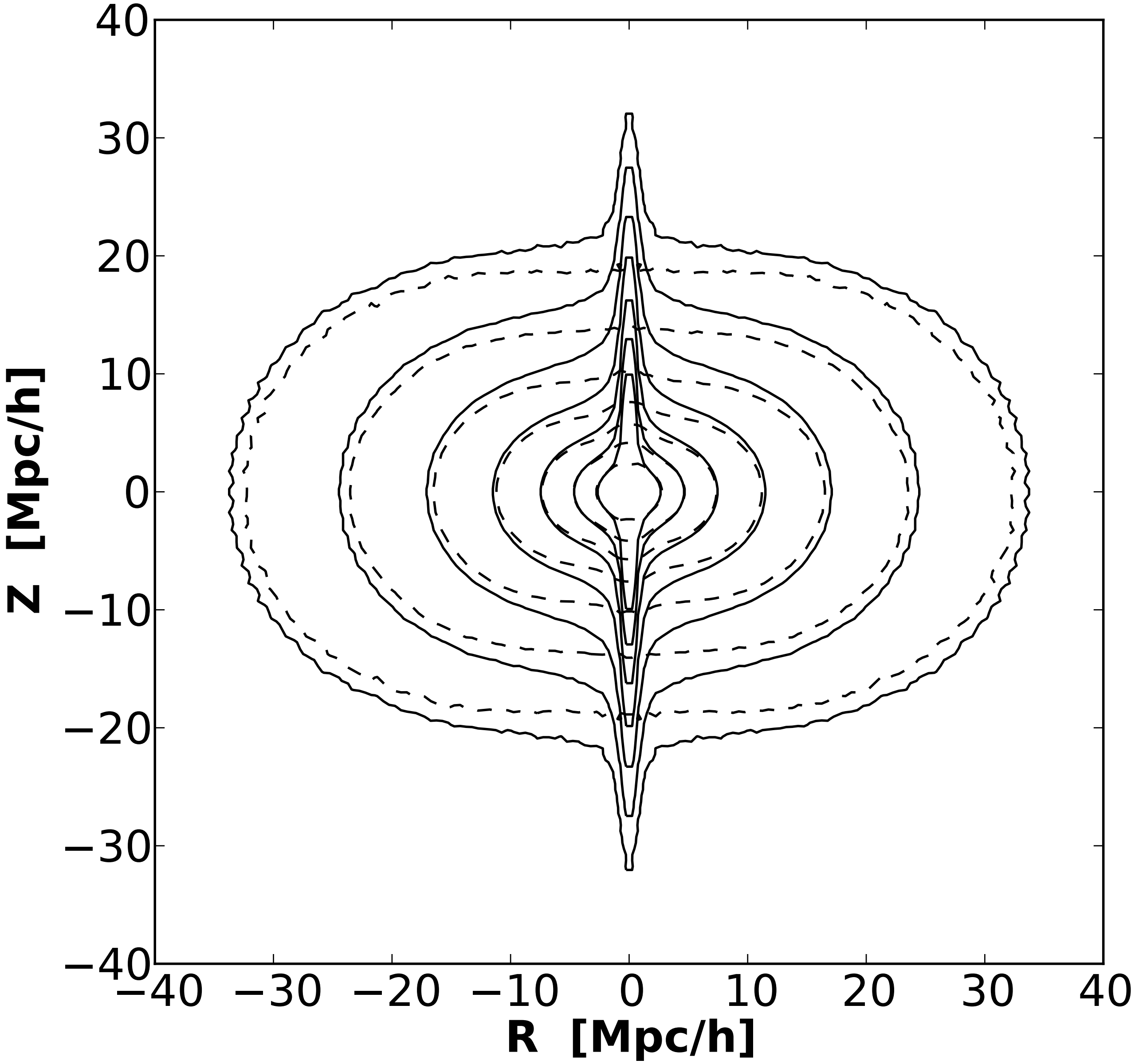
?

Want a fully non-linear “toy model”, like spherical top-hat collapse, to gain some intuition …

A model for the redshift-space clustering of halos

- We would like to develop a model capable of describing the redshift space clustering of halos over the widest range of scales.
- This will form the 1st step in a model for galaxies, but it also interesting in its own right.
- The model should try to treat the “non-linear mapping” part of the problem non-perturbatively.
- We will start with a toy model and then add realism …

The correlation function of halos



The correlation function of halo centers doesn’t have strong fingers of god, but still has “squashing” at large scales.

Note RSD is degenerate with A-P.

Halo model

- There are multiple insights into RSD which can be obtained by thinking of the problem in a halo model language.
- This has been developed in a number of papers
  - White (2001), Seljak (2001), Berlind et al. (2001), Tinker, Weinberg & Zheng (2006), Tinker (2007).
- This will take us too far afield for now …

Scale-dependent Gaussian streaming model

Let's go back to the exact result for a Gaussian field, a la Fisher:

$$1+\xi^s(R,Z) = \int \frac{dy}{\sqrt{2\pi\sigma_{12}^2(y)}} \exp\left[-\frac{(Z-y)^2}{2\sigma_{12}^2(y)}\right] \times$$
$$\left[1+\xi^r(r)+\frac{y}{r}\frac{(Z-y)v_{12}(r)}{\sigma_{12}^2(y)} - \frac{1}{4}\frac{y^2}{r^2}\frac{v_{12}^2(r)}{\sigma_{12}^2(y)}\left(1-\frac{(Z-y)^2}{\sigma_{12}^2(y)}\right)\right]$$

Looks convolution-like, but with a scale-dependent v_{12} and σ. Also, want to resum v_{12} into the exponential …

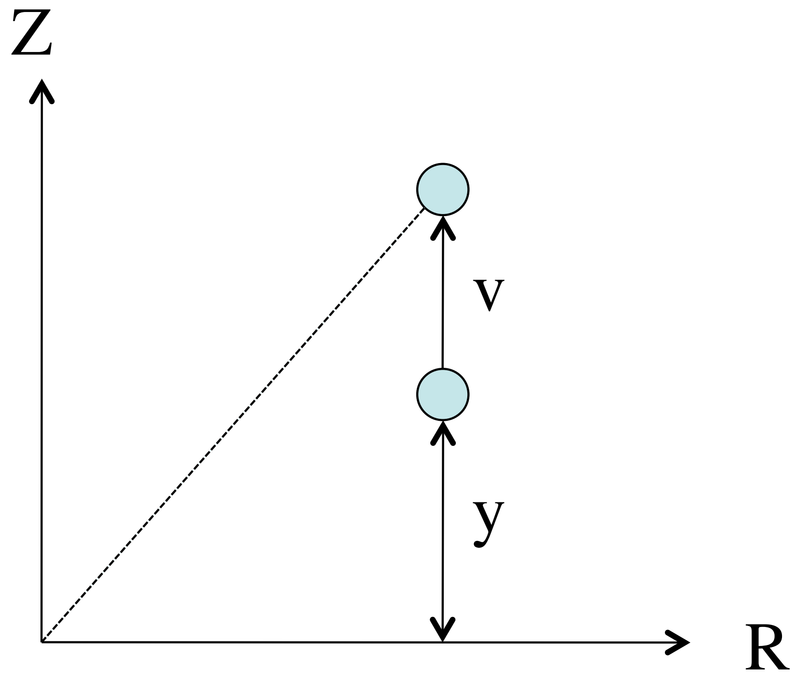
Scale-dependent Gaussian streaming model/ansatz

$$1 + \xi(R,Z) = \int dy\ \ [1+\xi(r)]\,\mathcal{P}\left(v = Z - y, \mathbf{r}\right)$$

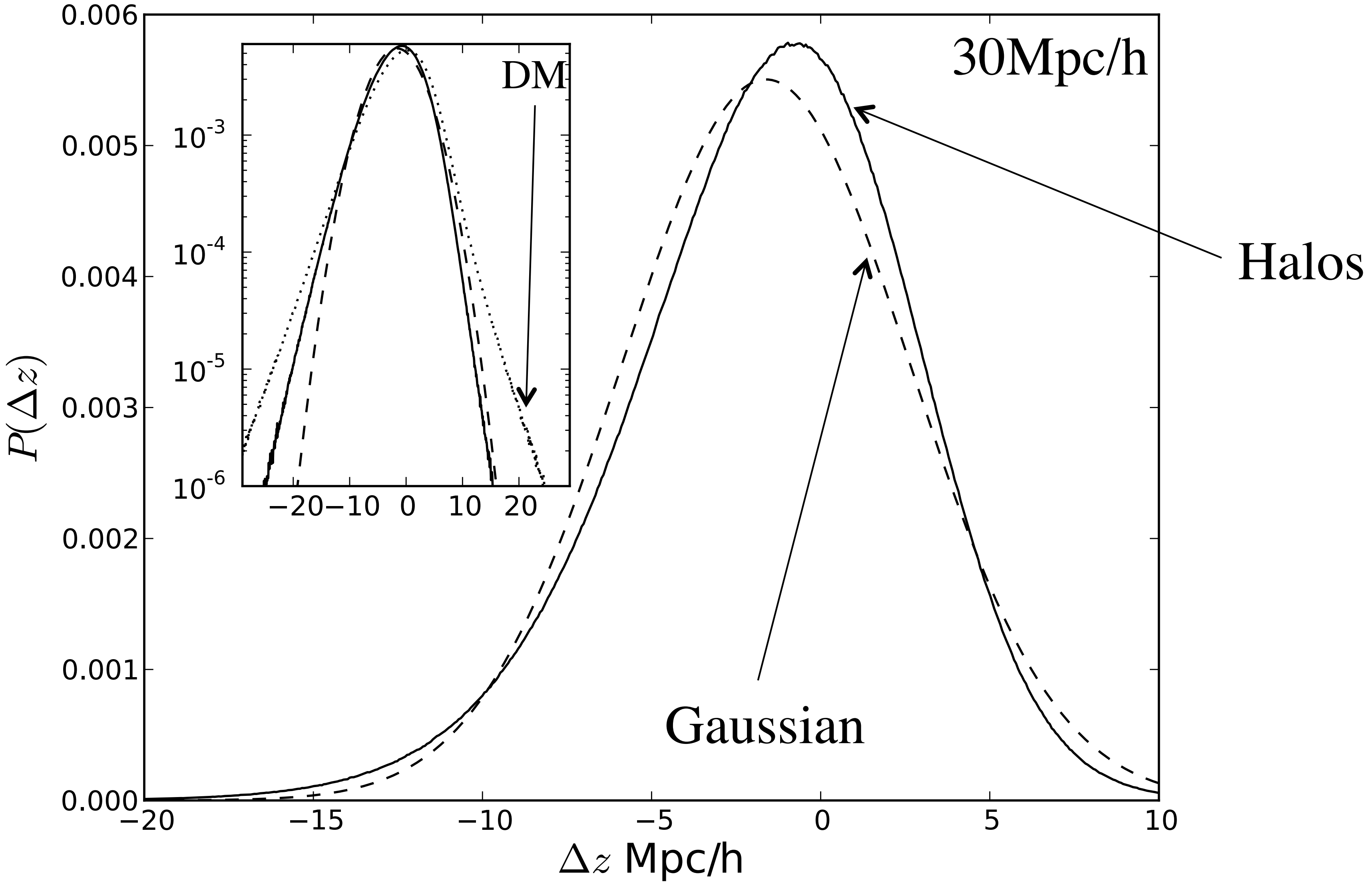
Note: *not* a convolution because of (important!) ***r*** dependence or kernel.

Non-perturbative mapping.

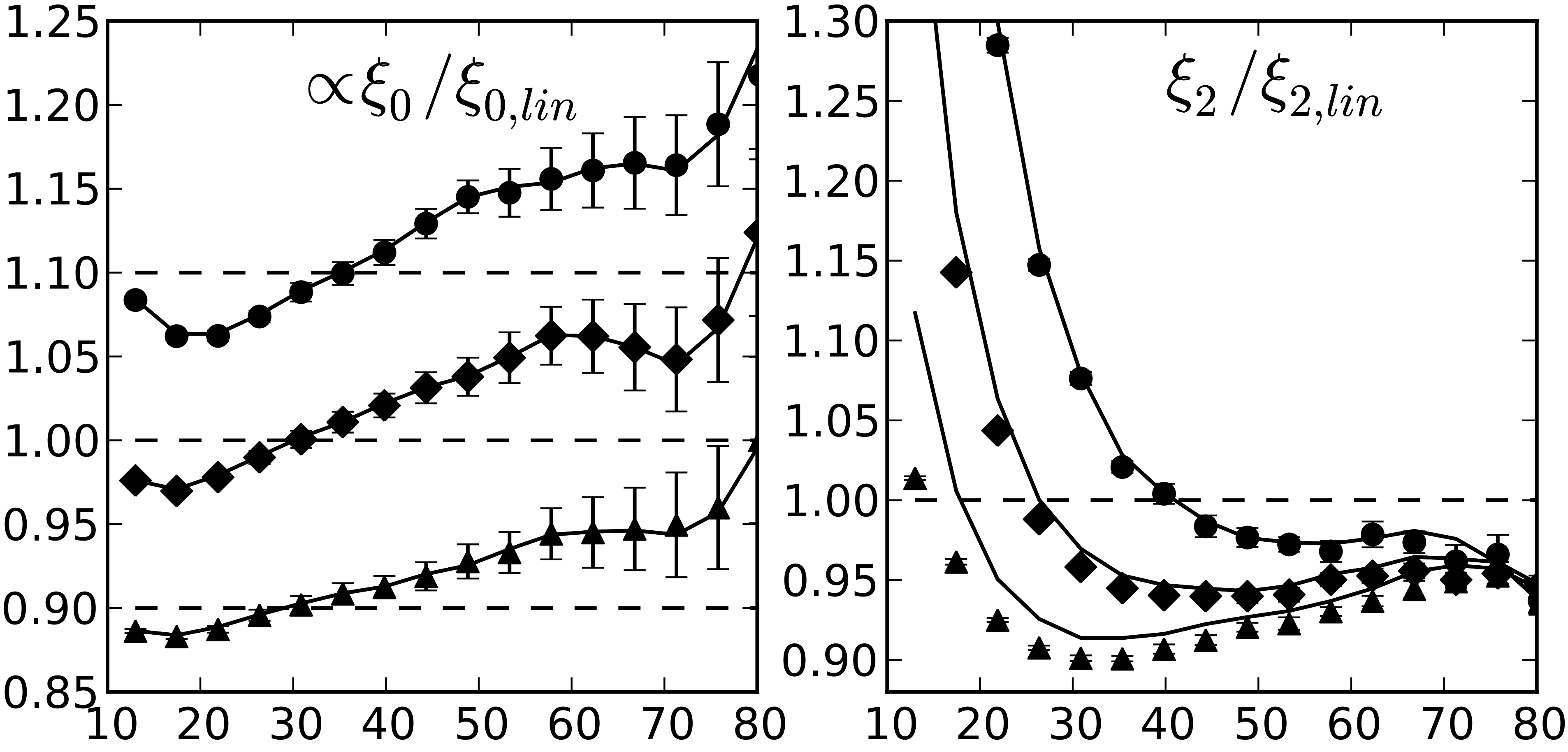
If lowest moments of *P* set by linear theory, agrees at linear order with Kaiser.
Approximate *P* as Gaussian …



Gaussian ansatz

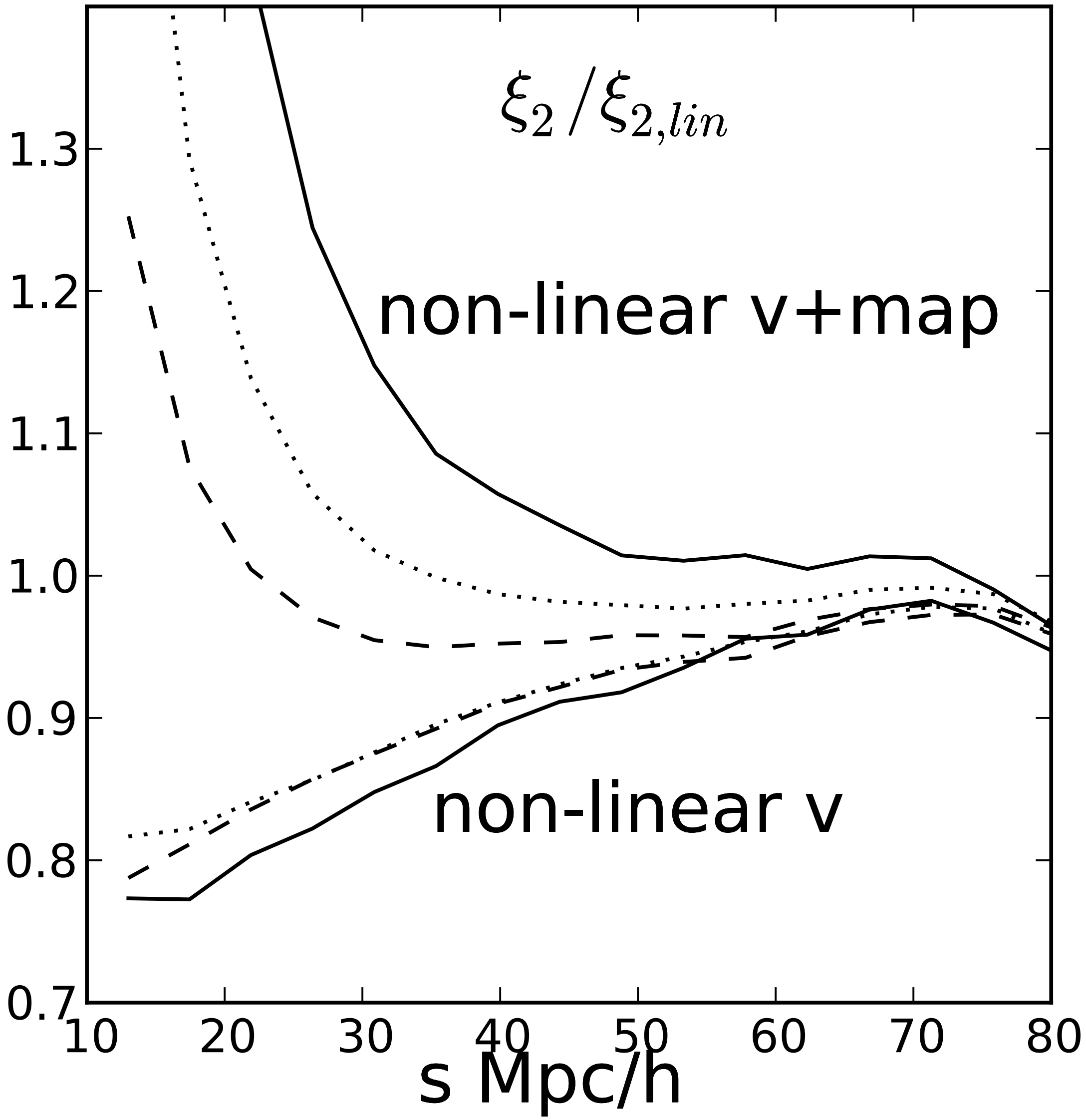


Testing the ansatz



Reid & White (2011)

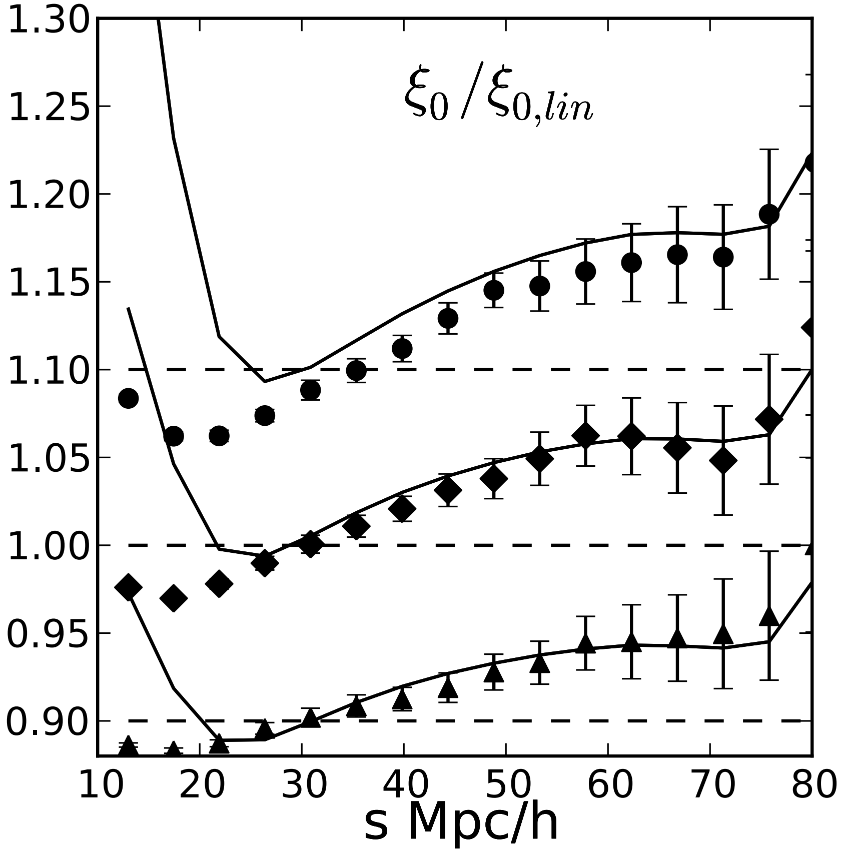
The mapping

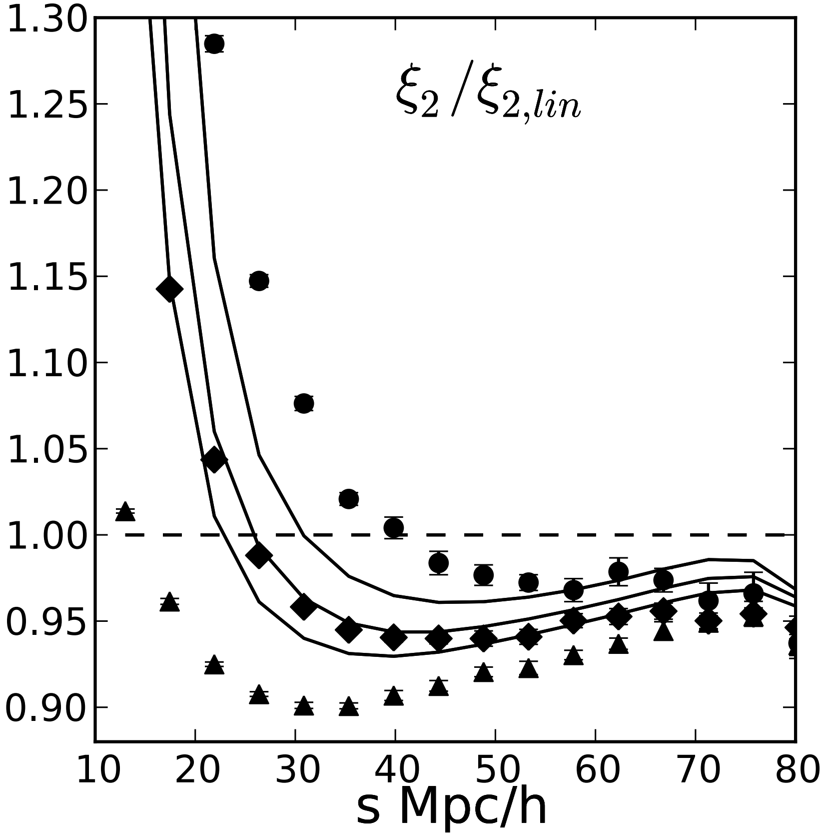


Note, the behavior of the quadrupole is particularly affected by the non-linear mapping. The effect of non-linear velocities is to suppress ξ_2 (by ~10%, significant!). The mapping causes the enhancement. This effect is tracer/ bias dependent!

An analytic model

This has all relied on input from N-body. Can we do an analytic model? Try “standard” perturbation theory* for the v_{12} and σ terms …

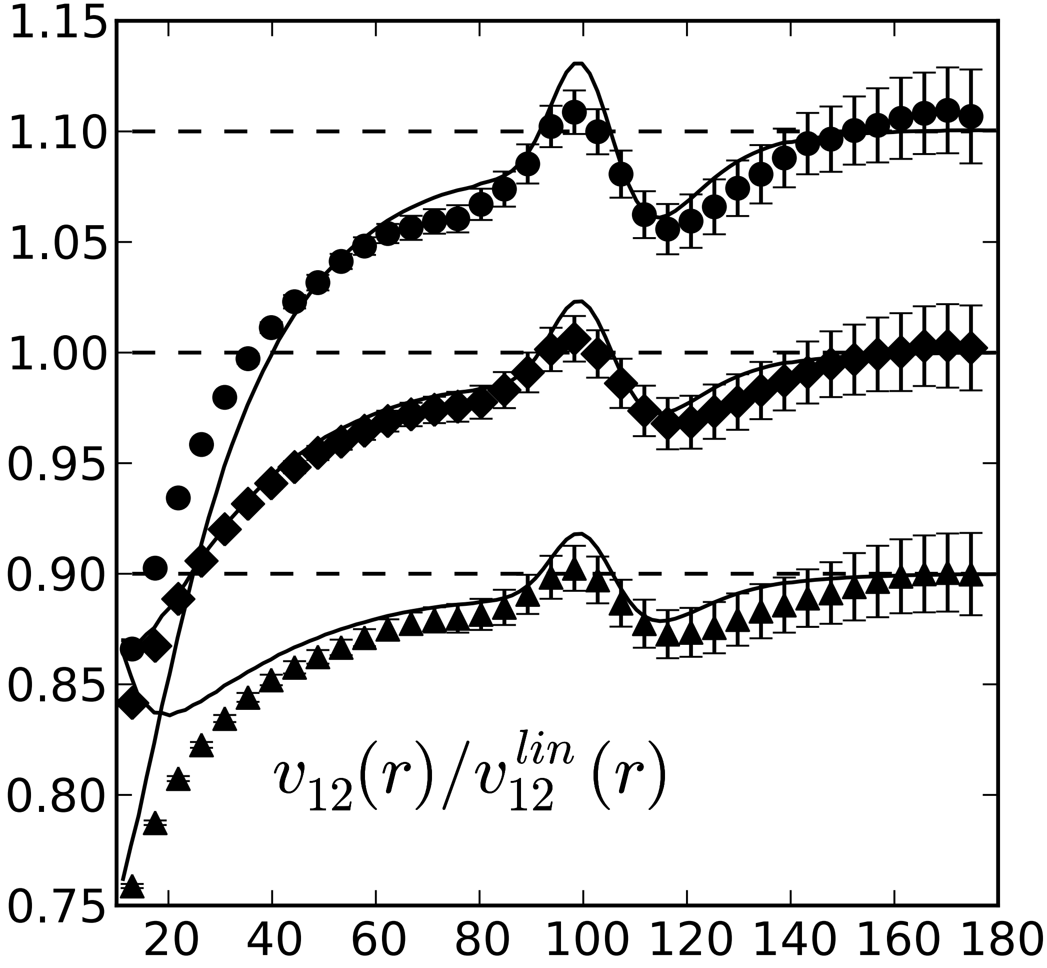


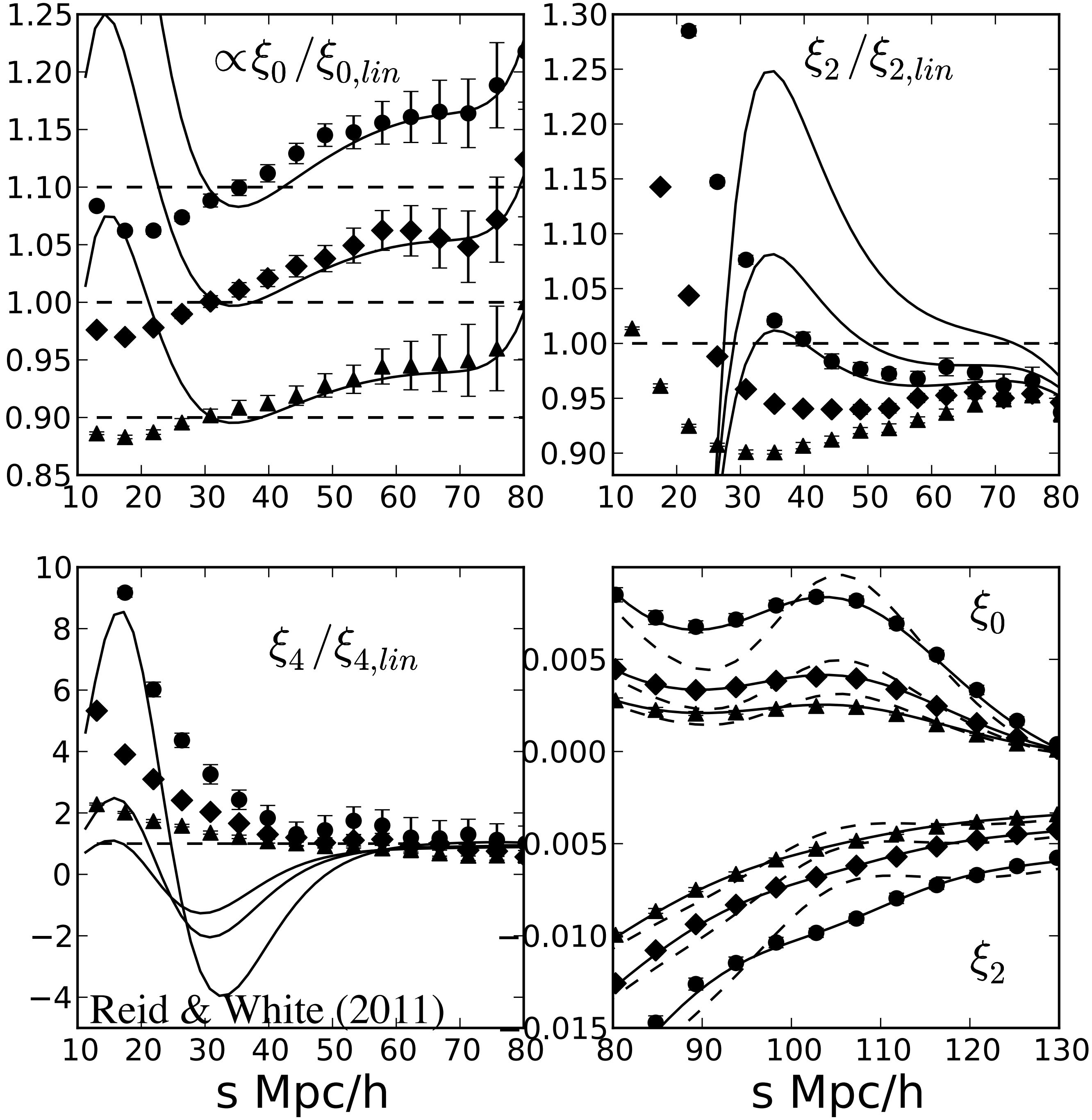


Reid & White 11

Many new SPT results

- Results for pair-weighted v_{12} and σ, including bispectrum terms are new.
- Assume *linear* bias.
- Error in model is dominated by error in slope of v_{12} at small *r*.





Perturbation theory can do a reasonable job on large scales, but breaks down surprisingly quickly.

Gaussian streaming model is better … but still suffers from problems on small scales.

Euleriam, Lagrangian & "Distribution functions" expansions

- There are several forms of PT for redshift-space, and numerous schemes to "resum" the expansions.
- "Standard" Eulerian PT expands δ and θ:

$$\delta_s(k) = \sum_{n=1}^{\infty} \int \Pi_i d^3k_i \delta^{(D)}(k - \sum_j k_j) Z_n(\{k\}) \delta_1(k_1) \cdots \delta_1(k_n)$$

- Lagrangian PT expands x=q+Ψ. and Ψ_s=RΨ.
- The distribution function approach has been championed recently by Seljak & McDonald (and collaborators).

$$\rho_s(k) = \int d^3\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} \int d^3\mathbf{q}\ f(\mathbf{x},\mathbf{q}) e^{ik_{||}u_{||}}$$

The “b^3” term?

- One of the more interesting things to come out of this ansatz is the existence of a “b^3” term.
  - Numerically quite important when b~2.
  - Another reason why mass results can be very misleading.
  - But hard to understand (naively) from

$$1+\xi^s(R,Z)=\left\langle \int dy\ (1+\delta_1)(1+\delta_2)\int \frac{d\kappa}{2\pi}e^{i\kappa(Z-y-v_{12})}\right\rangle$$

  - Where does it come from?

Lagrangian perturbation theory

- As I have mentioned there are a variety of approaches to PT and RSD via PT.
  - This is not the place for a review of PT methods.
- A different approach to PT, which has been radically extended recently by Matsubara (and is *very* useful for BAO):
  - Buchert89, Moutarde++91, Bouchet++92, Catelan95, Hivon++95.
  - Matsubara (2008a; PRD, 77, 063530)
  - Matsubara (2008b; PRD, 78, 083519)
- Relates the current (Eulerian) position of a mass element, **x**, to its initial (Lagrangian) position, **q**, through a displacement vector field, Ψ.

Lagrangian perturbation theory

$$\begin{aligned}
\delta(\mathbf{x}) &= \int d^3q \; \delta_D(\mathbf{x}-\mathbf{q}-\mathbf{\Psi}) - 1 \\
\delta(\mathbf{k}) &= \int d^3q \; e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1 \right) \; .
\end{aligned}$$

$$\frac{d^2\mathbf{\Psi}}{dt^2} + 2H\frac{d\mathbf{\Psi}}{dt} = -\nabla_x \phi\left[\mathbf{q}+\mathbf{\Psi}(\mathbf{q})\right]$$

$$\begin{aligned}
\mathbf{\Psi}^{(n)}(\mathbf{k}) &= \frac{i}{n!}\int \prod_{i=1}^{n} \left[\frac{d^3k_i}{(2\pi)^3}\right] \; (2\pi)^3 \delta_D\left(\sum_i \mathbf{k}_i - \mathbf{k}\right) \\
&\times \quad \mathbf{L}^{(n)}(\mathbf{k}_1, \cdots, \mathbf{k}_n, \mathbf{k}) \delta_0(\mathbf{k}_1) \cdots \delta_0(\mathbf{k}_n)
\end{aligned}$$

Kernels

$$\mathbf{L}^{(1)}(\mathbf{p}_1) = \frac{\mathbf{k}}{k^2} \tag{1}$$

$$\mathbf{L}^{(2)}(\mathbf{p}_1, \mathbf{p}_2) = \frac{3}{7}\frac{\mathbf{k}}{k^2}\left[1 - \left(\frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{p_1 p_2}\right)^2\right] \tag{2}$$

$$\mathbf{L}^{(3)}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \cdots \tag{3}$$

$$\mathbf{k} \equiv \mathbf{p}_1 + \cdots + \mathbf{p}_n$$

Standard LPT

- If we expand the exponential and keep terms consistently in δ_0 we regain a series $\delta=\delta^{(1)}+\delta^{(2)}+\ldots$ where $\delta^{(1)}$ is linear theory and e.g.

$$\begin{aligned}\delta^{(2)}(\mathbf{k}) &= \frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_0(\mathbf{k}_1)\delta_0(\mathbf{k}_2) \\ &\times \left[\mathbf{k}\cdot\mathbf{L}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})+\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2)\right]\end{aligned}$$

- which regains “SPT”.
  - The quantity in square brackets is F_2.

$$F_2(\mathbf{k}_1,\mathbf{k}_2)=\frac{5}{7}+\frac{2}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2}+\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)}{2}\left(k_1^{-2}+k_2^{-2}\right)$$

LPT power spectrum

- Alternatively we can use the expression for $\delta_{\mathbf{k}}$ to write

$$P(k) = \int d^3q \ e^{-i\vec{k}\cdot\vec{q}} \left(\left\langle e^{-i\vec{k}\cdot\Delta\vec{\Psi}} \right\rangle - 1 \right)$$

- where $\Delta\Psi=\Psi(\mathbf{q})-\Psi(0)$. [Note translational invariance.]
- Expanding the exponential and plugging in for $\Psi^{(n)}$ gives the usual results.

- **BUT** Matsubara suggested a different and very clever approach.

Cumulants

- The cumulant expansion theorem allows us to write the expectation value of the exponential in terms of the exponential of expectation values.
- Expand the terms $(\mathbf{k}\Delta\boldsymbol{\Psi})^N$ using the binomial theorem.
- There are two types of terms:
  - Those depending on Ψ at same point.
    - This is independent of position and can be factored out of the integral.
  - Those depending on Ψ at different points.
    - These can be expanded as in the usual treatment.

Example

- Imagine Ψ is Gaussian with mean zero.
- For such a Gaussian: $\langle e^{\Psi}\rangle = \exp[\sigma^2/2]$.

$$P(k) = \int d^3q e^{-i\mathbf{k}\cdot\mathbf{q}} \left(\left\langle e^{-ik_i \Delta\Psi_i(\mathbf{q})} \right\rangle - 1 \right)$$

$$\left\langle e^{-i\mathbf{k}\cdot\Delta\boldsymbol{\Psi}(q)} \right\rangle = \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta\Psi_i(\mathbf{q})\Delta\Psi_j(\mathbf{q}) \right\rangle \right]$$

$$k_i k_j \left\langle \Delta\Psi_i(\mathbf{q})\Delta\Psi_j(\mathbf{q}) \right\rangle = 2k_i^2 \langle \Psi_i^2(\mathbf{0}) \rangle - 2k_i k_j \xi_{ij}(\mathbf{q})$$

Keep exponentiated,
call Σ^2.

Expand

Resummed LPT

- The first corrections to the power spectrum are then:

$$P(k) = e^{-(k\Sigma)^2/2}\left[P_L(k) + P^{(2,2)}(k) + \widetilde{P}^{(1,3)}(k)\right],$$

- where $P^{(2,2)}$ is as in SPT but part of $P^{(1,3)}$ has been "resummed" into the exponential prefactor.
- The exponential prefactor is identical to that obtained from
  - The peak-background split (Eisenstein++07)
  - Renormalized Perturbation Theory (Crocce++08).
- Does a great job of explaining the broadening and shifting of the BAO feature in ξ(r) and also what happens with reconstruction.
- But breaks down on smaller scales …

Aside

- The exponential suppression goes all the way back to the Zel'dovich approximation and the dinosaurs.
- Physically it expresses the fact that matter "streams" a distance Σ and this "smears" any correlations.
  - Diffusion-like idea.
- This also explains why the density field today decorrelates (on small scales) with the initial field.
  - Objects move, so that the density today at $\boldsymbol{x}$ is well correlated with the initial field at $\boldsymbol{q}$, not at $\boldsymbol{x}$.
  - See e.g. Tassev & Zaldarriaga for further discussion.

Beyond real-space mass

- One of the more impressive features of Matsubara's approach is that it can gracefully handle both biased tracers and redshift space distortions.
- In redshift space, in the plane-parallel limit,

$$\mathbf{\Psi} \to \mathbf{\Psi} + \frac{\widehat{\mathbf{z}} \cdot \dot{\mathbf{\Psi}}}{H} \, \widehat{z} = R \, \mathbf{\Psi}$$

- In PT $\mathbf{\Psi}^{(n)} \propto D^n \Rightarrow R_{ij}^{(n)} = \delta_{ij} + nf \, \widehat{z}_i \widehat{z}_j$

- Again we're going to leave the zero-lag piece exponentiated so that the prefactor contains

$$k_i k_j R_{ia} R_{jb} \delta_{ab} = (k_a + fk\mu\widehat{z}_a)\,(k_a + fk\mu\widehat{z}_a) = k^2 \left[1 + f(f+2)\mu^2\right]$$

- while the ξ(r) piece, when FTed, becomes the usual Kaiser expression plus higher order terms.

Beyond real-space mass

- One of the more impressive features of Matsubara's approach is that it can gracefully handle both biased tracers and redshift space distortions.

- For bias local in Lagrangian space:

$$\delta_{\rm obj}(\mathbf{x}) = \int d^3q\ F\left[\delta_L(\mathbf{q})\right]\delta_D(\mathbf{x}-\mathbf{q}-\mathbf{\Psi})$$

- we obtain

$$P(k) \quad = \quad \int d^3q\ e^{-i\mathbf{k}\cdot\mathbf{q}}\left[\int \frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi}\ F(\lambda_1)F(\lambda_2)\left\langle e^{i[\lambda_1\delta_L(\mathbf{q}_1)+\lambda_2\delta_L(\mathbf{q}_2)]+i\mathbf{k}\cdot\Delta\mathbf{\Psi}}\right\rangle - 1\right]$$

- which can be massaged with the same tricks as we used for the mass.
- If we assume halos/galaxies form at peaks* of the initial density field ("peaks bias") then explicit expressions for the integrals of F exist.

*…and assume the peak-background split.

Peaks bias

- Expanding the exponential pulls down powers of λ.
- FT of terms like $\lambda^n F(\lambda)$ give $F^{(n)}$
- The averages of F' and F'' over the density distribution take the place of "bias" terms
  - b_1 and b_2 in standard perturbation theory*.
- If we assume halos form at the peaks of the initial density field and use the peak-background split we can obtain:

$$b_1 = \frac{\nu^2 - 1}{\delta_c} \quad , \quad b_2 = \frac{\nu^4 - 3\nu^2}{\delta_c^2} \approx b_1^2$$

*but "renormalized".

Example: Zel'dovich

- To reduce long expressions, let's consider the lowest order expression
  - Zel'dovich approximation.

$$\mathbf{\Psi}(\mathbf{q}) = \mathbf{\Psi}^{(1)}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3}\ e^{i\mathbf{k}\cdot\mathbf{q}}\ \frac{i\mathbf{k}}{k^2}\ \delta_0(\mathbf{k})$$

- Have to plug this into 1+ξ formula, Taylor expand terms in the exponential, do λ integrals, …

Example: Zel'dovich

- One obtains

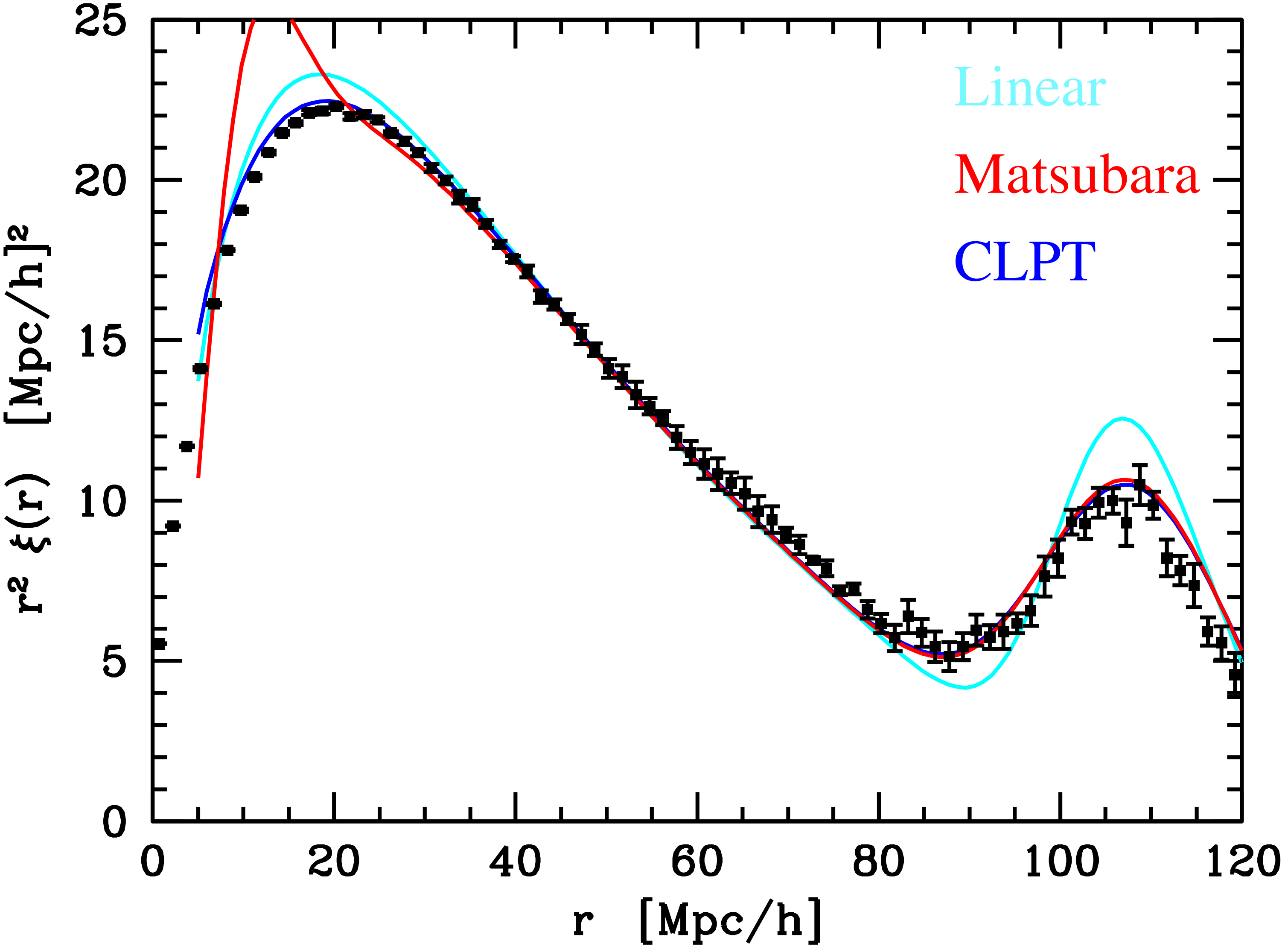
$$1+\xi_X(r)=\int \frac{d^3q}{(2\pi)^{3/2}|A|^{1/2}}\; e^{-\frac{1}{2}(q-r)^T A^{-1}(q-r)}$$

$$\times\left[1-\cdots 2\underbrace{\langle F'\rangle\langle F''\rangle}_{\substack{\uparrow\\ b^3}}\xi_R\underset{\substack{\uparrow\\ v}}{U_i}g_i+\cdots\right]$$

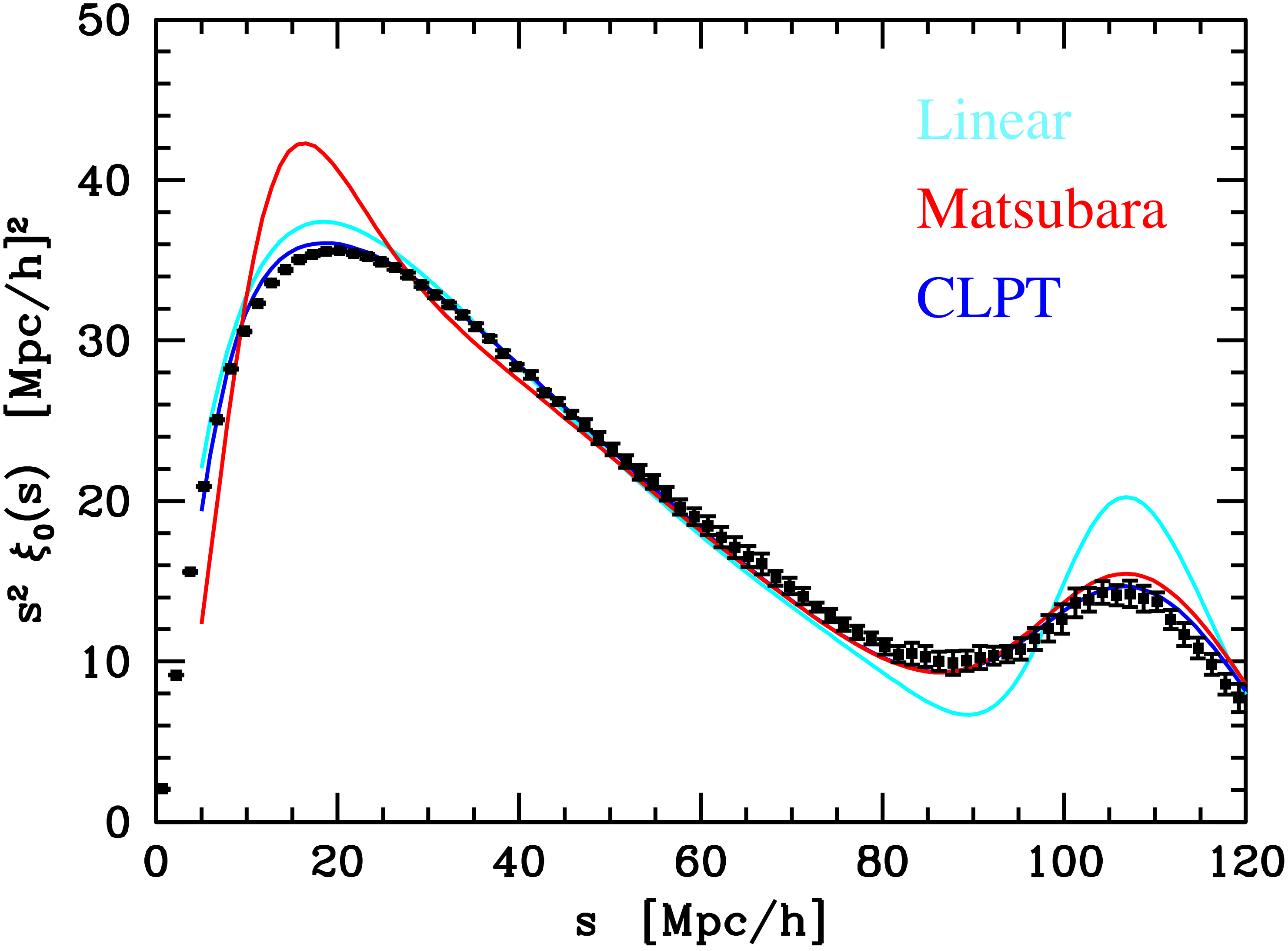
Convolution LPT?

- Matsubara separates out the q-independent piece of the 2-point function $<\Delta\Psi_i\Delta\Psi_j>$
- Instead keep all of $<\Delta\Psi_i\Delta\Psi_j>$ exponentiated.
  - Expand the rest.
  - Do some algebra.
  - Evaluate convolution integral numerically.
- Guarantees we recover the Zel'dovich limit as 0th order CLPT (for the matter).
  - Eulerian and LPT require an ∞ number of terms.
  - Many advantages: as emphasized recently by Tassev & Zaldarriaga

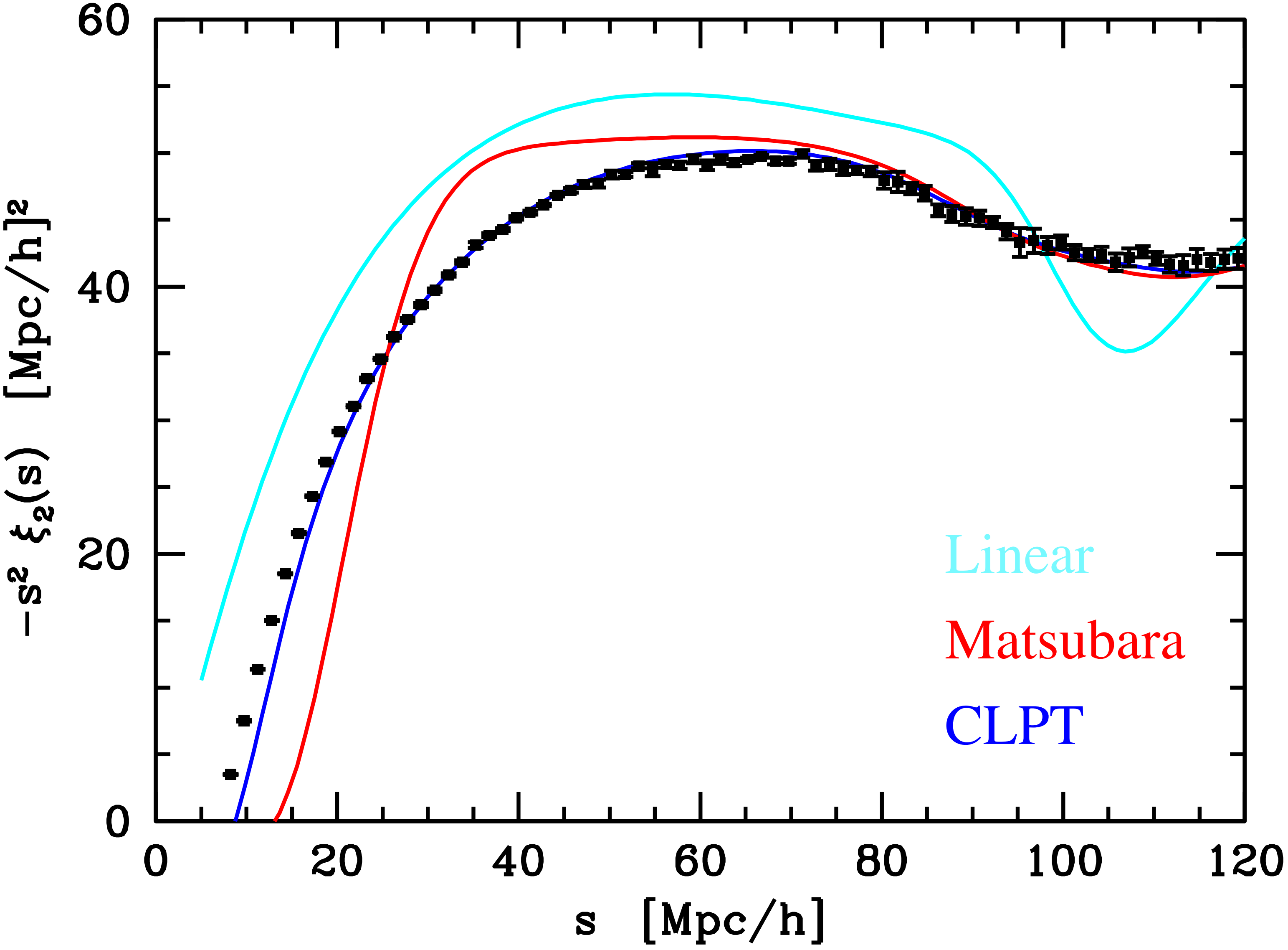
Matter: Real: Monopole



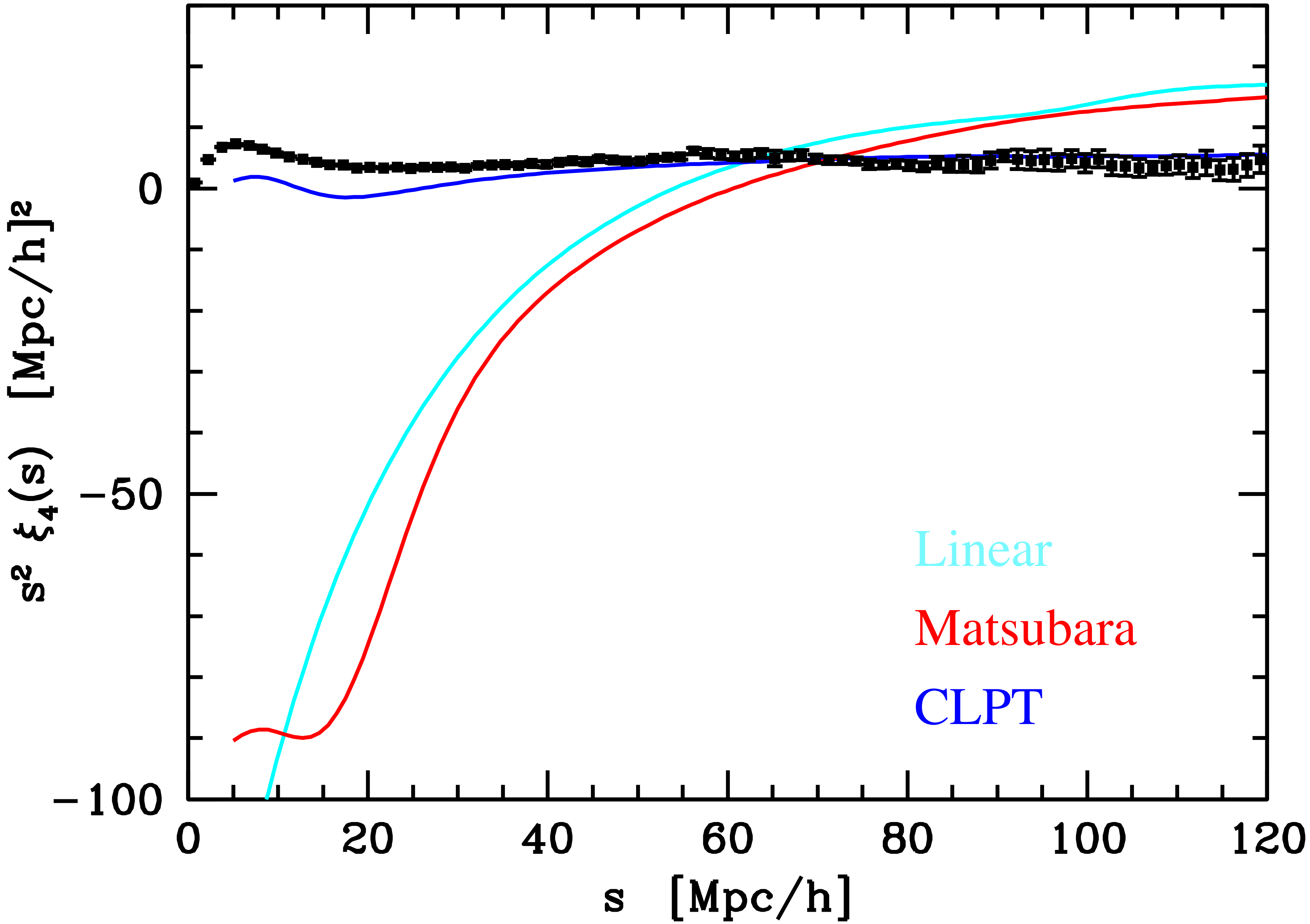
Matter: Red: Monopole



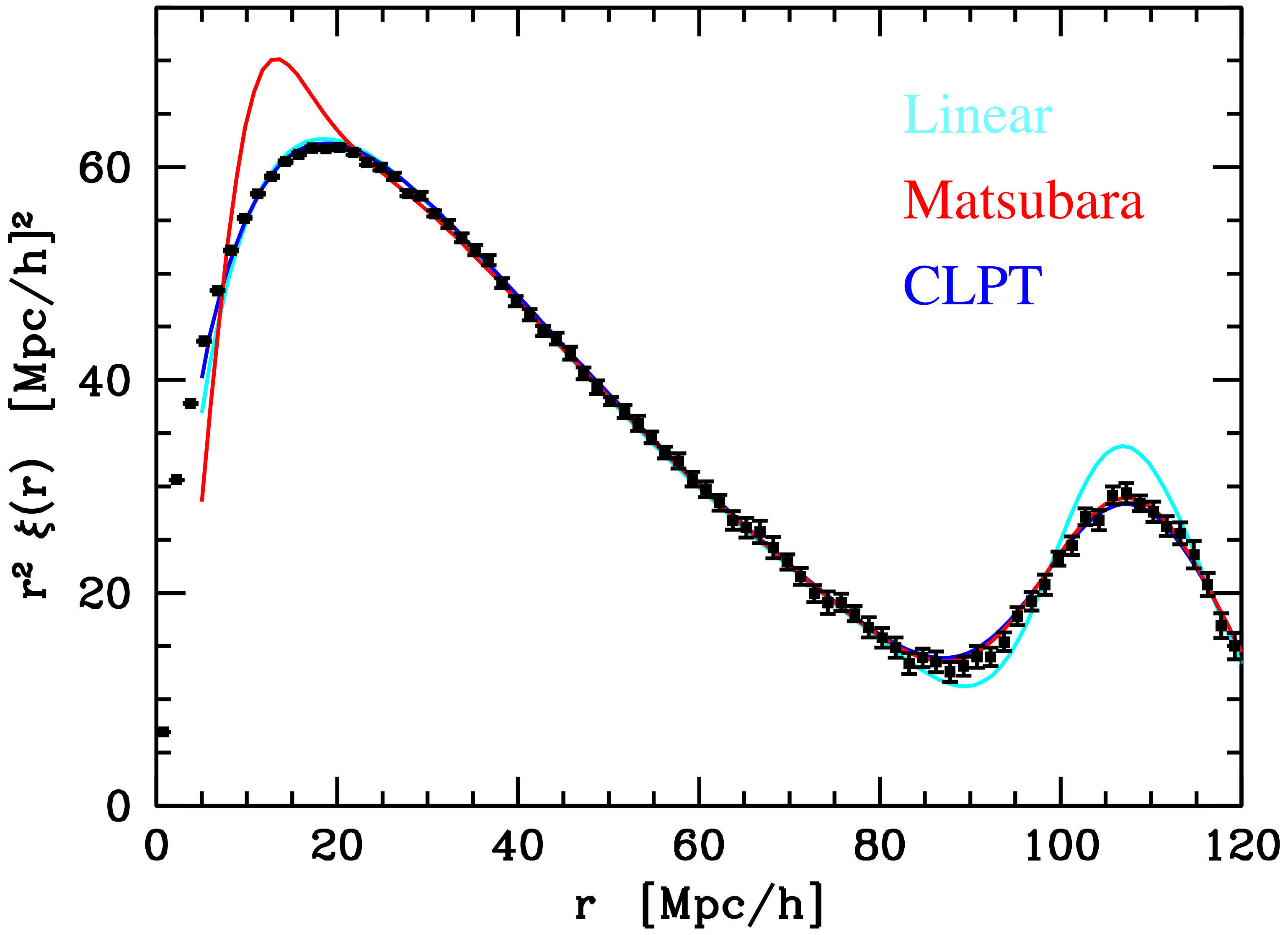
Matter: Quadrupole



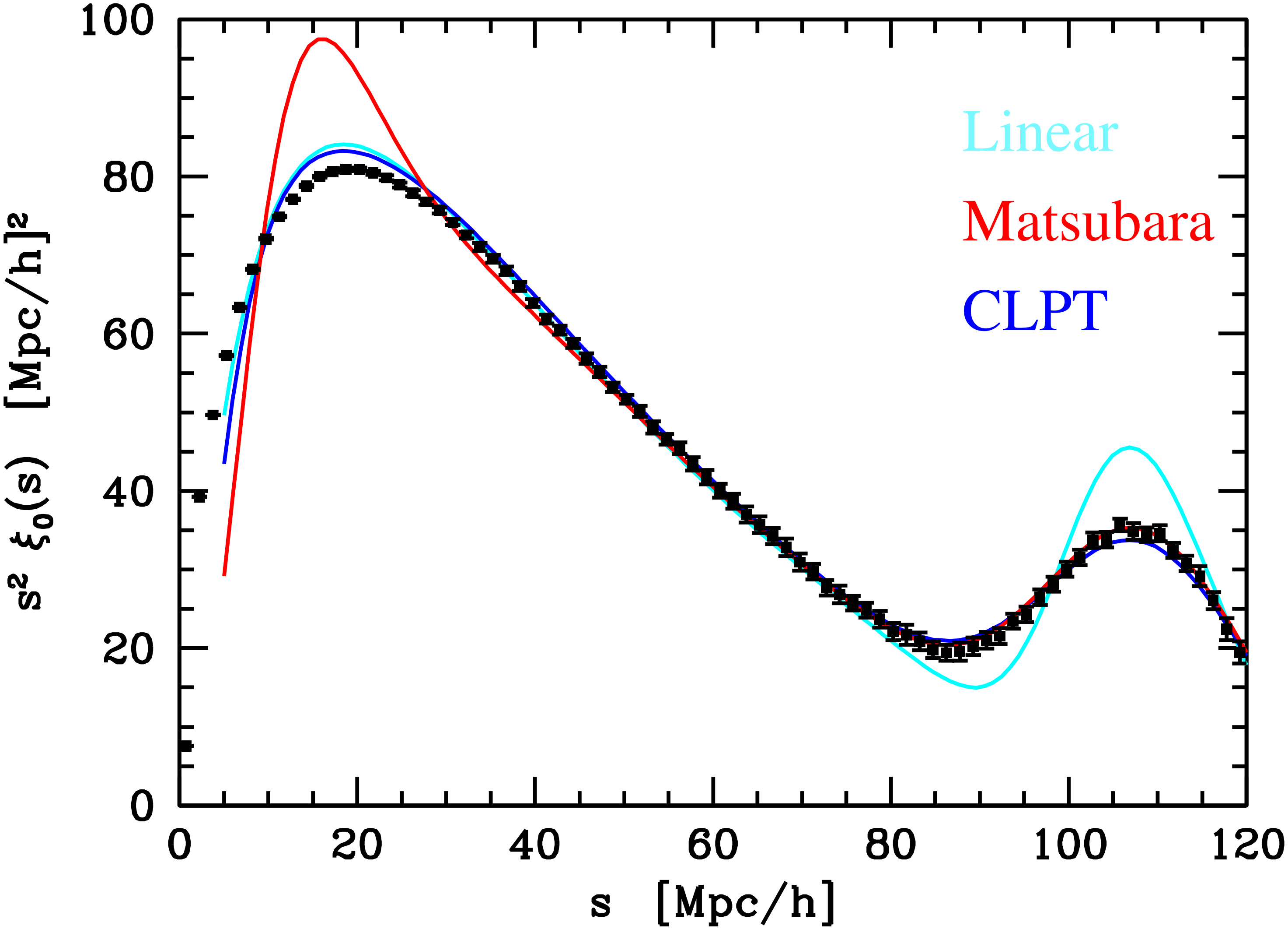
Matter: Hexadecapole



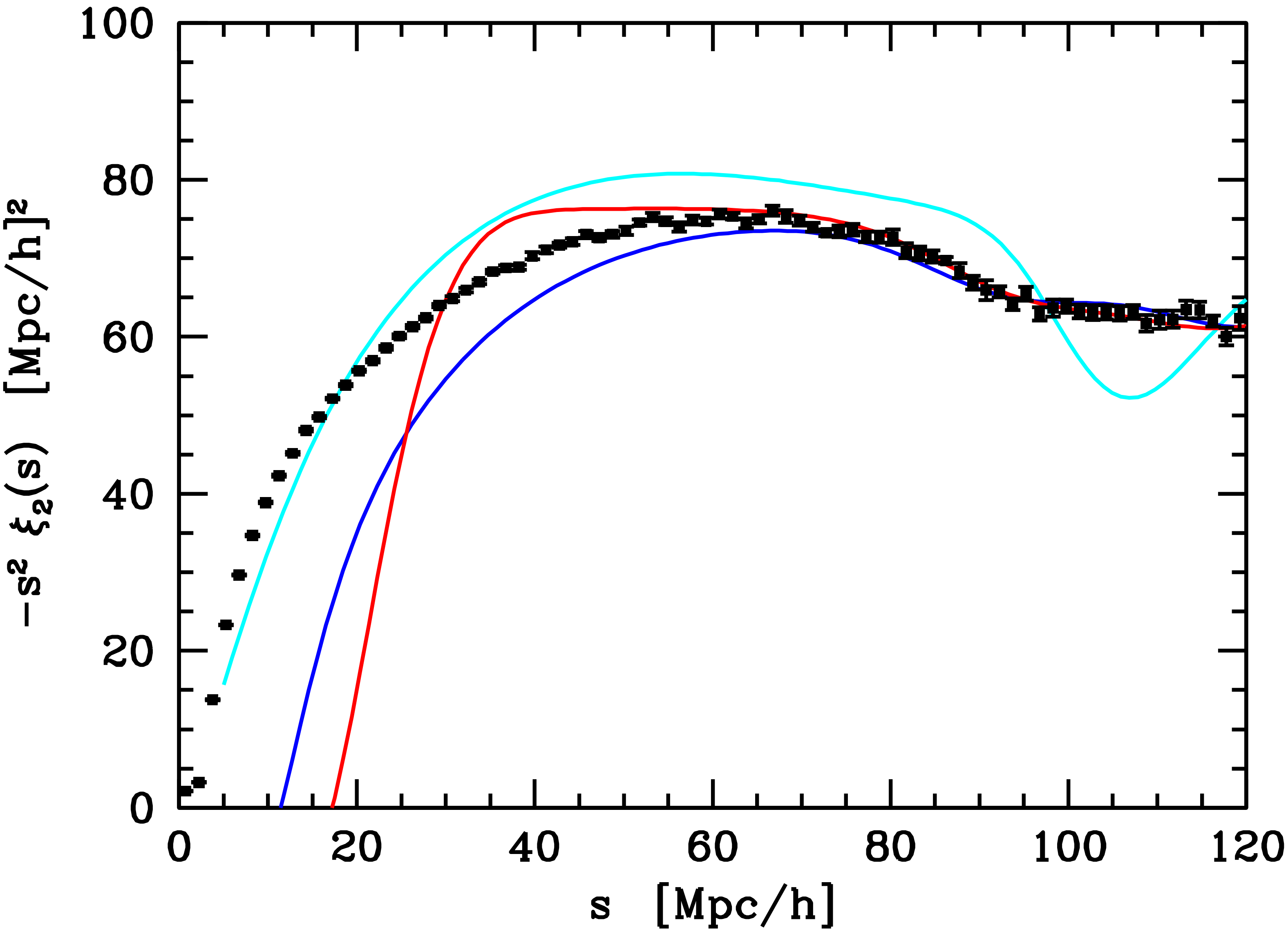
Halos: Real: Monopole



Halos: Red: Monopole



Halos: Quadrupole



From halos to galaxies

- In principle just another convolution
  - Intra-halo PDF.
- In practice need to model cs, $ss^{(1h)}$ and $ss^{(2h)}$.
- A difficult problem in principle, since have fingers-of-god mixing small and large scales.
  - Our model for ξ falls apart at small scales…
- On quasilinear scales things simplify drastically.
  - Classical FoG unimportant.
  - Remaining effect can be absorbed into a single Gaussian dispersion which can be marginalized over.

Conclusions

- Redshift space distortions arise in a number of contexts in cosmology.
  - Fundamental questions about structure formation.
  - Constraining cosmological parameters.
  - Testing the paradigm.
- Linear theory doesn't work very well.
- "Standard" PT doesn't work well either.
  - Need some form of resummation scheme.
  - Need some kind of calibration with N-body?
- Two types of non-linearity.
  - Non-linear dynamics and non-linear maps.
- Bias dependence can be complex.

The End