Bispectrum of the Sunyaev-Zeldovich Effect

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with
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CMB Science- Secondary Anisotropies

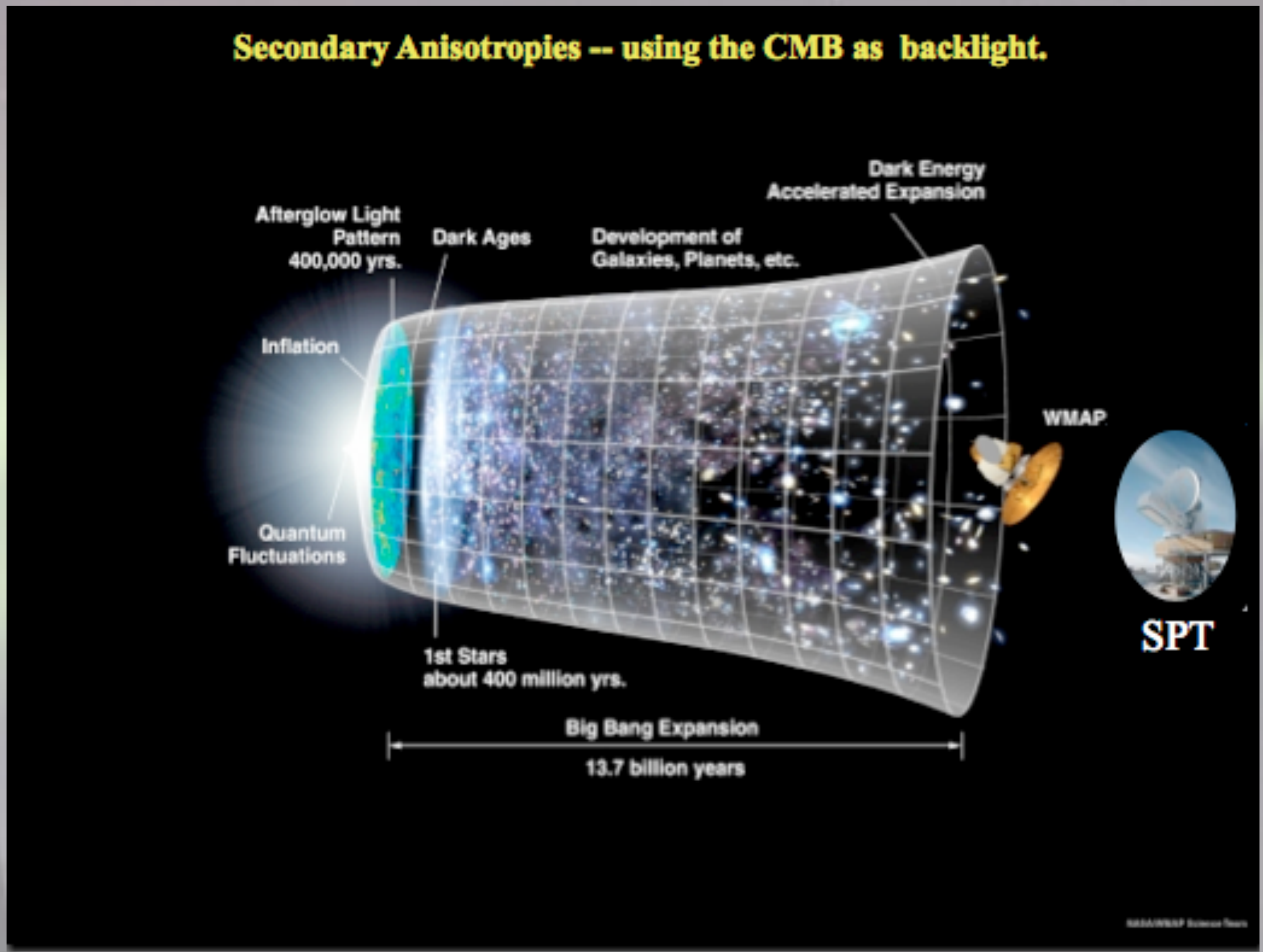
- Secondary Anisotropies => late universe, small scale phoenomena
- Sunyaev-Zeldovich Effect

  ---Thermal SZ

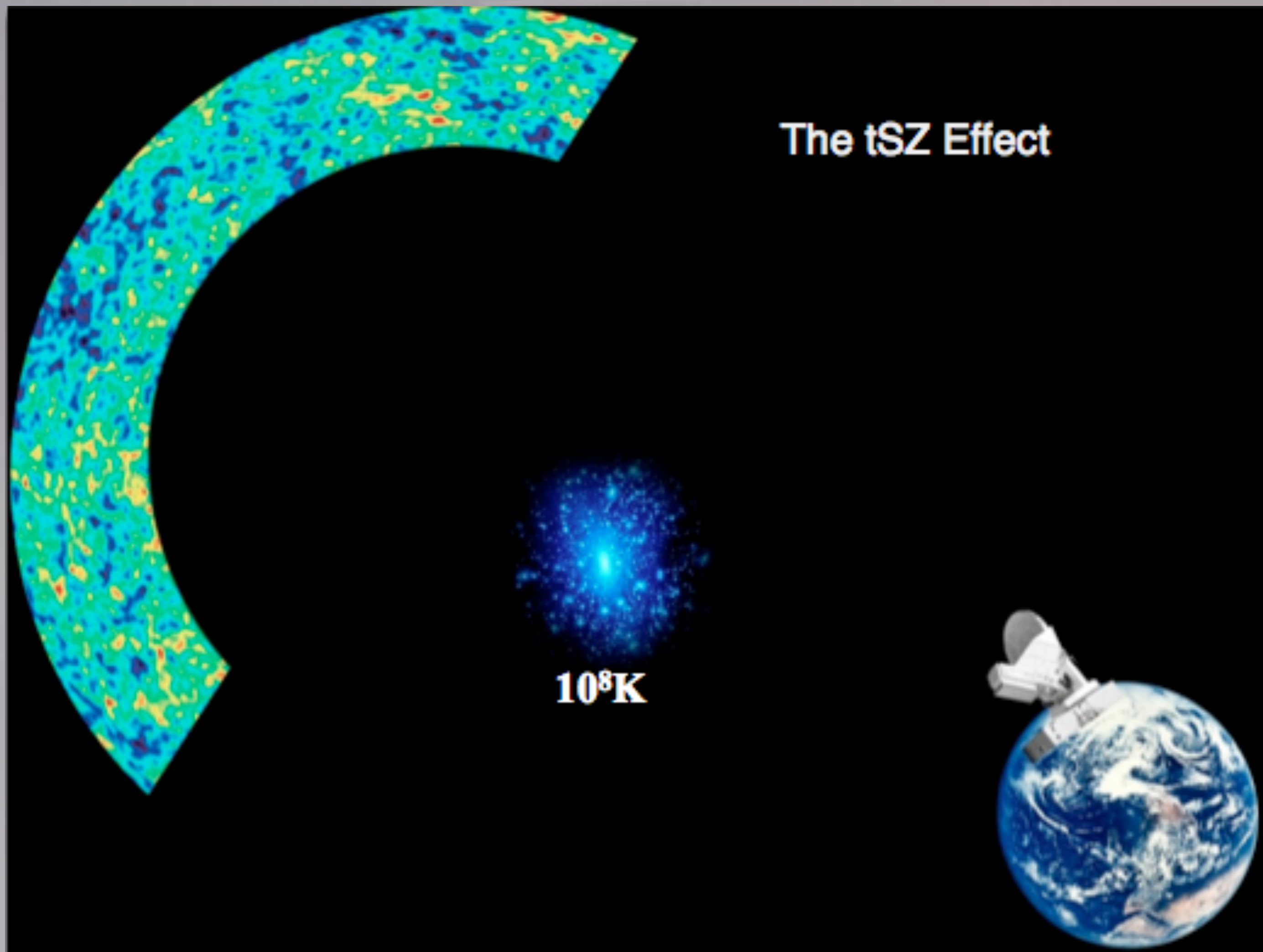
  --- kinetic SZ
- CMB Lensing
- point sources

  ---radio (AGN), dusty high-z star forming galaxy

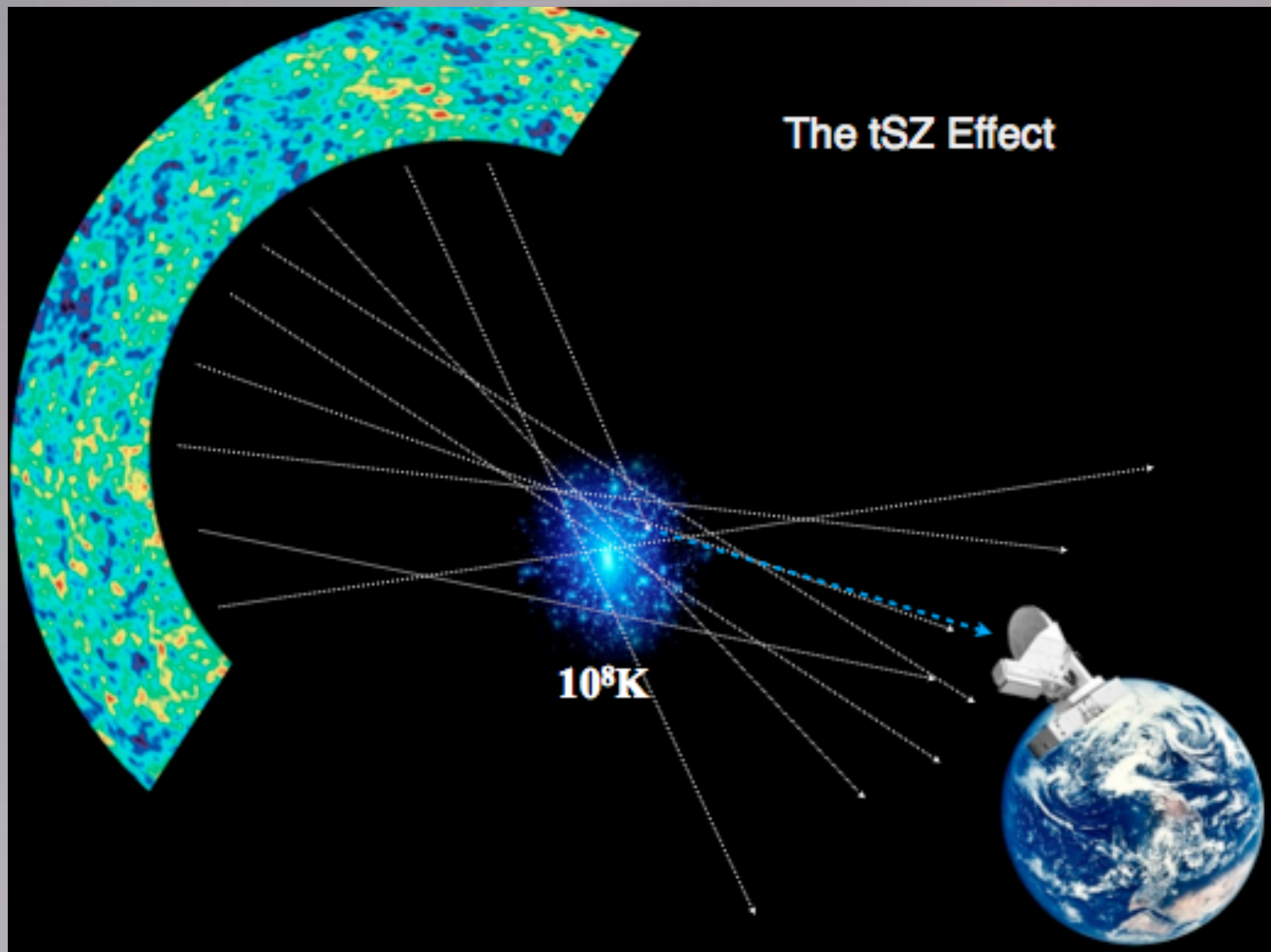
  ---Poisson , clustered

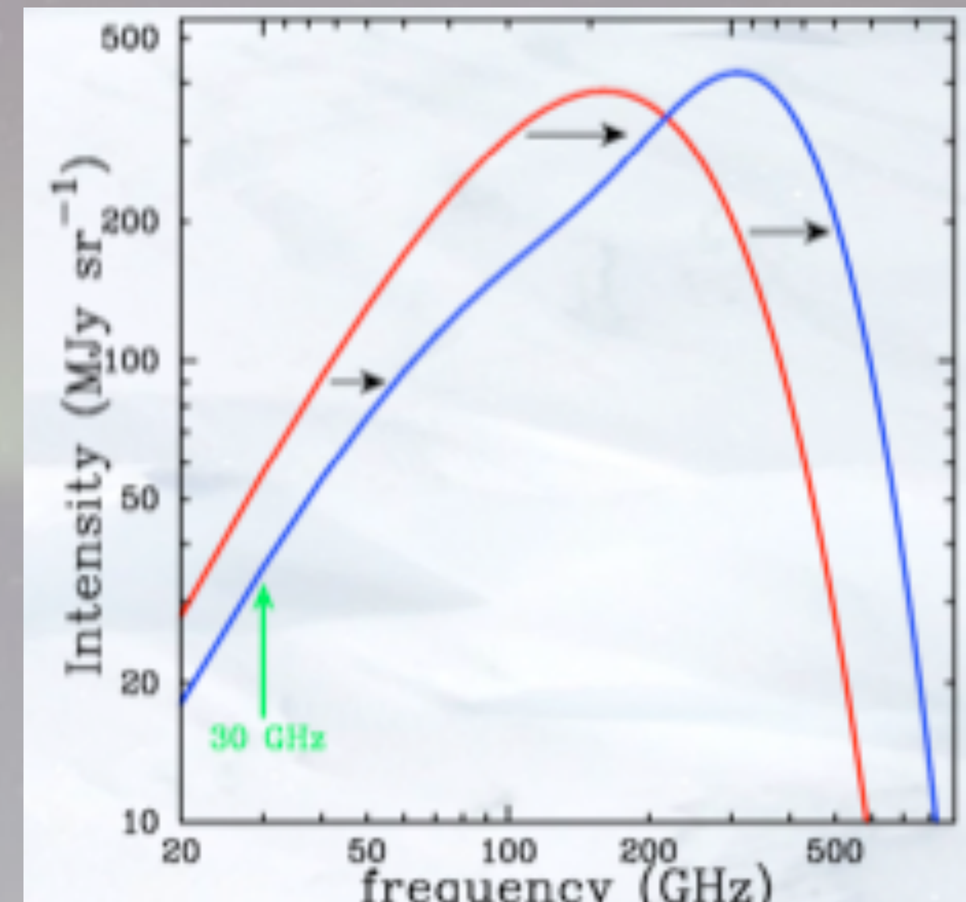


Secondary Anisotropies-Sunyaev-Zeldovich Effect



Secondary Anisotropies-Sunyaev-Zeldovich Effect



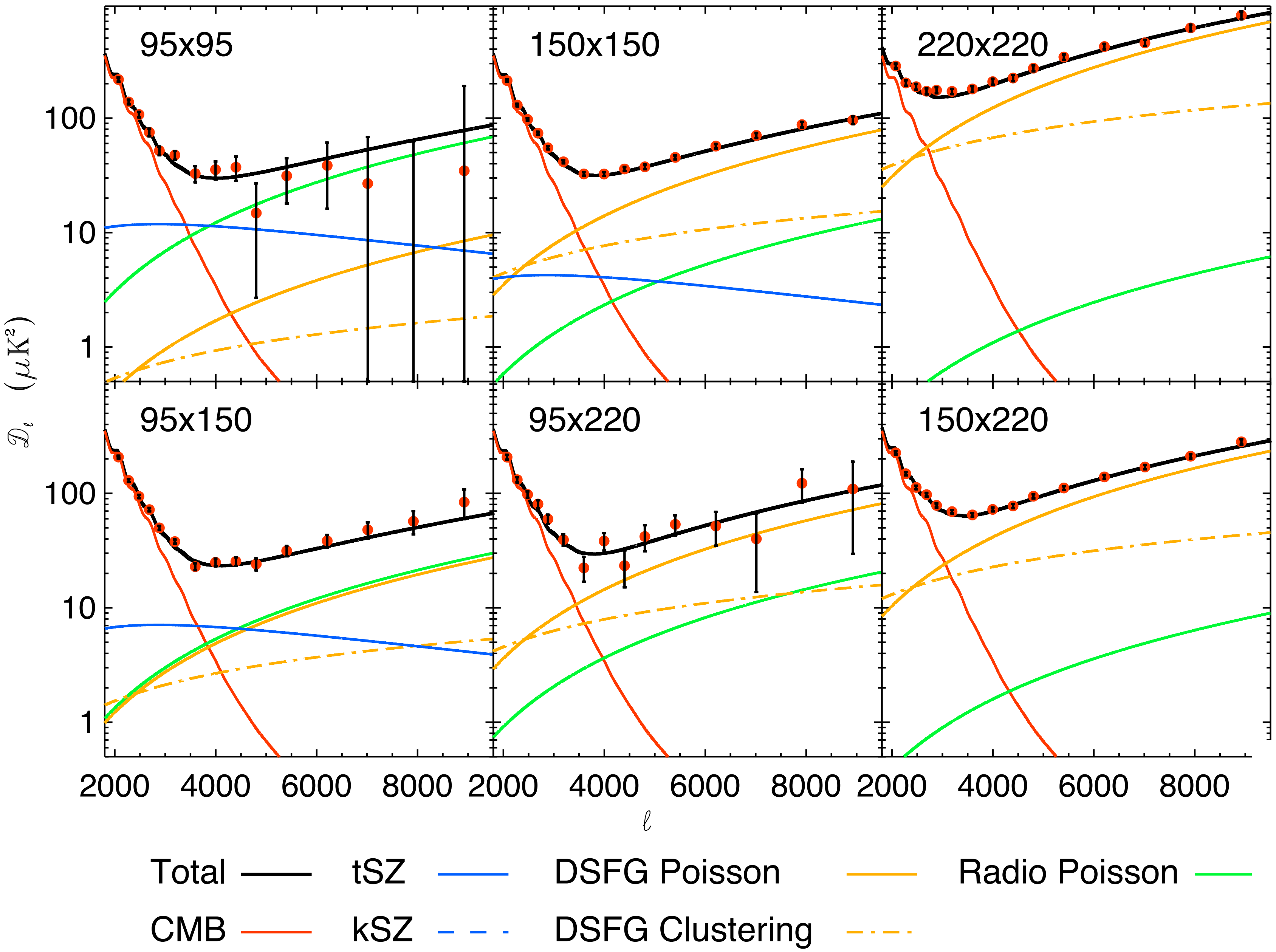


- 1-2 % CMB photons passing through galaxy clusters get inverse Compton scattered to higher energy

- Surface Brightness is independent of redshift

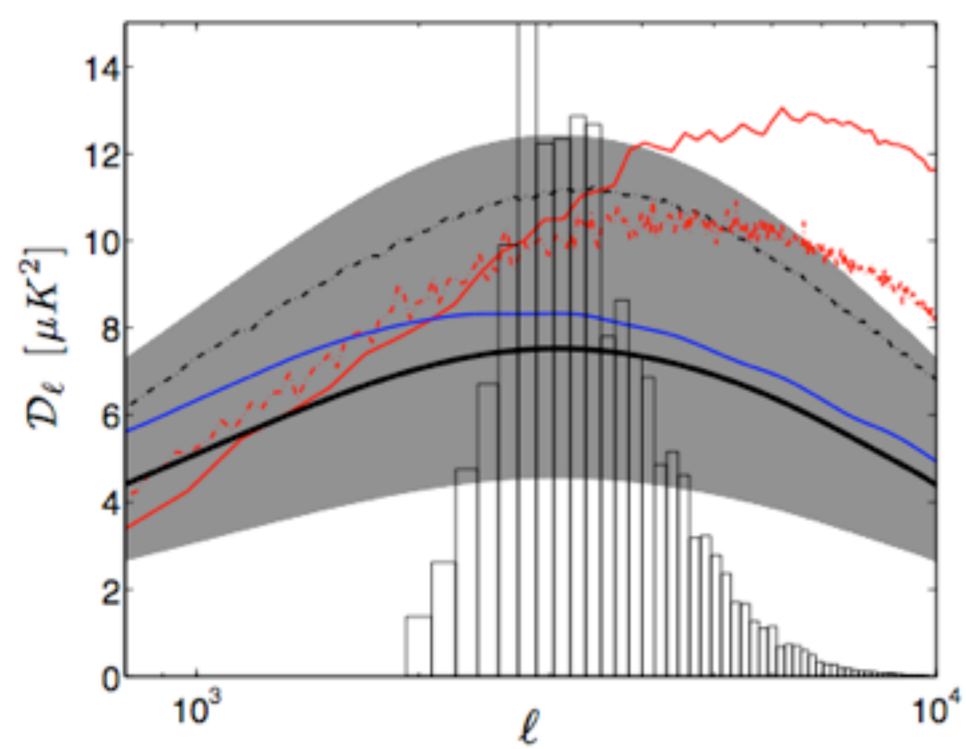
$$\frac{\Delta T_{cmb}}{T_{cmb}} = f_\nu(x)y = \left(\frac{k_B\sigma_T}{m_e c^2}\right)\int n_e(l)T_e(l)dl$$

SZ spectrum studies



Reichardt et al , SPT team

Theory uncertainty



Theory uncertainty is the limiting factor!

Pressure Profile: Theoretical Model

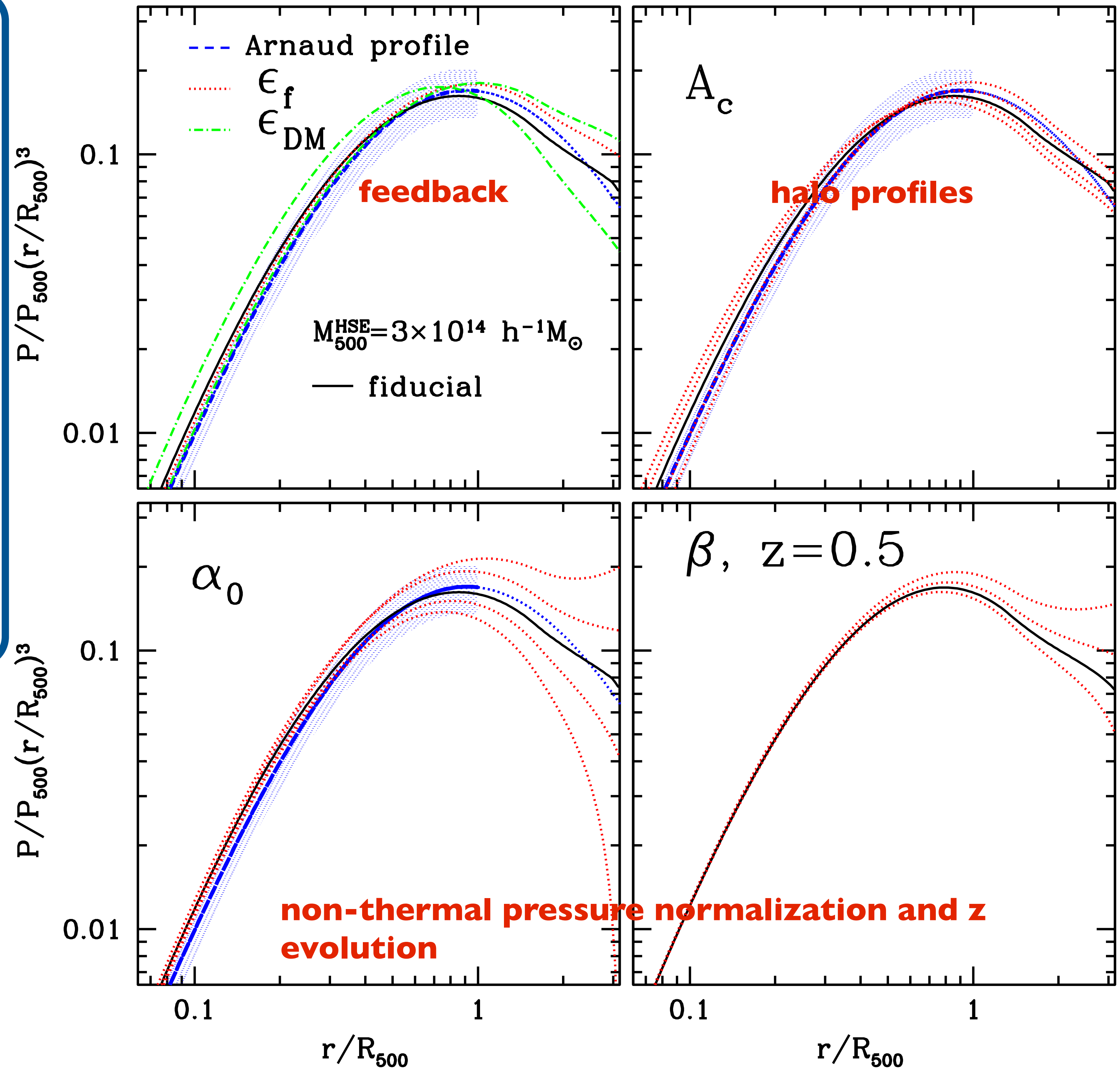
Modeling pressure profiles is the key to understand the SZ effect !

- gas reside on dark matter halos in hydrostatic equilibrium
- pressure and gas density are related via a power law
- fraction of gas turns to stars
- fraction of star energy goes back to intra-cluster medium via feedback processes.
- non-thermal pressure due to gas motion: $\frac{P_{nt}}{P_{tot}}(z) = \alpha(z)\,(r/R_{500})^{n_{nt}}$

Bode & Ostriker 06, ..., Shaw, Nagai, Bhattacharya & Lau 2010

Pressure Profile: Theory vs. Observation

- we reproduce Arnaud profile of massive clusters at low-z -> low-z gas parameters are constrained for massive clusters
- don't have high-z pressure profile observations-> high z parameters unconstrained
- galaxy groups -> not constrained!

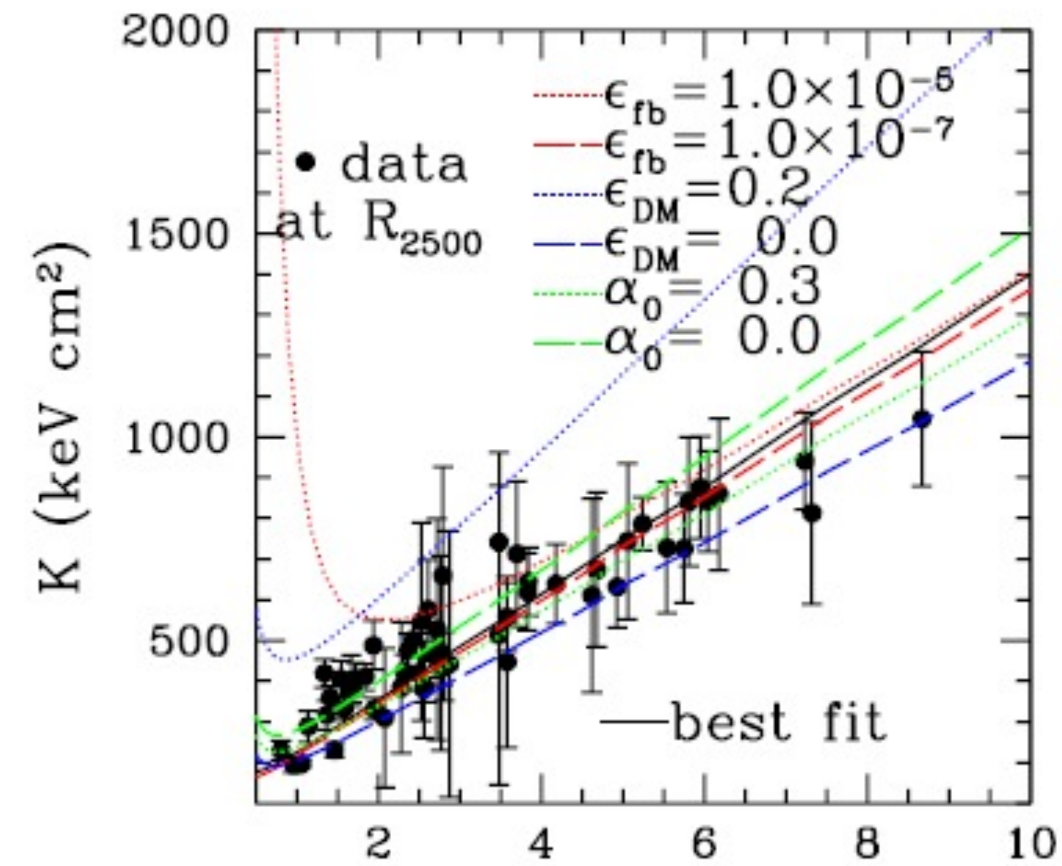


+other low-z X ray data:

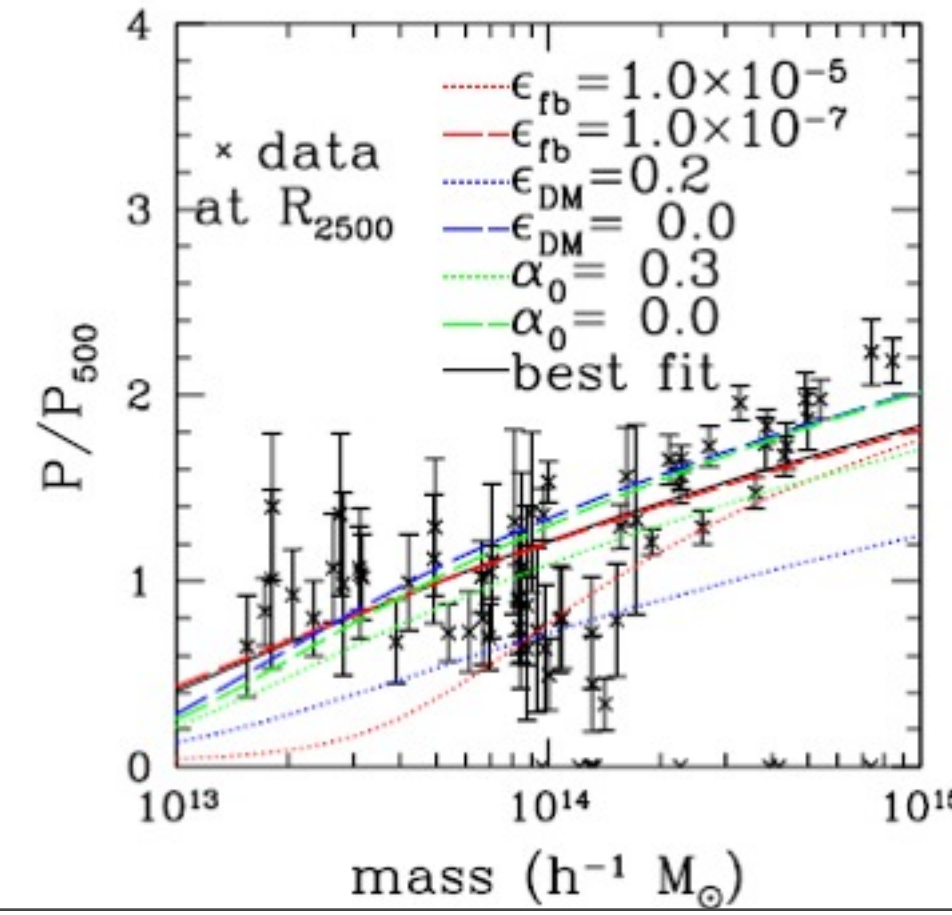
- Low redshift groups (Sun et al) and cluster (Vikhlinin et al, Pratt et al, Arnaud et al)

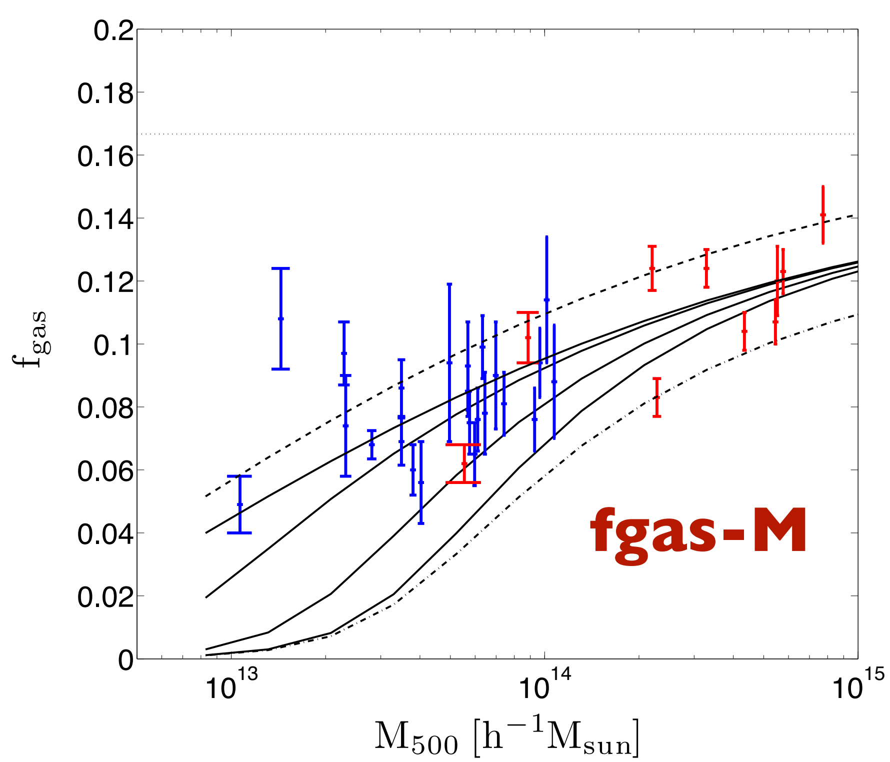
- **Xray scaling relations:**
  -> Vikhlinin et al , Sun et al entropy-temperature relation
  -> Pratt et al , Sun et al pressure-mass relation
  -> Vikhlinin et al , Sun et al gas fraction-mass relation

K-T

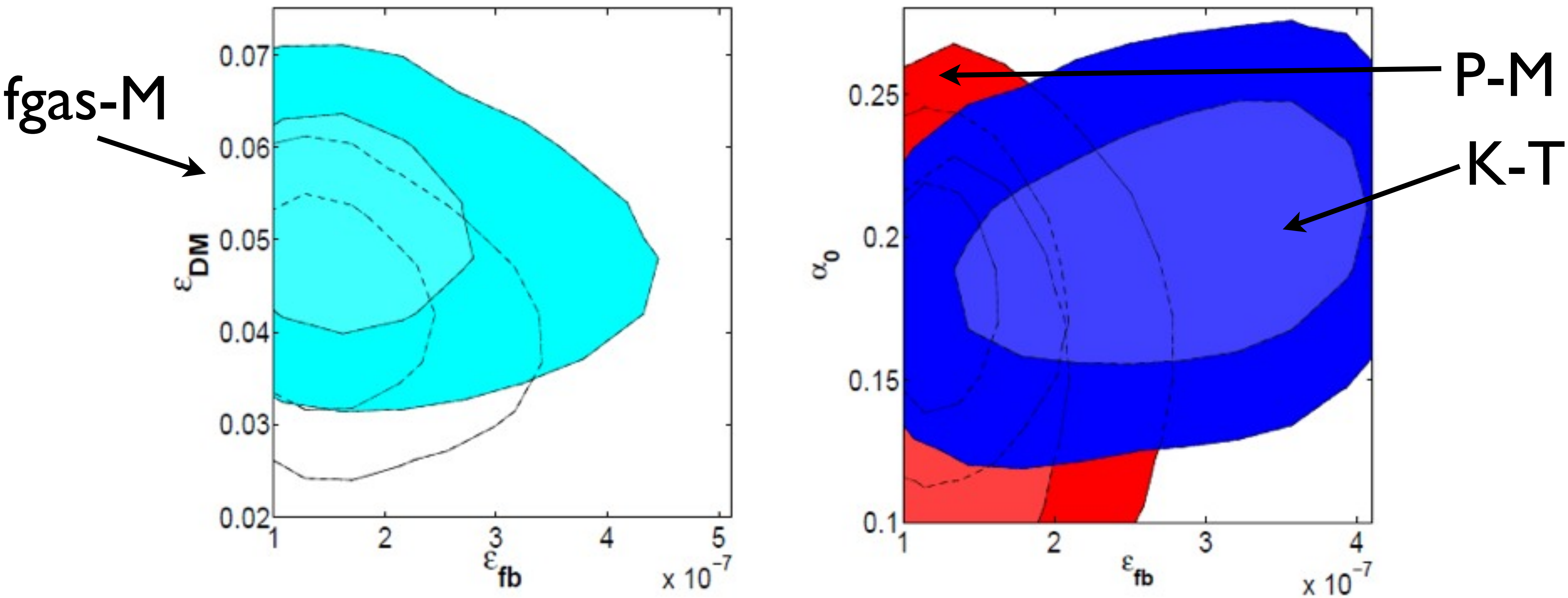


P-M





exploration of ICM parameter space



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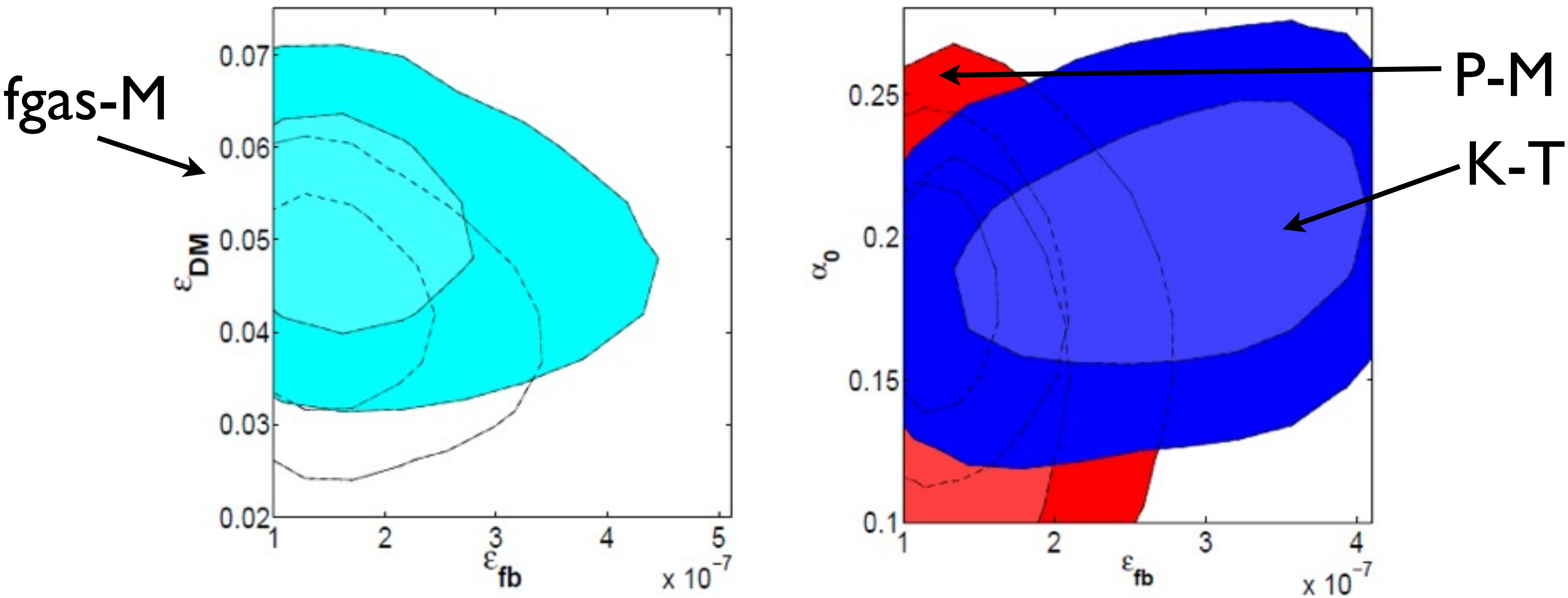
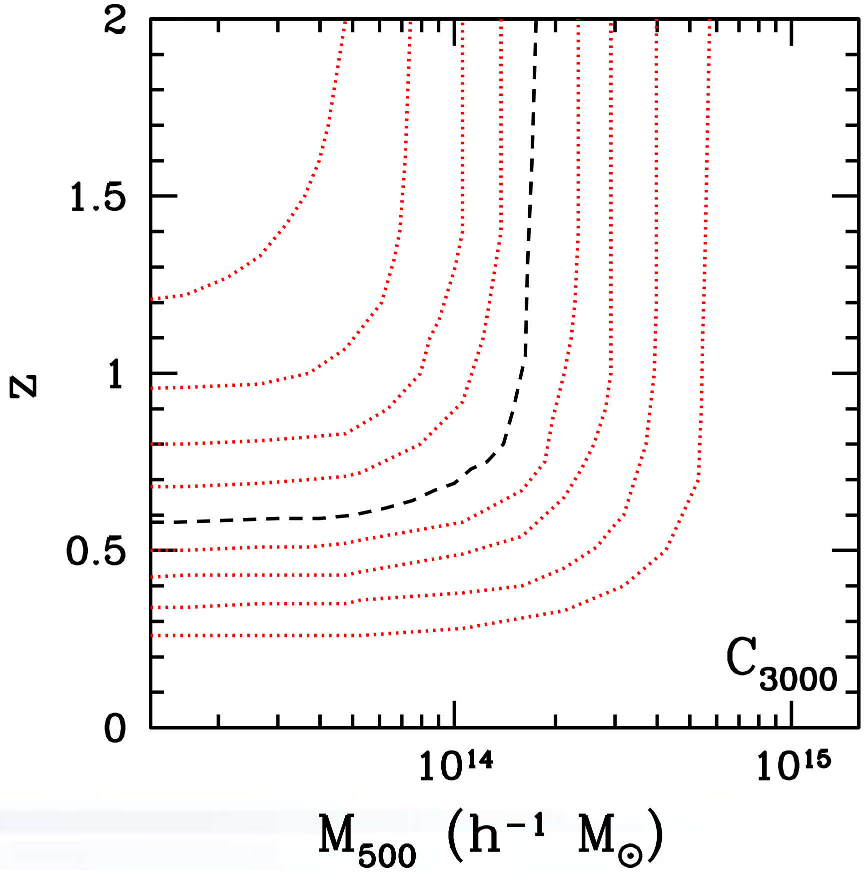


Parameters	True Mass			
	mean	std. dev.	1σ (68% CL)	2σ (95% CL)
ϵ_{fb} $(\times 10^{-7})$	1.82	0.78	1.16-2.52	1.04-3.93
ϵ_{DM}	0.051	0.005	0.046-0.056	0.039-0.061
α_0	0.197	0.026	0.171-0.223	0.144-0.246

SZ Power Spectrum

$$C_\ell = f(x_\nu)^2 \int dz \frac{dV}{dz} \int d\ln M \frac{dn(M,z)}{d\ln M} \tilde{y}(M,z,\ell)^2$$

- Cl ~ σ_8 ^8
- Measurement uncertainty of the power spectrum amplitude ~ 20%
- Power spectrum gets about 40% signal from high-z galaxy groups
- Theoretical uncertainty ~ 40-50%



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Cosmology (Geometry, growth factor)

Astrophysics: Fourier transform of pressure profile

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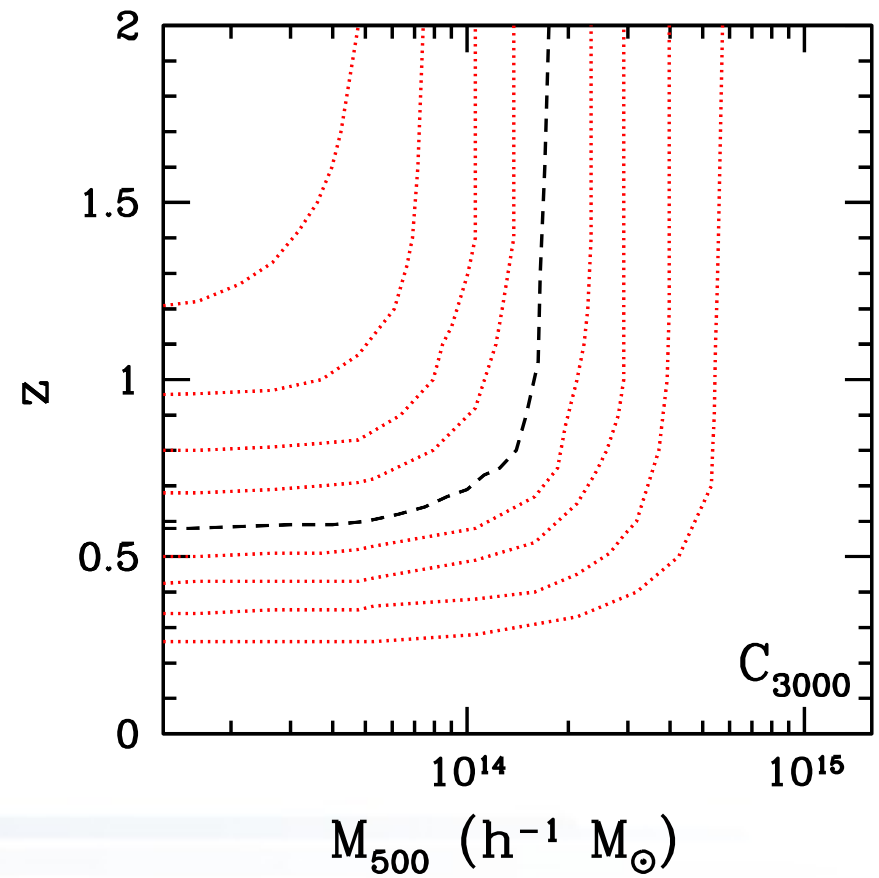

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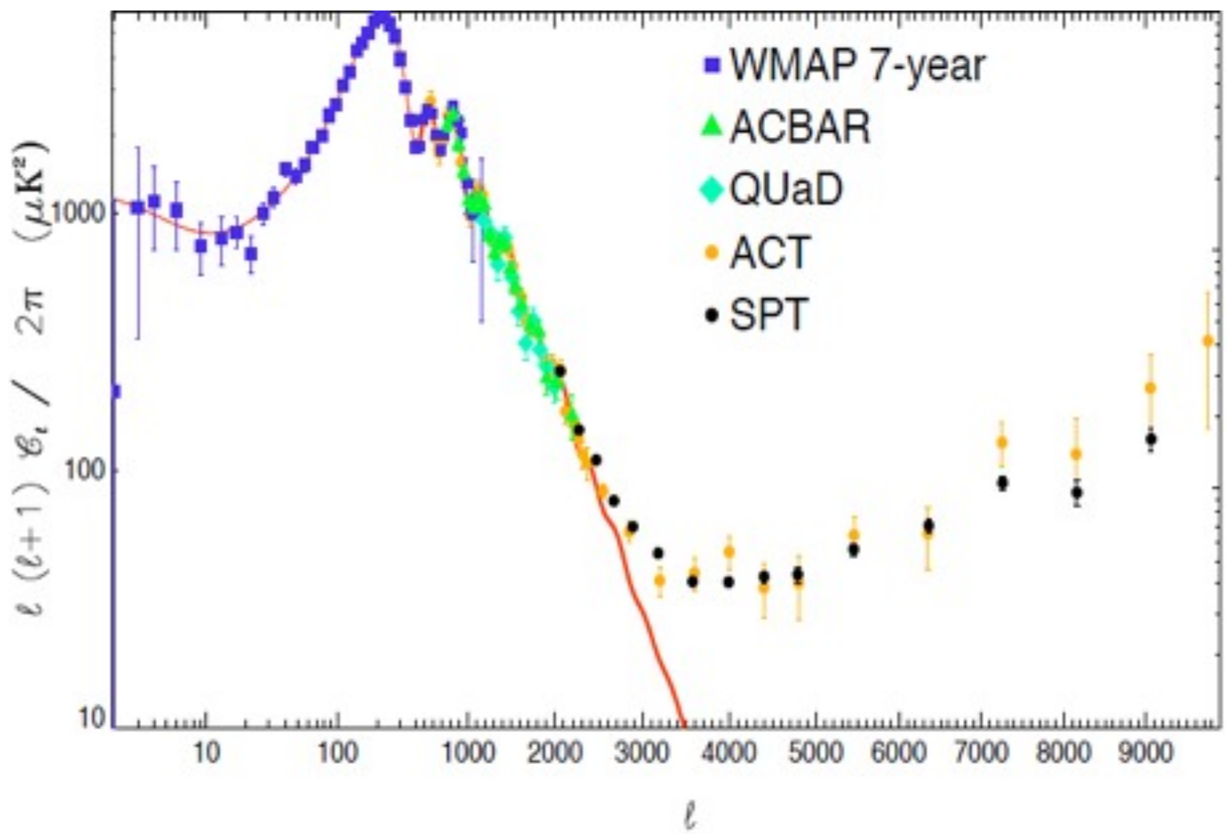
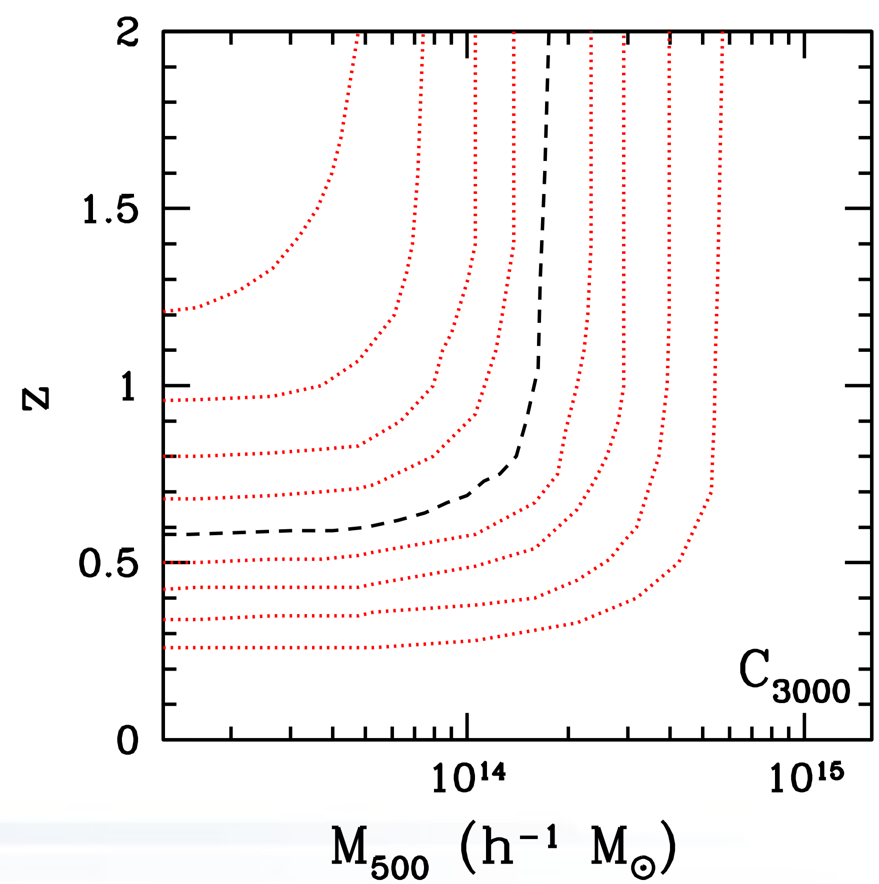




Bispectrum Primer

- 3-pt function on the sky in Fourier space
- CMB temperature (harmonic space) in a particular direction in the sky: $a_{\ell m} = \int d^2\hat{\mathbf{n}} \frac{\Delta T}{T} Y^*_{lm}(\hat{\mathbf{n}})$
- angular bispectrum then is: $\langle a_{l_1 m_1} a_{l2 m_2} a_{l_3 m_3} \rangle$
- average over m satisfying:

$$m_1 + m_2 + m_3 = 0,\ \ell_1 + \ell_2 + \ell_3 = \text{even, and } |l_i - l_j| \leq l_k \geq l_i + l_j$$

- **SZ bispectrum (flat sky limit):**

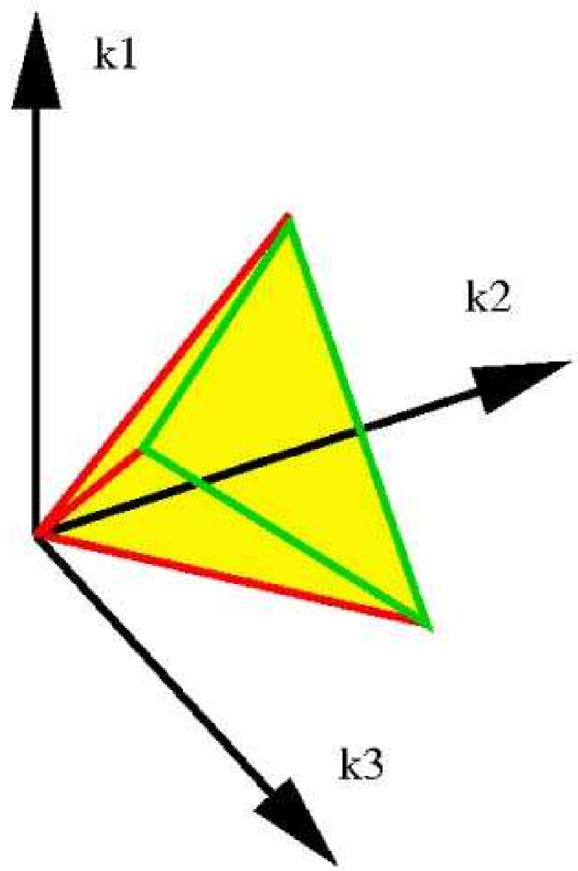
$$b(\ell_1 \ell_2 \ell_3) = f(x_\nu)^3 \int dz \frac{dV}{dz} \int d\ln M \frac{dn(M,z)}{d\ln M} \times \tilde{y}(M,z,\ell_1)\tilde{y}(M,z,\ell_2)\tilde{y}(M,z,\ell_3)$$

and its noise: $N(\ell_1 \ell_2 \ell_3) = C(\ell_1)C(\ell_2)C(\ell_3)$

where C(l) is the total power spectrum

(point sources +SZ+ lensed CMB+beam noise)

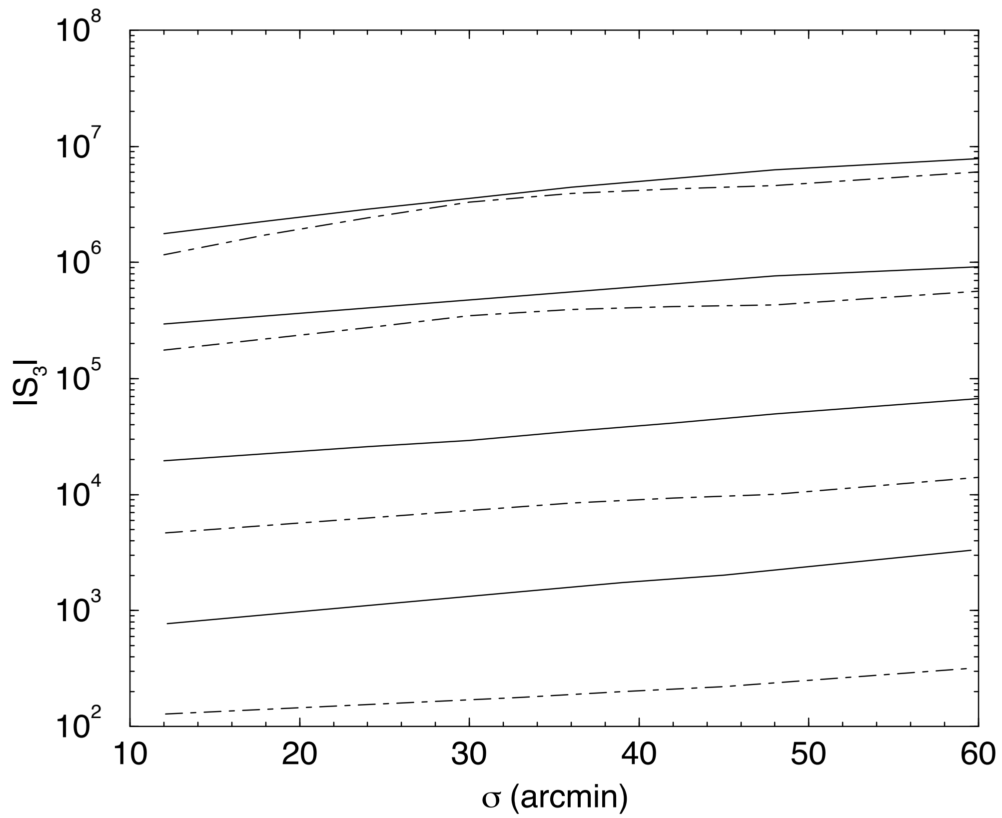
FT of the pressure profiles



- Komatsu &Spergel 2000; Cooray & Hu 1999, 2000; Hu 2001

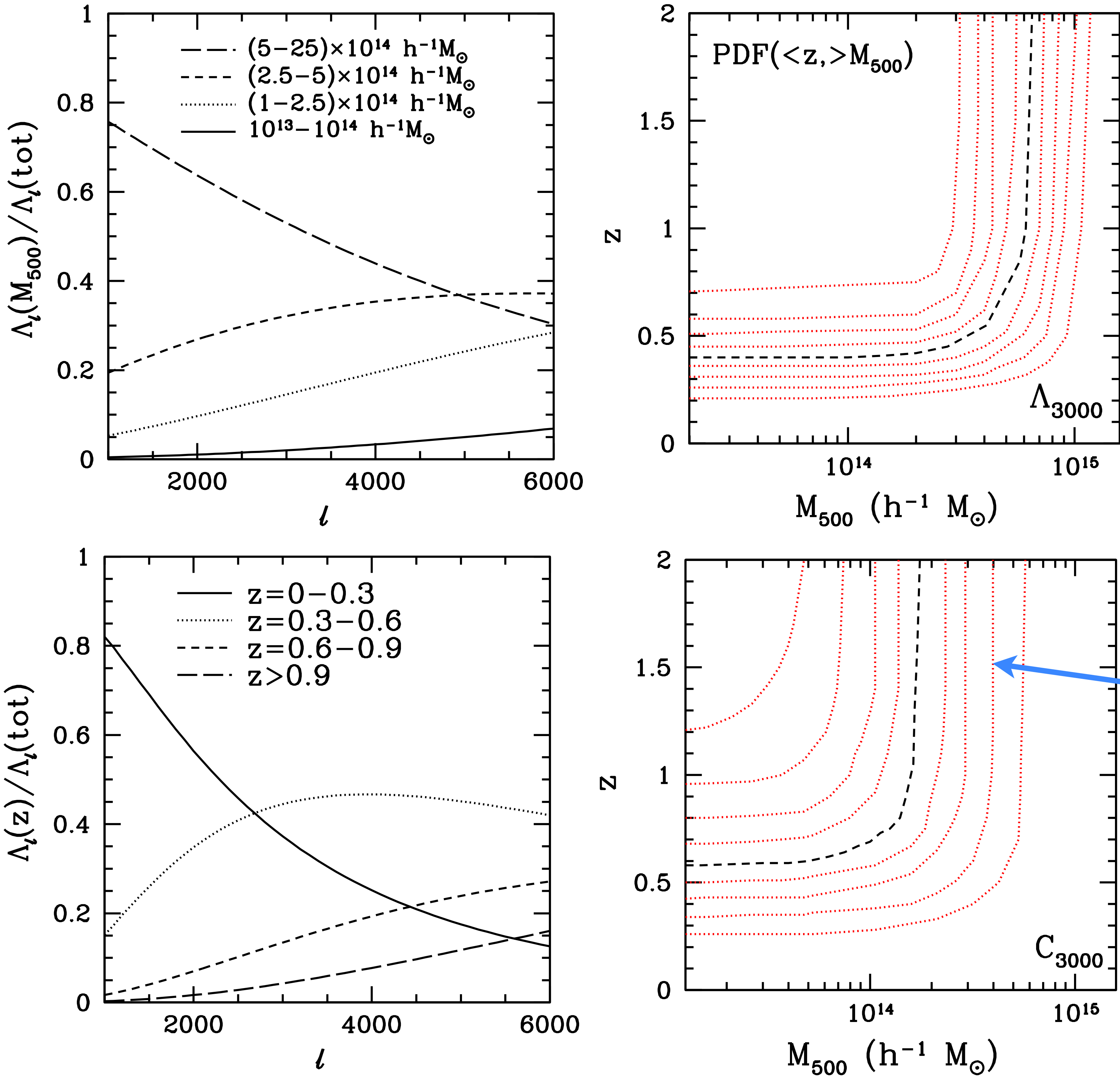
From Bispectrum to Skewness Spectrum

- skewness is the simplest 3-pt statistics in real space(equivalent to variance in 2-pt space): $S_3 \equiv \left\langle \left(\frac{\Delta T(\hat{\mathbf{n}})}{T} \right)^3 \right\rangle$
- Skewness is the sum over all possible triangles in harmonic space, then FT to real space.
- in real space, a skewness function is the skewness measured over a certain angular scale (Cooray 2000)
- Problem with real space skewness function is different sources of non-Gaussianity.
- Solution ? define a skewness spectrum in harmonic space (Munshi & Heavens 2008)



Cooray 2000; Rubino-Martin & Sunyaev 03; Hill & Sherwin 12

Where the SZ skewness spectrum signal comes from?

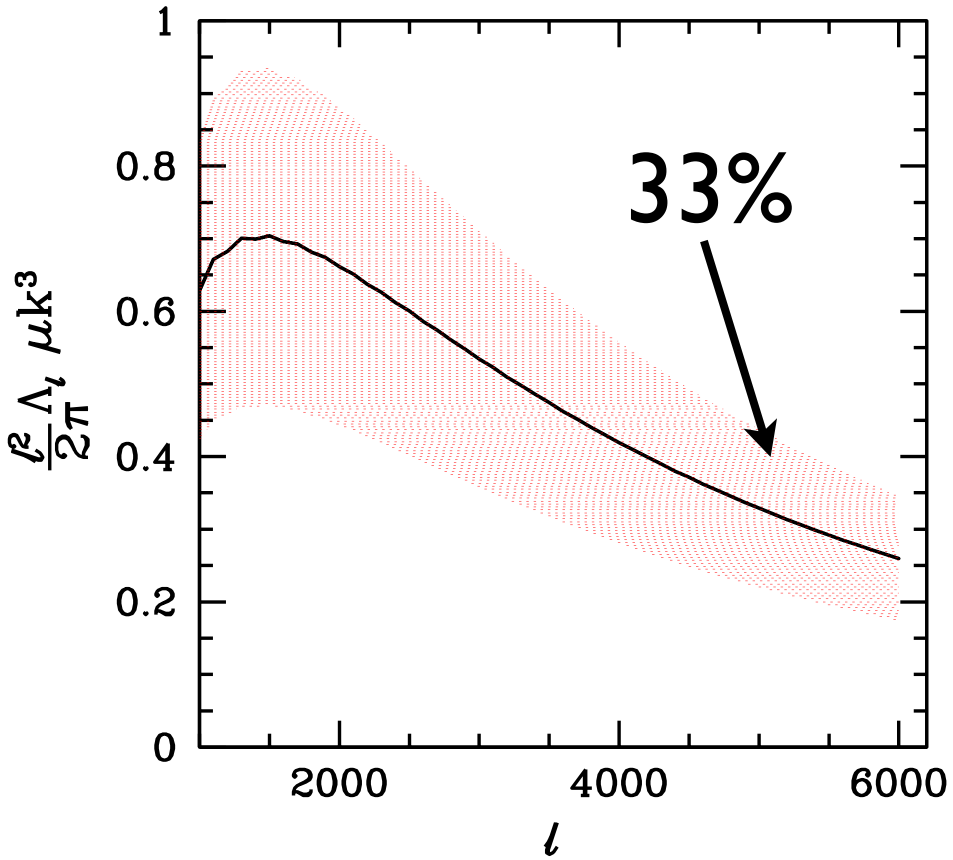


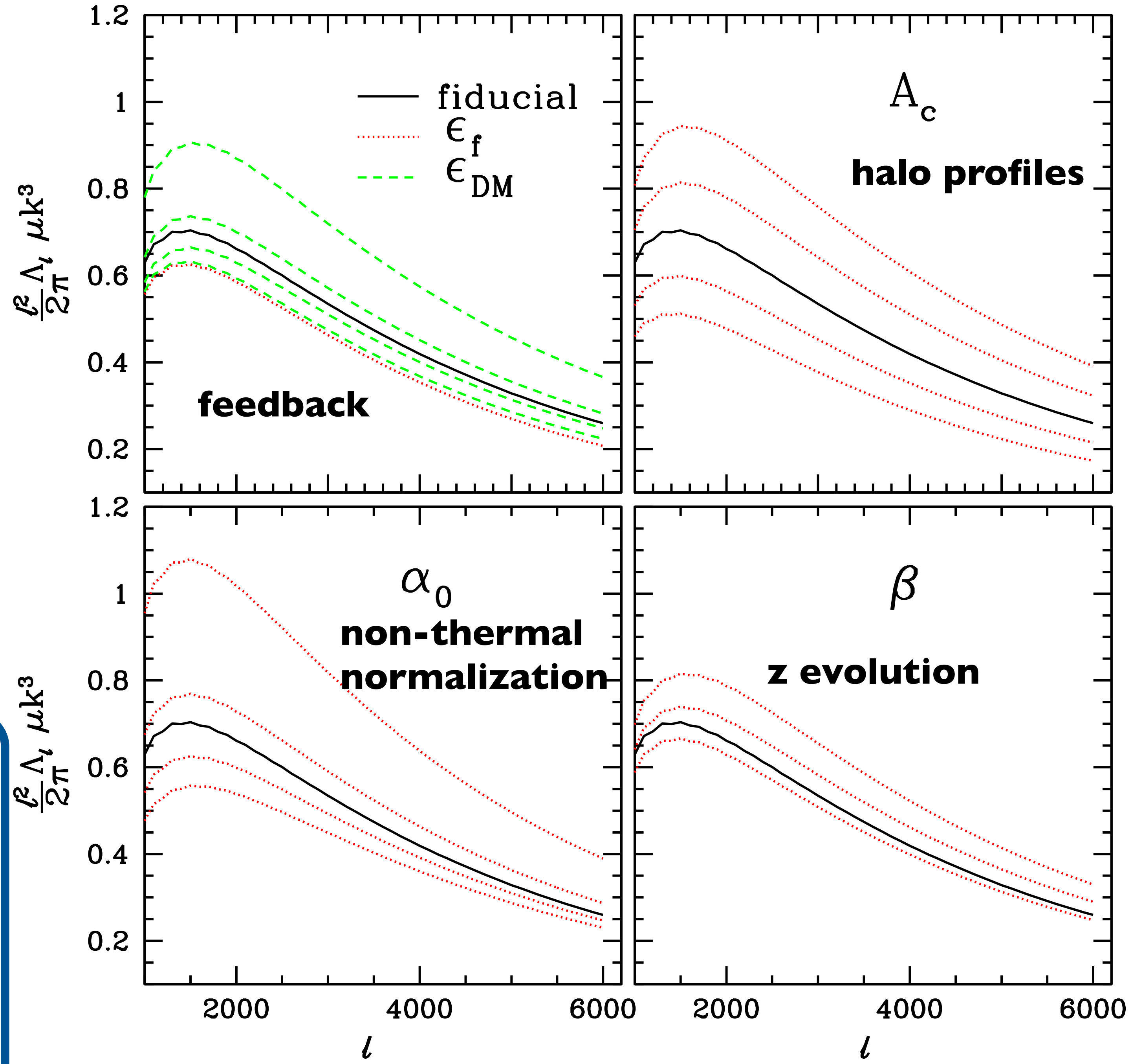
about 95% of the signal comes from M500> 1e14 & z<1

SZ power spectrum distribution

(also Trac et al 2011)

Astrophysical Uncertainties of the Skewness Spectrum:



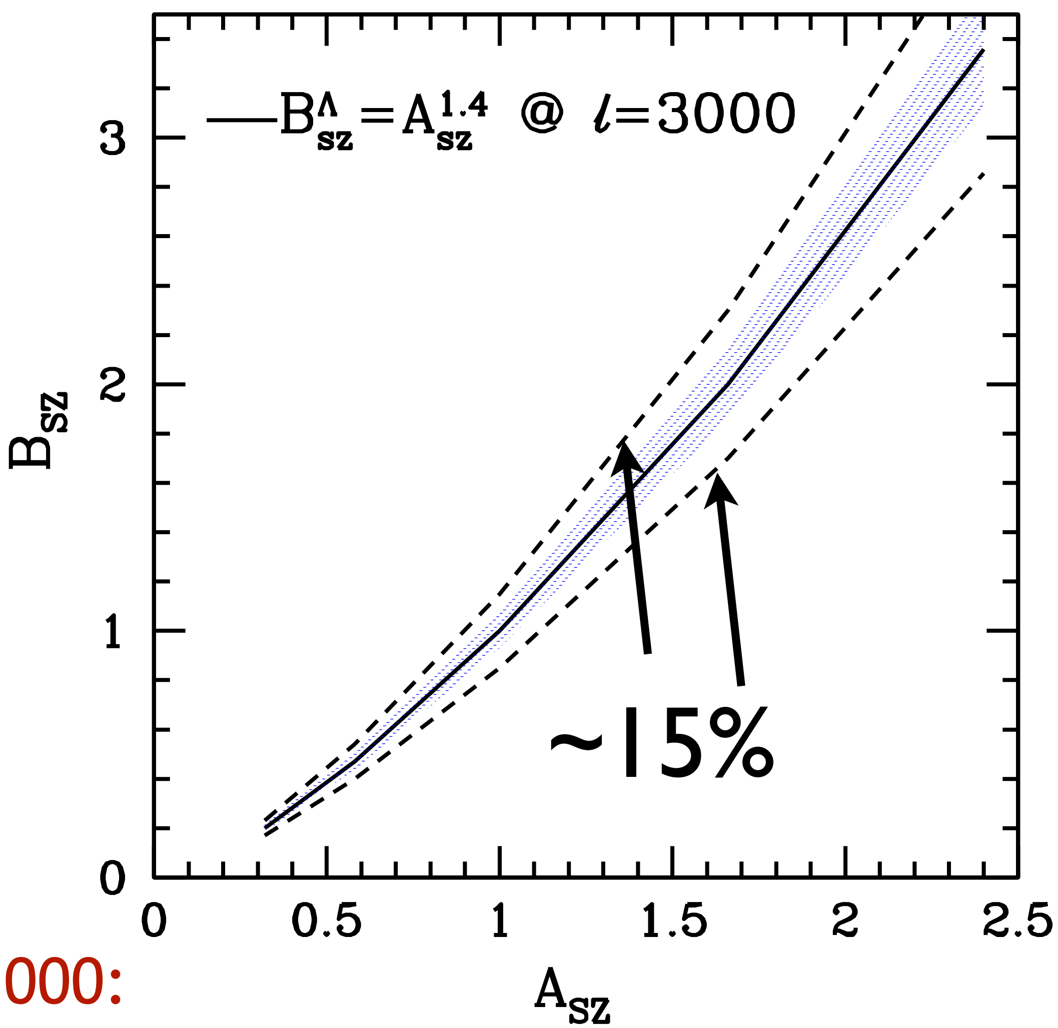


- theoretical uncertainty in the skewness spectrum ~33% (compared to the 40-50% uncertainty in the SZ power spectrum)

Power Spectrum vs. Bispectrum

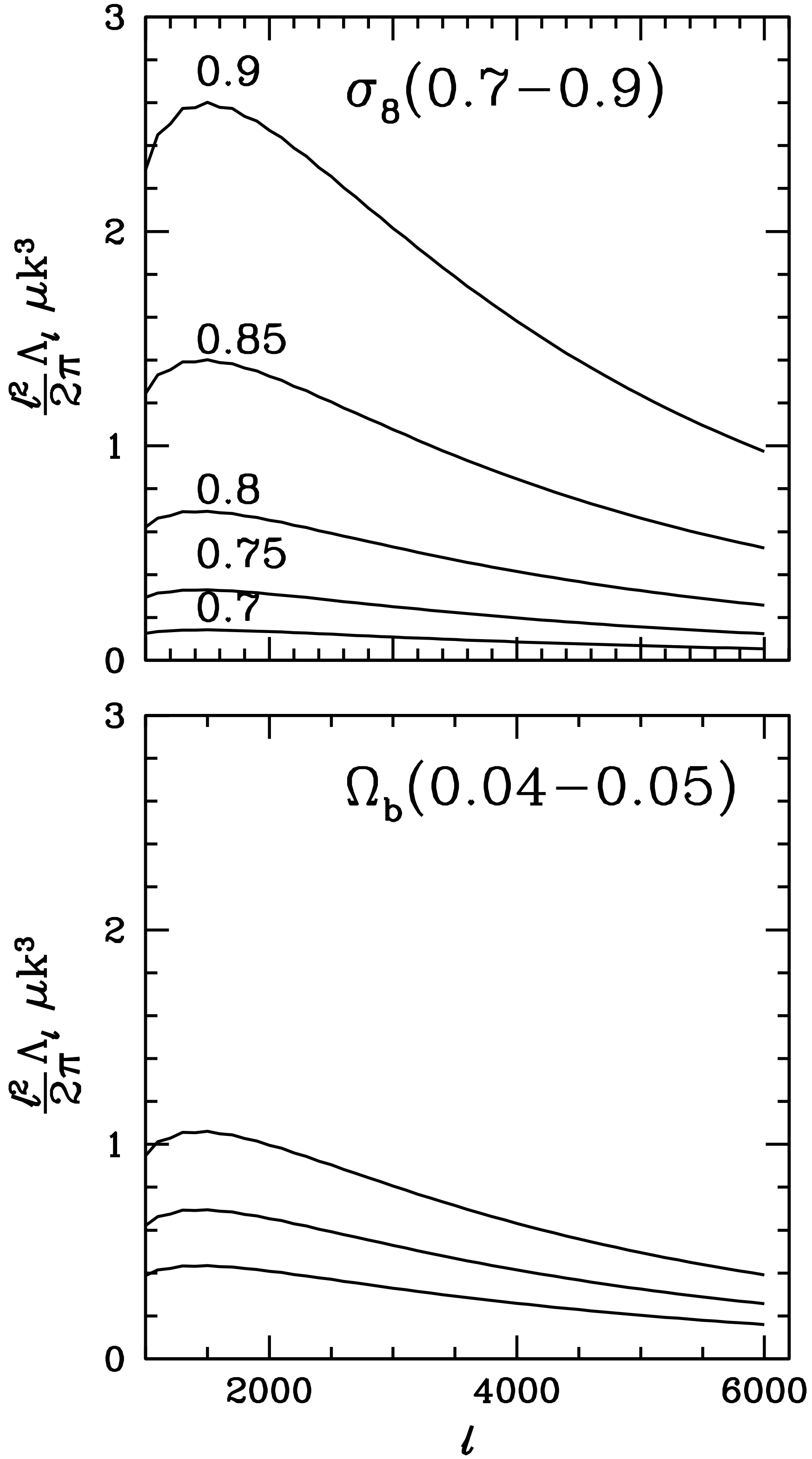
- SZ power spectrum gets about 30-50% signal from groups and high-z clusters vs. bispectrum gets only 5% signal from such objects.
- power spectrum measures a combination of kinetic+thermal SZ (and point sources) vs. kinetic SZ bispectrum =0, so bispectrum measures thermal SZ only.
- define the SZ skewness spectrum amplitude @ l=3000: $B_{sz} = \Lambda_{4000}(\sigma_8)/\Lambda_{4000}(0.8)$
- define the SZ power spectrum amplitude @l=3000:

$$A_{sz} = C_{3000}(\sigma_8)/C_{3000}(0.8)$$



- Asz-Bsz relation is extremely robust. Change in gas physics changes it only by 15%. The power spectrum over the same range changes by factor of 4.
- A combination of Asz and Bsz amplitude can break the degeneracy of the thermal and the kinetic SZ amplitude.

Cosmology dependence of SZ Bispectrum



Scaling of the bispectrum amplitude

$$B^{\Lambda}_{SZ} \propto \left(\frac{\sigma_8}{0.8}\right)^{11.4} \left(\frac{\Omega_b}{0.04}\right)^{4} \left(\frac{h}{0.71}\right)^{2} \left(\frac{w_0}{-1.0}\right)^{-0.95} \times \left(\frac{n_s}{0.96}\right)^{-1.5} \left(\frac{\Omega_m}{0.26}\right)^{-0.46}$$

Cosmic Complementarity->

=> SZ bispectrum contrains $\sigma_8\Omega_b$^0.36 complementary to clusters which constrains $\sigma_8\Omega_m$^0.4

=> mask out the clusters used in the SZ mass function:

=> measure bispectrum of the map=> constrain σ_8

=> independent of the cluster constraints

=> joint constraints from bispectrum +abundance

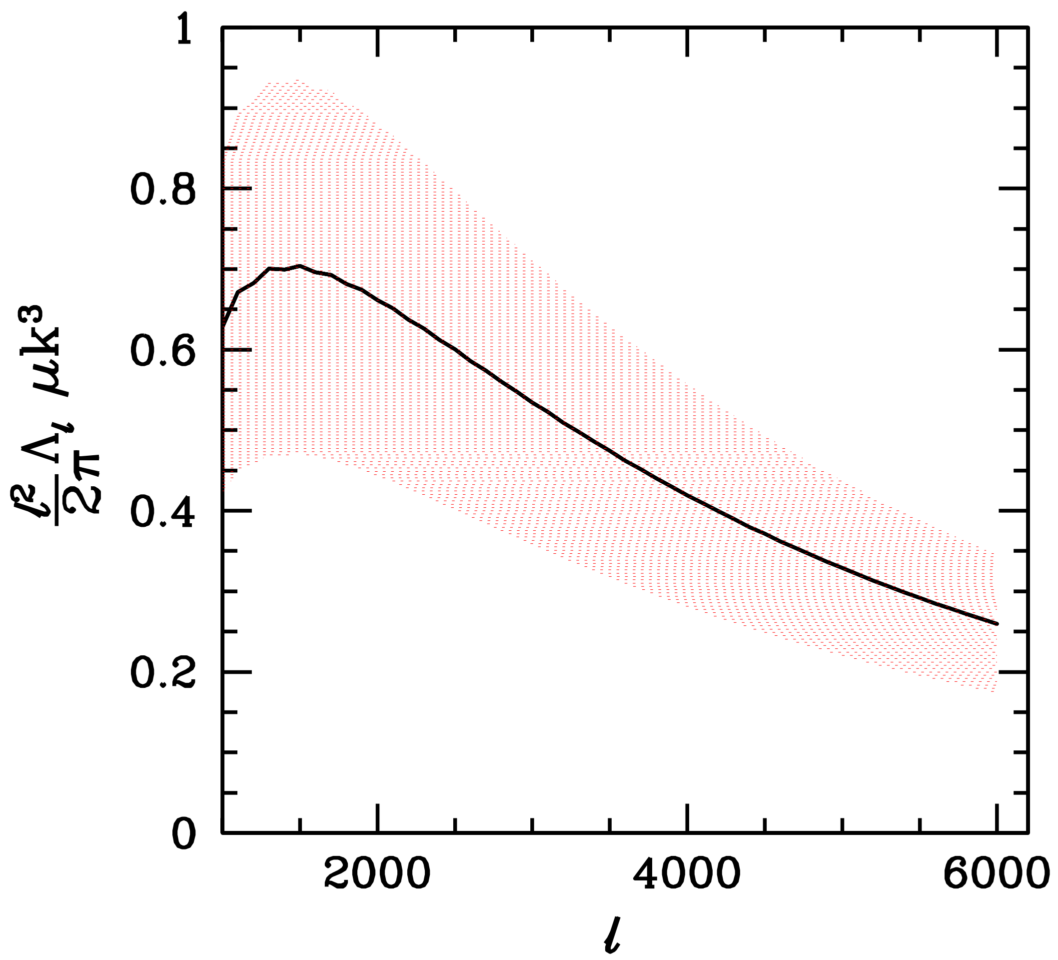
The SZ Skewness Spectrum

- define the SZ skewness spectrum as sum over the two smaller sides and expressed as a function of the largest l:

$$\Lambda(\ell) = \sqrt{\sum_{\ell_1 \ell_2} b^2(\ell \ell_1 \ell_2)}.$$

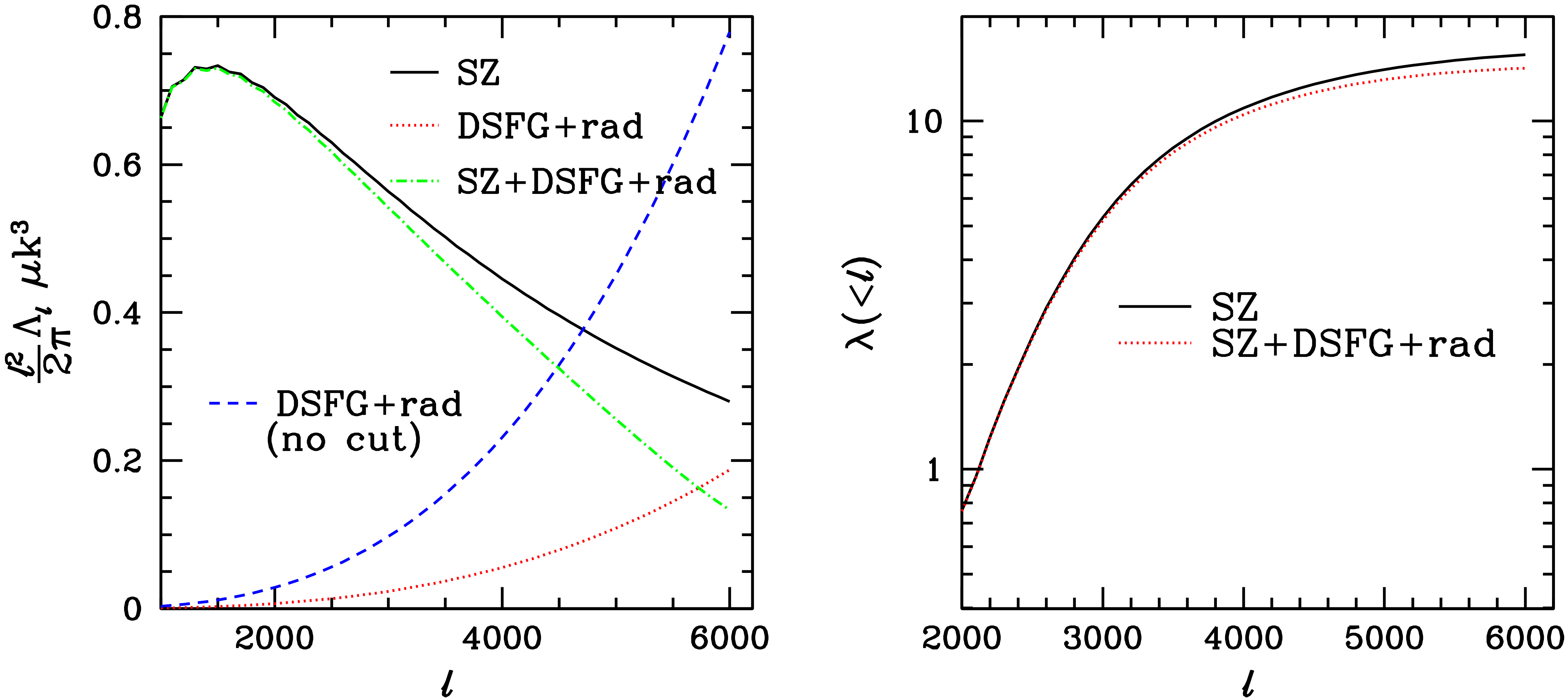
- and signal-to-noise integrated to certain l:

$$\lambda(< \ell) = \sqrt{\Sigma_{\ell_1}^{\ell} \Sigma_{\ell_2 \ell_3} \frac{b^2(\ell_1 \ell_2 \ell_3)}{N^2(\ell_1 \ell_2 \ell_3)}}$$

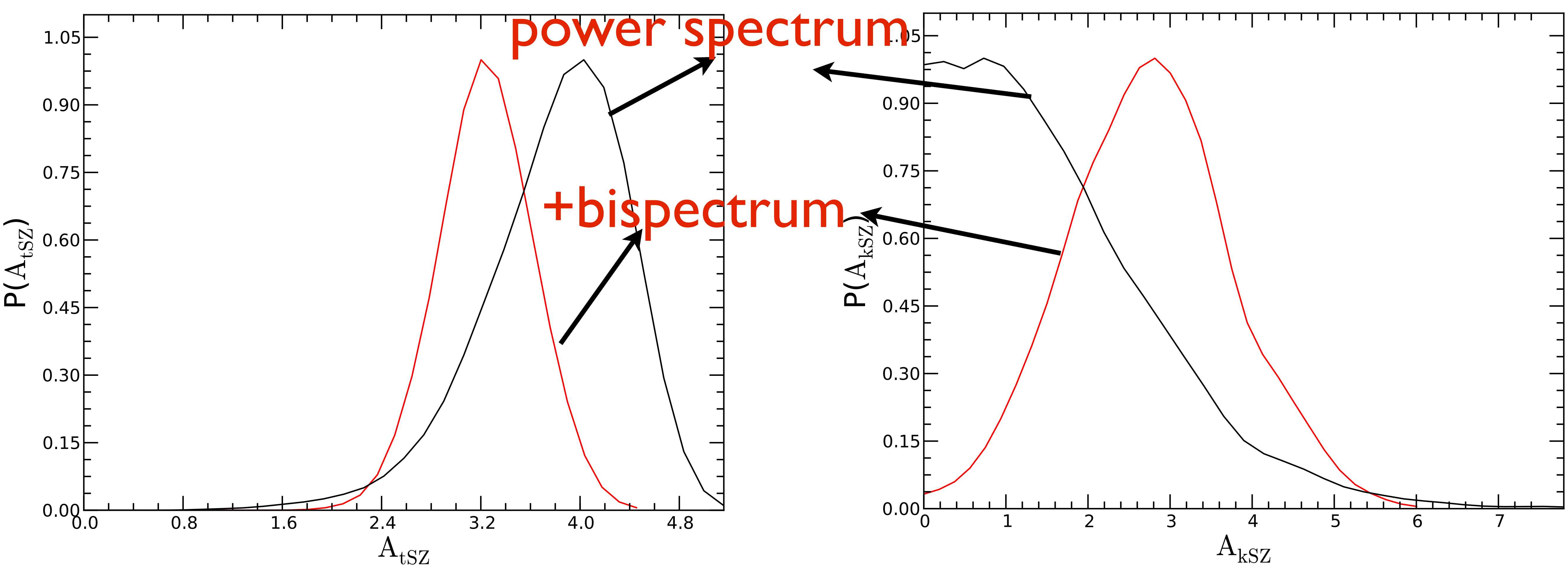


Measurement Prospects From Current Data

- @150 GHz, 1.2', 18 microk-arcmin, 2500 deg^2
- total S/N~16
- Bispectrum of point sources (dusty and radio galaxies) are a contamination (or signal to detect!)



What can we learn from bispectrum+ power spectrum combined



- Adding bispectrum to the power spectrum data improves the constraints on the thermal SZ amplitude by factor of 2.
- kinetic SZ amplitude can be detected at 2 sigma level

ICM parameter constraints from future experiments

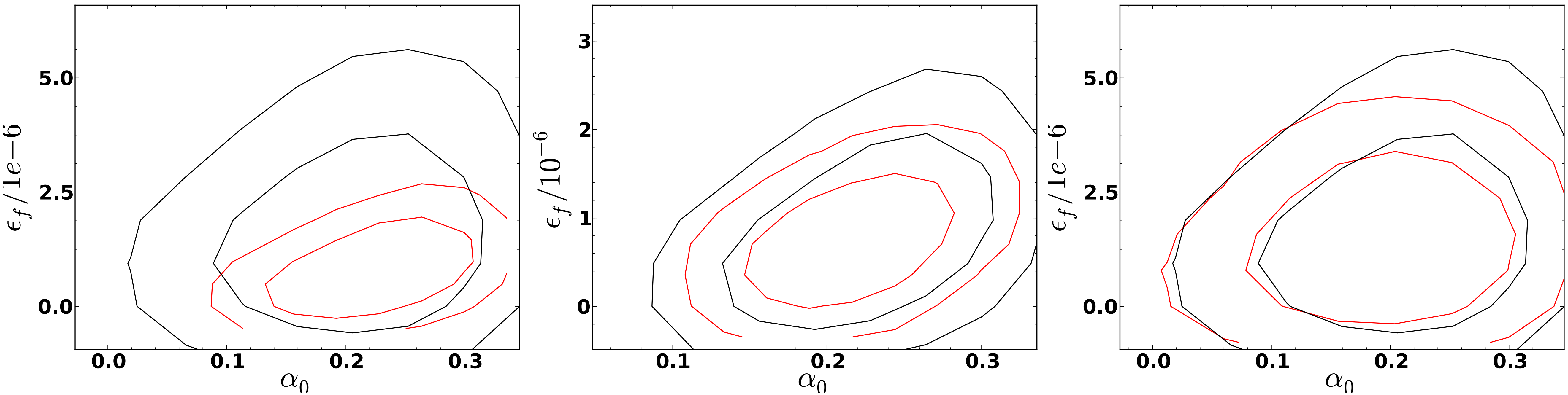
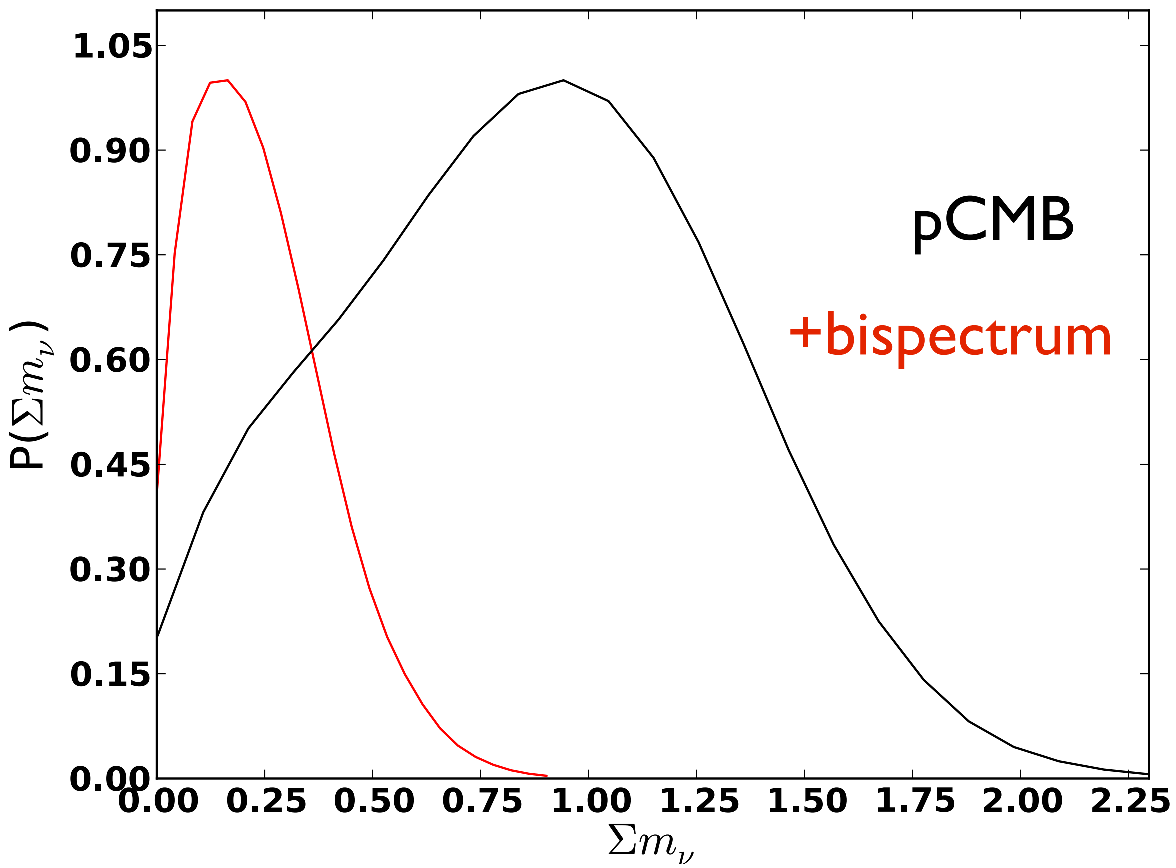
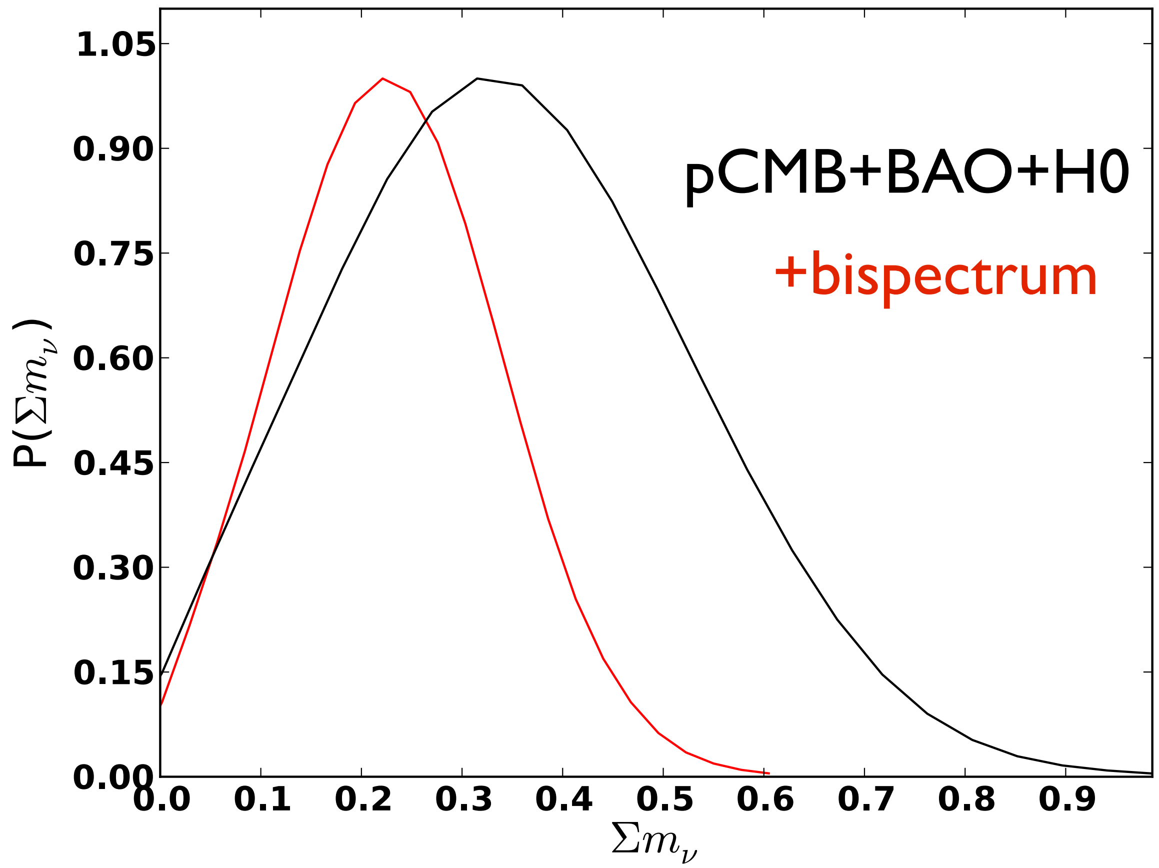


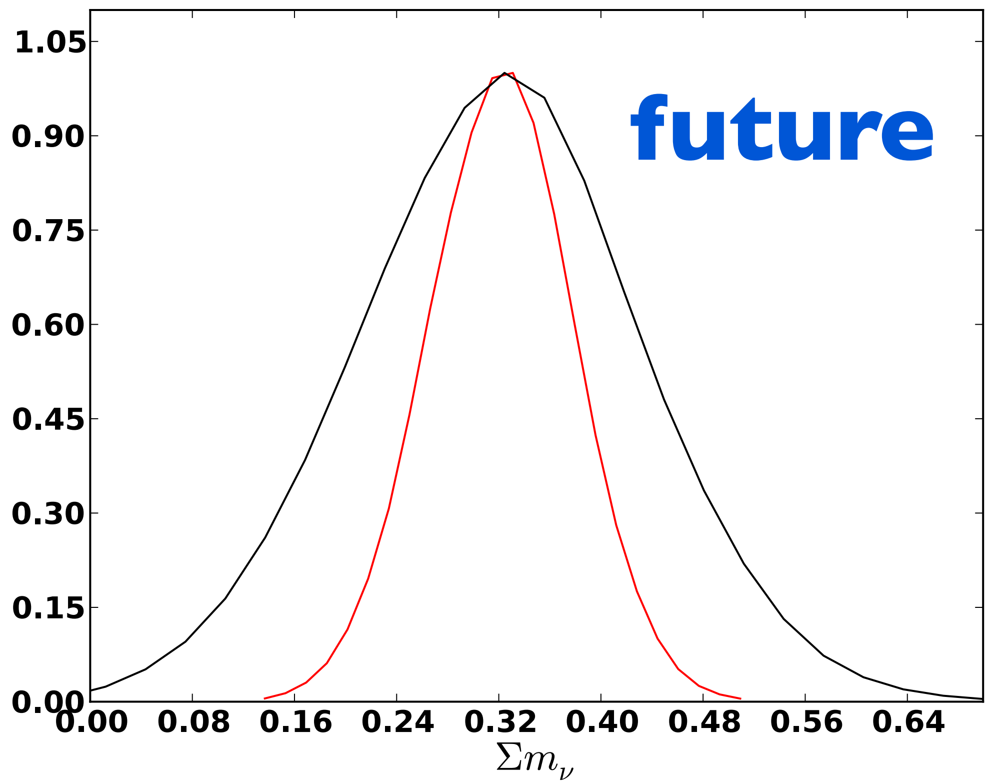
Table 5: Parameter constraints (1 (2)σ limits) from MCMC analysis of the power spectrum measurements

	CCAT (1 Khr, 2000 deg^2)		SUPERCCAT (100 Khr, 20,000 deg^2)	
survey	α_0	$\epsilon_f(10^{-6})$	α_0	$\epsilon_f(10^{-6})$
fix cosmo, medsz, fg	0.15– 0.23(0.13 – 0.29)	1.35– 4.45(0.36 – 5.8)	0.18-0.25(0.15 – 0.29)	0.34–1.17(0 – 1.58)
w7, medsz, fg	0.07– 0.25 (0.015– 0.29)	0.86– 3.8 (0.2 – 5.65)	0.17– 0.27(0.12 – 0.29)	0.24– 1.41(0 – 2.12)
w7, fg	0.073-0.25 (0–0.29)	1.6– 6.2(0.32 – 8.6)	0.14-0.27(0.07 –0.29)	0.4-2.9(0 – 4.56)
w7, nofg	0.075– 0.25(0.02 – 0.29)	0.67-4.0 (0.11 – 6.2)	0.13– 0.25 (0.067 – 0.29)	0.45– 2.5(0.07–3.61)

Constraints on the sum of the neutrino mass

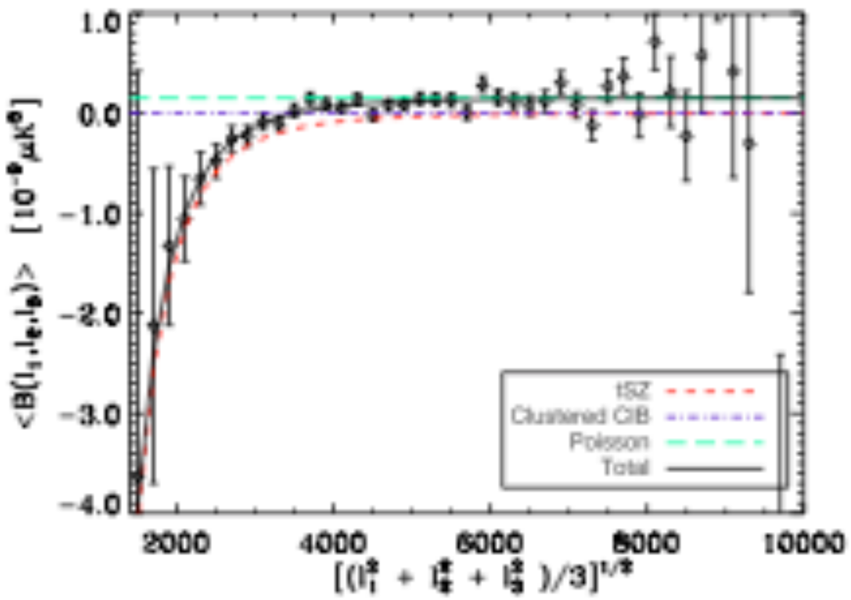




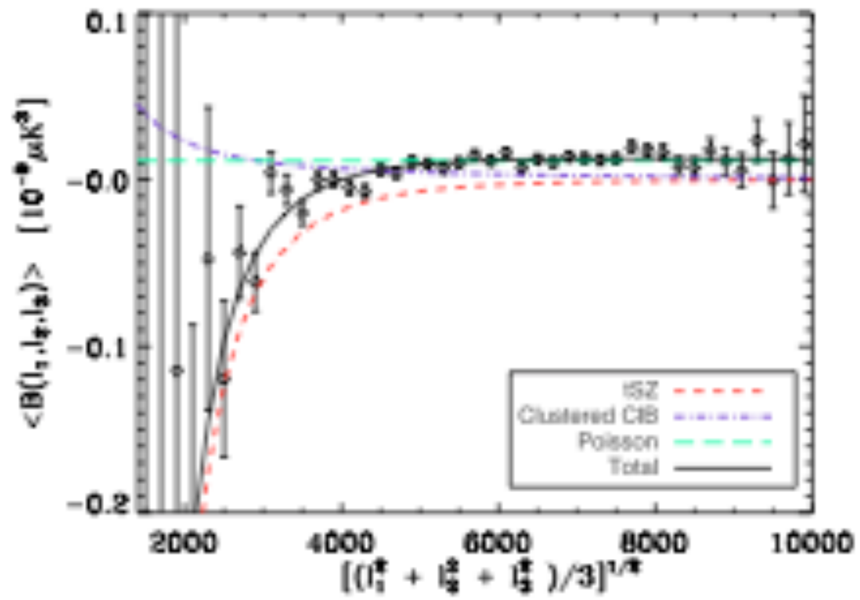


adding bispectrum
->improves neutrino mass constraints by factor of 3 compared to WMAP alone
-> and by about 50% compared to WMAP +BAO+H0
-> Future: adding bispectrum can constrain neutrino mass with 0.06-0.1 eV accuracy.

SPT Bispectrum Measurements

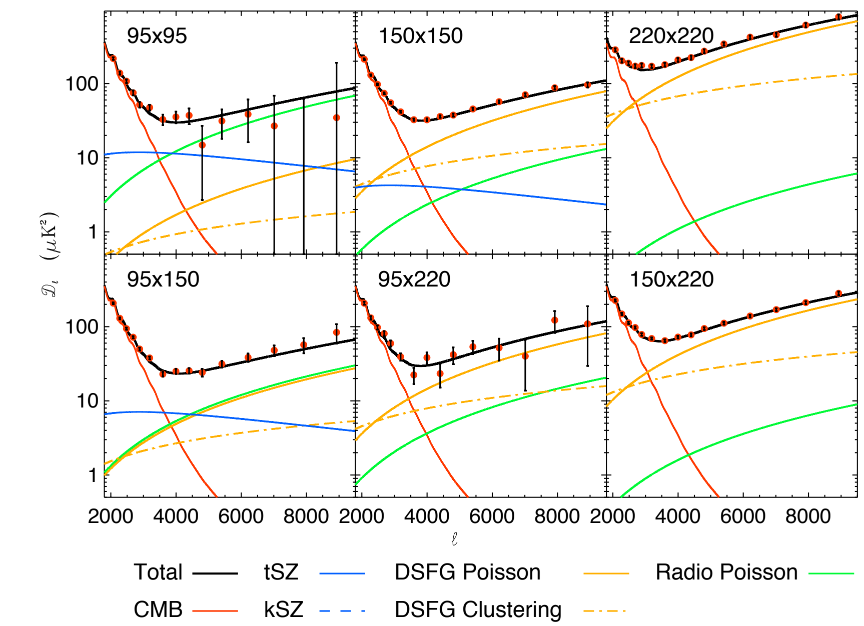


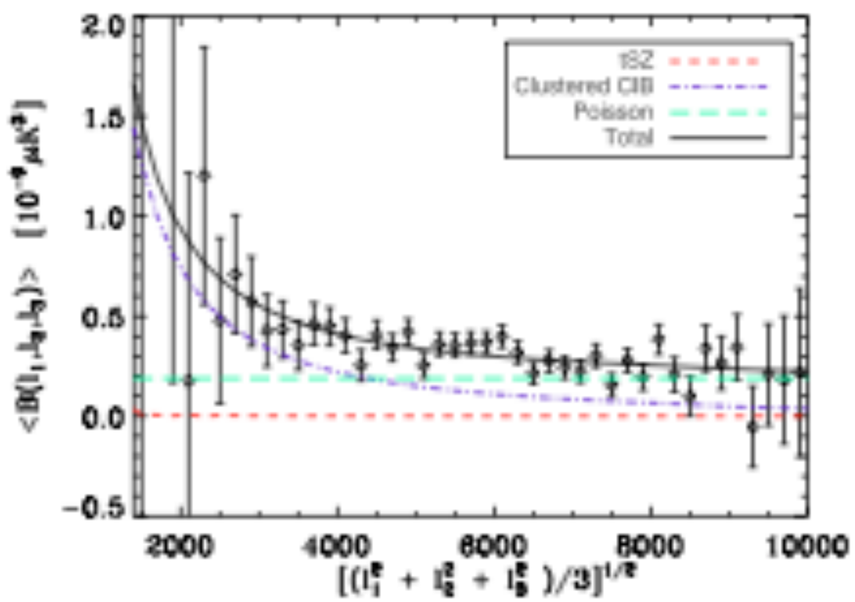
(a) 95 GHz 1d bispectrum, with best-fit model overplotted



(b) 150 GHz 1d bispectrum, with best-fit model overplotted

Crawford et al, SPT team





(c) 220 GHz 1d bispectrum, with best-fit model overplotted

=> SPT 3 frequency channels- 95, 150, 220 GHz
=> cover 800 sq. deg
=> detect SZ bispectrum to > 10 σ

Also , Wilson et al, ACT measure skewness ~5 σ
Planck measurements of SZ bispectrum

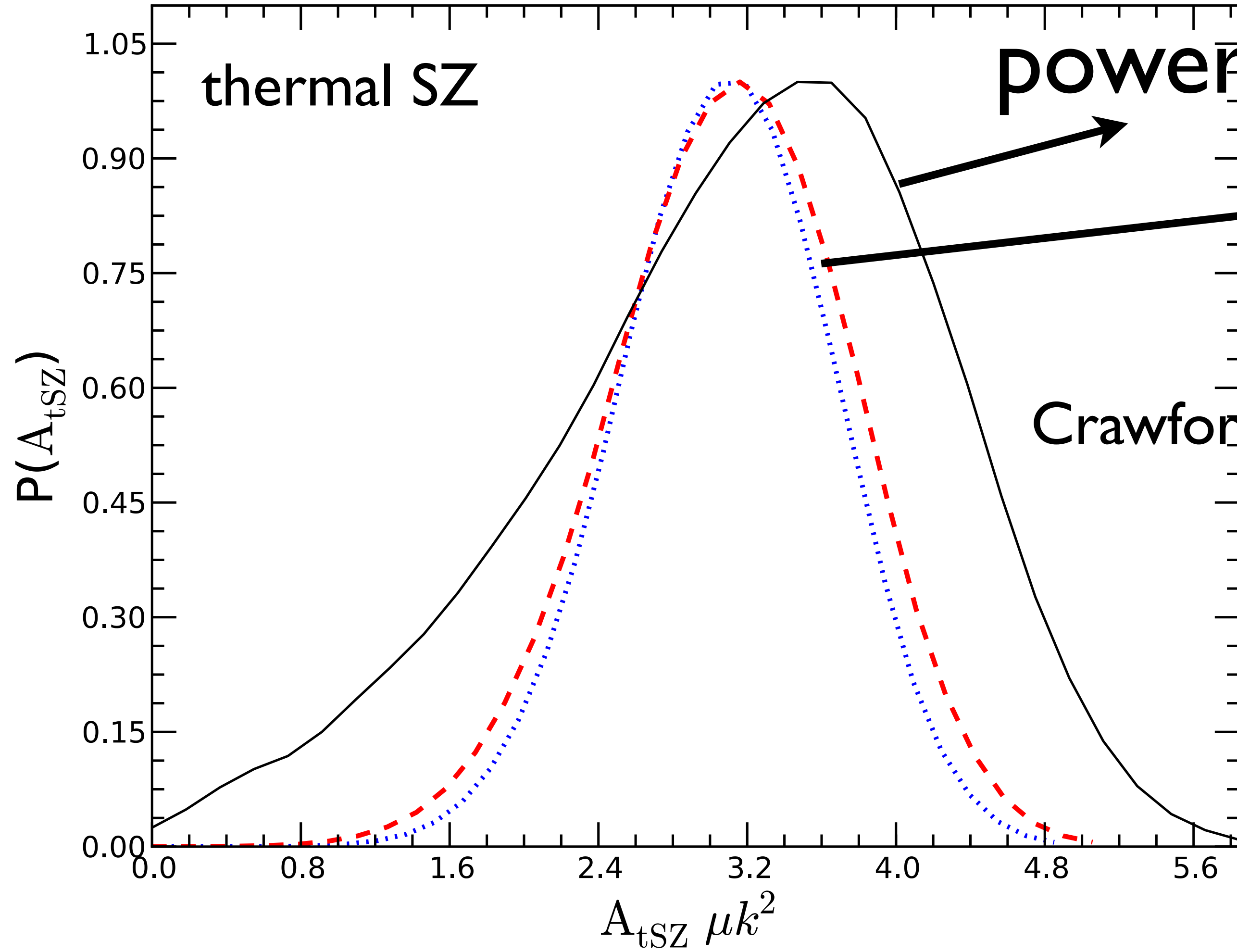
Current SPT measurements: thermal SZ and σ_8

two theoretical
modeling assumptions
10% (nominal) and 18% (extreme)

nominal- one that fits the X-ray data

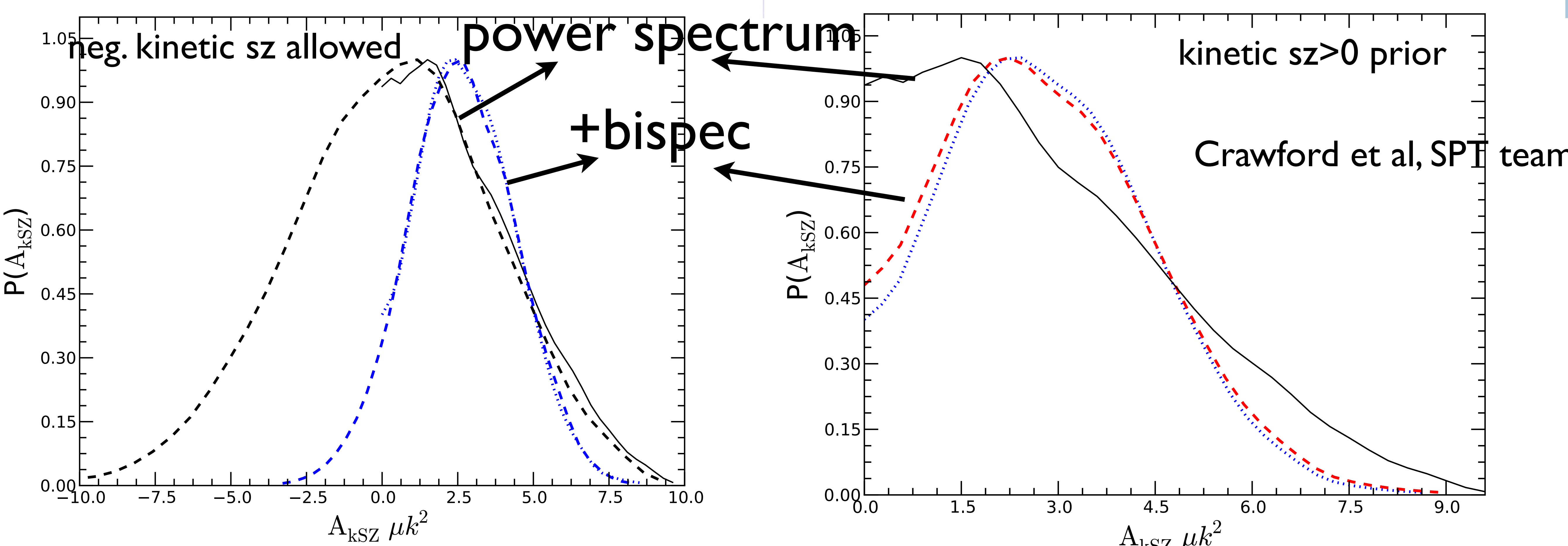
extreme- one that test the physical boundary of our model

power spectrum changes by factor~4 in the extreme scenario



=> bispectrum measures σ_8=0.79 +/- 0.031
=> combine the bispectrum and power spectrum measurements to individually measure tSZ and kSZ amplitude
=> tSZ amplitude 2.96 µk^2 +/- 0.642 (nominal) +/- 0.768 (extreme)
=> improves the tSZ amplitude by factor ~2 compared to the power spectrum only case (uncertainty~1.05 µk^2)

Current SPT measurements: kinetic SZ



95% upper limit for Aksz

=> power, neg. kinetic sz allowed = 6.50 μk^2

=> power+bispec, neg. kinetic sz allowed = 5.57 μk^2

=> power, Aksz>0 prior= 6.68 μk^2

=> power+bispec, Aksz>0 prior(nominal)= 5.59 μk^2

=> power+bispec, Aksz>0 prior(extreme)= 5.73 μk^2

=> detect Aksz =2.9 +/- 1.5 μk^2

=> Adding bispectrum improves upper limit of the kSZ amplitude by ~15% and we start to see the peak (Aksz > 0 prior case)

Science Cases:

- SZ bispectrum is a new (and powerful) technique to measure the thermal SZ amplitude
- More robust than power spectrum-> signal comes from massive clusters, theoretical uncertainty less compared to power spectrum, kSZ bispectrum is approximately 0, point sources bispectrum is comparable
- A combination of bispectrum+power spectrum measurement can improve the measurement of thermal and kinetic SZ amplitude individually.
- A measurement of the kSZ amplitude can provide useful insight to the reionization epoch.