

ISOCURVATURE PERTURBATIONS AND REHEATING

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Based on work with Ian Huston, [arXiv:1111.6919](https://arxiv.org/abs/1111.6919) and [arXiv:1302.4298](https://arxiv.org/abs/1302.4298)



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INFLATION: A VERY BRIEF
REVIEW

INFLATION

- ‘Invented’ to solve problems of the big bang:
 - Horizon problem
 - Monopole problem
 - Flatness problem
- Period of rapid accelerated expansion in the early universe

$$\text{Inflation} \iff \ddot{a} > 0 \iff P < -\frac{1}{3}\rho$$

INFLATION

- Accelerated expansion driven by scalar field

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

- Homogeneous field:

$$P_\varphi = \frac{1}{2} \dot{\varphi}^2 - V(\varphi)$$

$$\rho_\varphi = \frac{1}{2} \dot{\varphi}^2 + V(\varphi)$$

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accelerated expansion

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accelerated expansion

Gives primordial spectrum of density perturbations!

WHICH MODEL?

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

'simplest' models, specify potential



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increasing
complexity



etc...

Zero Parameter Models

- 3.1 Higgs Inflation (HI)

One Parameter Models

- 4.1 Radiatively Corrected Higgs Inflation (RCHI)
- 4.2 Large Field Inflation (LFI)
- 4.3 Mixed Large Field Inflation (MLFI)
- 4.4 Radiatively Corrected Massive Inflation (RCMI)
- 4.5 Radiatively Corrected Quartic Inflation (RCQI)
- 4.6 Natural Inflation (NI)
- 4.7 Exponential SUSY Inflation (ESI)
- 4.8 Power Law Inflation (PLI)
- 4.9 Kähler Moduli Inflation I (KMII)
- 4.10 Horizon Flow Inflation at first order (HF1I)
- 4.11 Coleman-Weinberg Inflation (CWI)
- 4.12 Loop Inflation (LI)
- 4.13 $(R + R^{2p})$ Inflation (RpI)
- 4.14 Double-Well Inflation (DWI)
- 4.15 Mutated Hilltop Inflation (MHI)
- 4.16 Radion Gauge Inflation (RGI)
- 4.17 MSSM Inflation (MSSMI)
- 4.18 Renormalizable Inflection Point Inflation (RIPI)
- 4.19 Arctan Inflation (AI)
- 4.20 Constant n_s A Inflation (CNAI)
- 4.21 Constant n_s B Inflation (CNBI)
- 4.22 Open String Tachyonic Inflation (OSTI)
- 4.23 Witten-O'Raifeartaigh Inflation (WRI)

Two Parameters Models

- 5.1 Small Field Inflation (SFI)
- 5.2 Intermediate Inflation (II)
- 5.3 Kähler Moduli Inflation II (KMIII)
- 5.4 Logamediate Inflation (LMI)
- 5.5 Twisted Inflation (TWI)
- 5.6 Generalized MSSM Inflation (GMSSMI)

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- 5.7 Generalized Renormalizable Point Inflation (GRIPI)
- 5.8 Brane SUSY breaking Inflation (BSUSYBI)
- 5.9 Tip Inflation (TI)
- 5.10 β exponential inflation (BEI)
- 5.11 Pseudo Natural Inflation (PSNI)
- 5.12 Non Canonical Kähler Inflation (NCKI)
- 5.13 Constant Spectrum Inflation (CSI)
- 5.14 Orientifold Inflation (OI)
- 5.15 Constant n_s C Inflation (CNCI)
- 5.16 Supergravity Brane Inflation (SBI)
- 5.17 Spontaneous Symmetry Breaking Inflation (SSBI)
- 5.18 Inverse Monomial Inflation (IMI)

Three parameters Models

- 6.1 Running-mass Inflation (RMI)
- 6.2 Valley Hybrid Inflation (VHI)
- 6.3 Dynamical Supersymmetric Inflation (DSI)
- 6.4 Generalized Mixed Inflation (GMLFI)
- 6.5 Logarithmic Potential Inflation (LPI)
- 6.6 Constant n_s D Inflation (CNDI)

Only single field models!

ISOCURVATURE/ENTROPY/
NON-ADIABATIC PRESSURE
PERTURBATIONS

ISOCURVATURE/ENTROPY/NON-ADIABATIC PRESSURE PERTURBATIONS

- Adiabatic system $\frac{\delta P}{\dot{P}} = \frac{\delta \rho}{\dot{\rho}}$
- Breaking requirement of adiabaticity allows the pressure perturbation to be expanded as

$$\frac{\delta P}{\dot{P}} \neq \frac{\delta \rho}{\dot{\rho}} \quad \longrightarrow \quad \delta P = \frac{\dot{P}}{\dot{\rho}} \delta \rho + \dots$$

- The non-adiabatic pressure perturbation can then be defined as $\delta P_{\text{nad}} = \delta P - c_s^2 \delta \rho$

ENTROPY PERTURBATIONS

- Thus require a fluid with equation of state depending on two independent variables, i.e.

$$P \equiv P(\rho, S)$$

- Inflationary fields can be written as fluids:
 - single field has barotropic equation of state and so, on large scales, no isocurvature
 - multiple fields, entropy perturbations can be non-zero, and may be important

WHY ISOCURVATURE? 1: VORTICITY

- In classical fluid dynamics, vorticity: $\vec{\omega} = \vec{\nabla} \times \vec{u}$
- Evolves according to

$$\frac{\partial \omega}{\partial t} = \nabla \times (\mathbf{v} \times \boldsymbol{\omega}) + \frac{1}{\rho^2} \nabla \rho \times \nabla P$$

- ‘Source term’ zero if $\vec{\nabla} P$ and $\vec{\nabla} \rho$ are parallel
 - i.e. barotropic fluid, no source term
 - including entropy allows for a source term

VORTICITY IN COSMOLOGY

- Relativistic generalisation, consider vorticity tensor in a perturbed FLRW universe
- Linear perturbations have no source term and decay with expansion

$$\omega'_{1ij} - 3\mathcal{H}c_s^2 w_{1ij} = 0$$

Kodama & Sasaki (1984)

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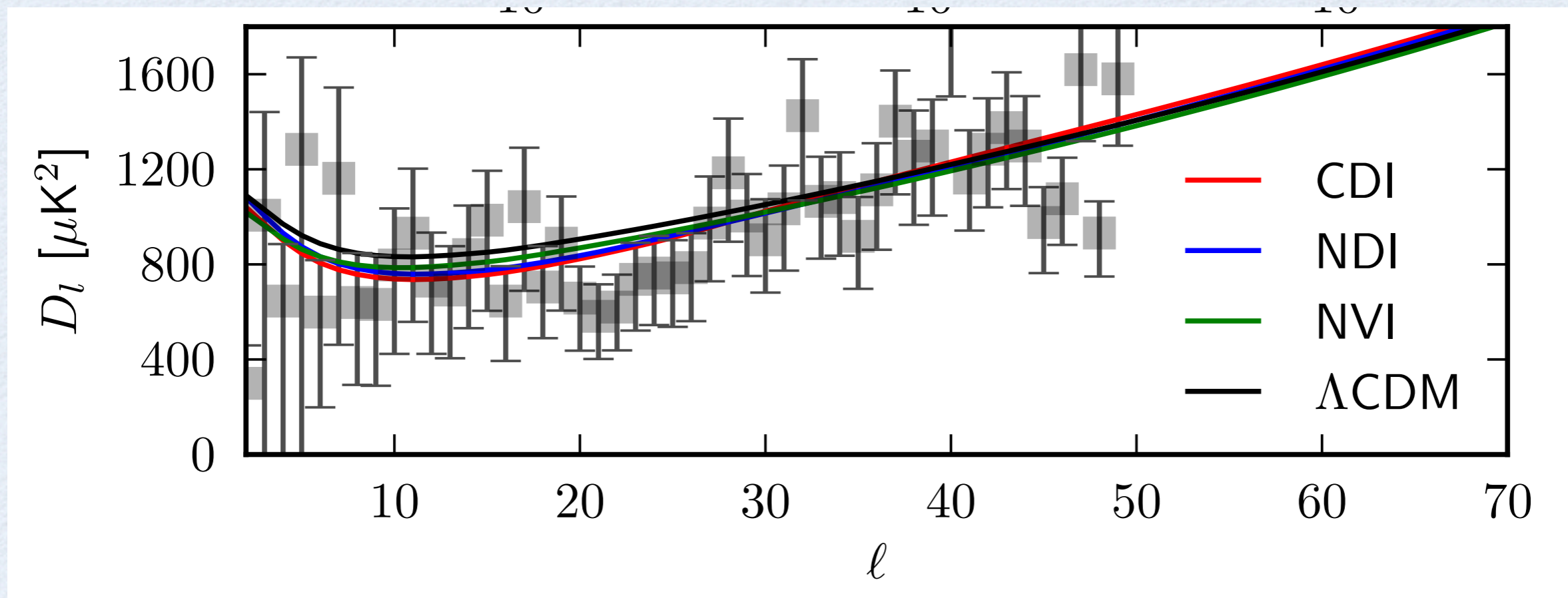
Kodama & Sasaki (1984)

- Beyond linear theory source term depends upon the entropy perturbations

$$\omega'_{2ij} - 3\mathcal{H}c_s^2\omega_{2ij} = \frac{2a}{\rho_0 + P_0} \left\{ 3\mathcal{H}V_{1[i}\delta P_{\text{nad}1,j]} + \frac{\delta\rho_{1,[j}\delta P_{\text{nad}1,i]}}{\rho_0 + P_0} \right\}$$

AJC, Malik & Matravars (2009)

WHY ISOCURVATURE? 2: CMB?



ISOCURVATURE PERTURBATIONS

- We consider multi-field inflationary models with Lagrangian

$$\mathcal{L}(\varphi, \chi) = \frac{1}{2}(\dot{\varphi}^2 + \dot{\chi}^2) + U(\varphi, \chi)$$

with double quartic, quadratic and product exponential potentials

- Study isocurvature perturbations
 - after inflation
 - after period of perturbative reheating

MULTI-FIELD INFLATION

- Double quadratic

$$V(\varphi, \chi) = \frac{1}{2}m_\varphi^2\varphi^2 + \frac{1}{2}m_\chi^2\chi^2$$

- Double quartic

$$V(\varphi, \chi) = \Lambda^4 \left[\left(1 - \frac{\chi_0^2}{v^2} \right)^2 + \frac{\varphi^2}{\mu^2} + \frac{2\varphi^2\chi^2}{\varphi_c^2 v^2} \right]$$

Kodama et al. (2011)
Avgoustidis et. al. (2011)

- Product exponential

$$V(\varphi, \chi) = V_0\varphi^2 e^{-\lambda\chi^2}$$

Byrnes, Choi & Hall (2008)

- Solve background field dynamics and linear perturbations numerically $\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0$

- Compute non-adiabatic pressure perturbation

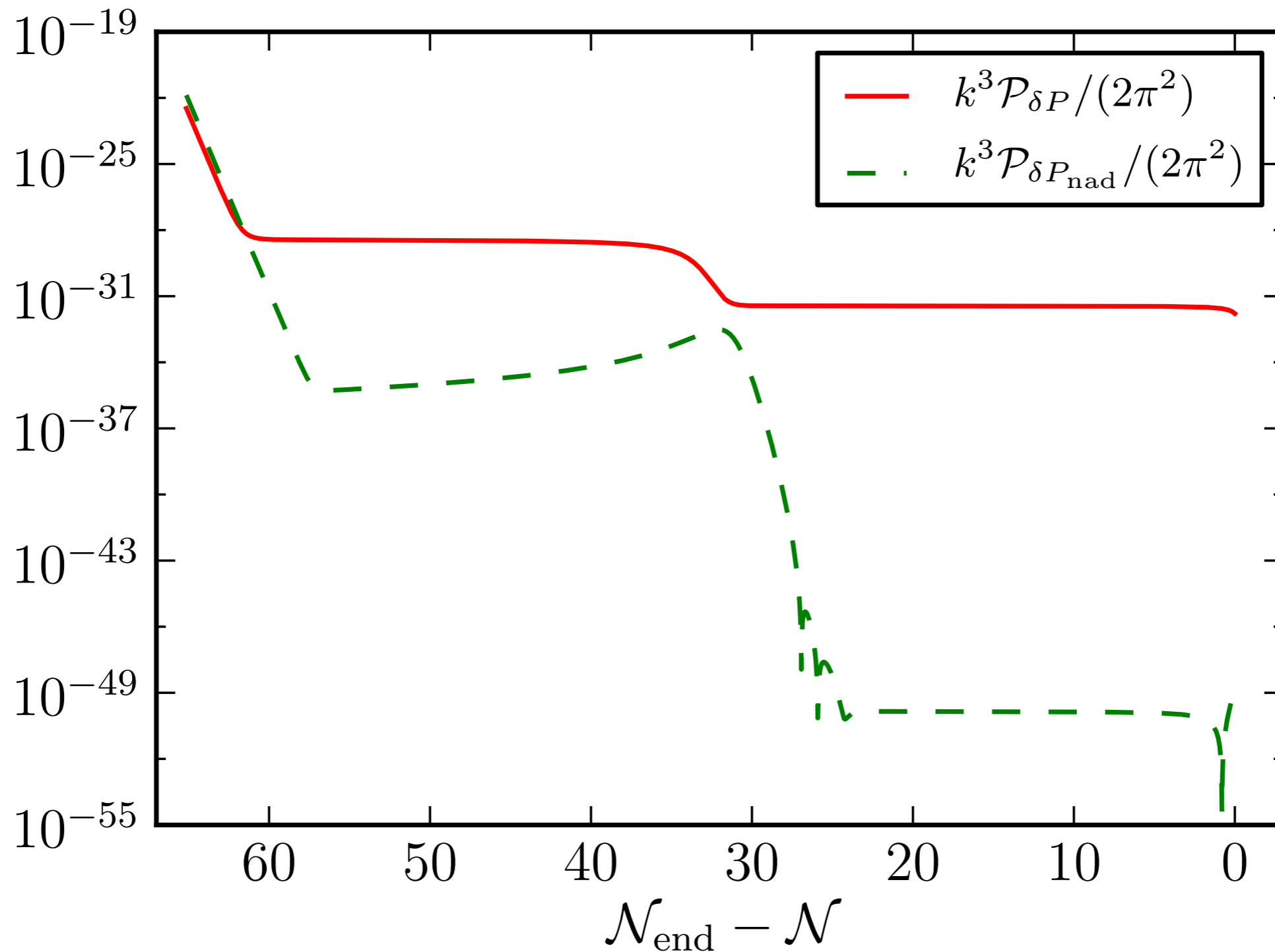
$$\delta P_{\text{nad}} = \delta P - c_s^2 \delta \rho$$

by defining the pressure and energy density perturbation of the scalar fields as

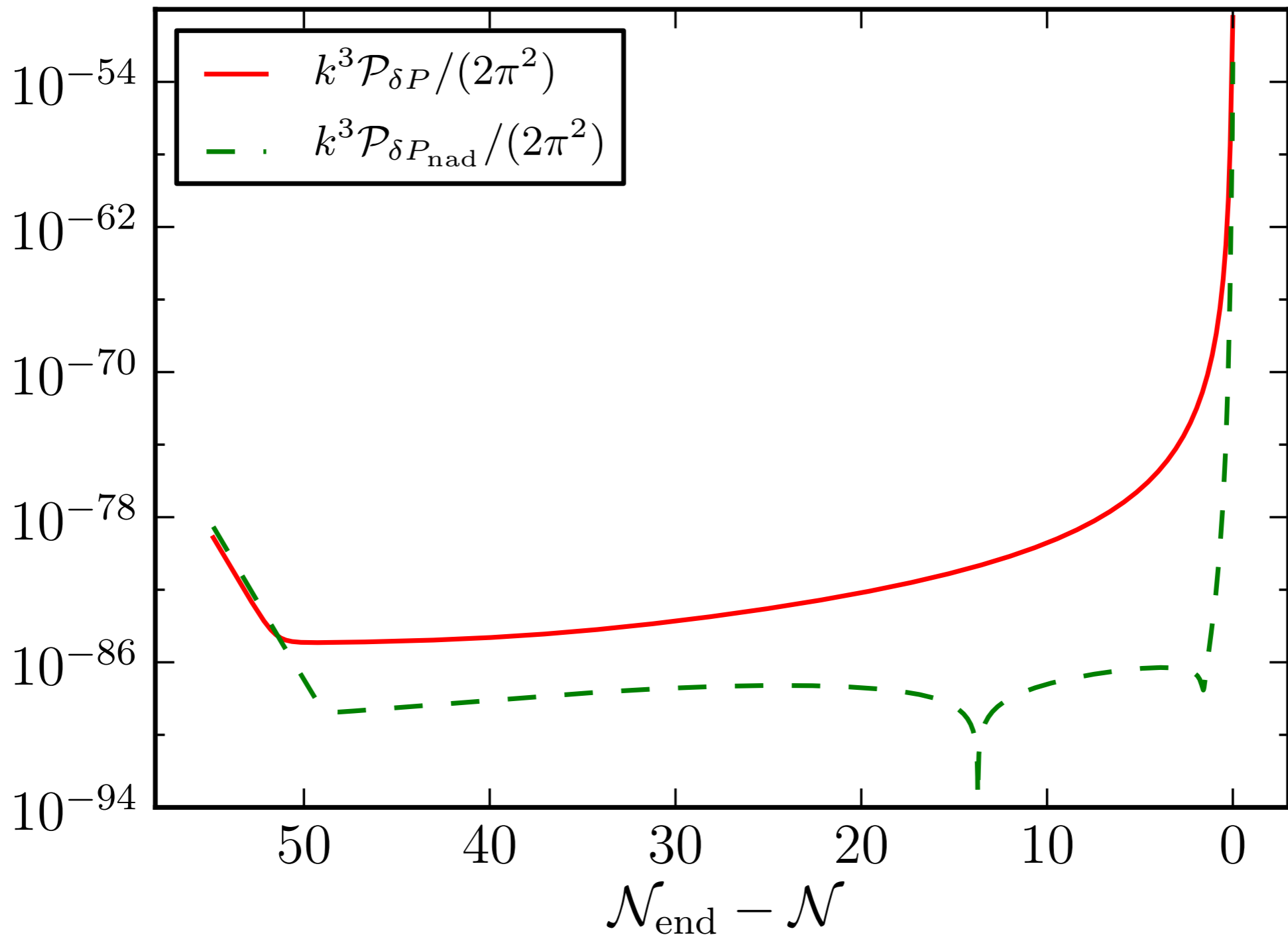
$$\delta P = \sum_{\alpha} \left(\dot{\varphi}_{\alpha} \delta \dot{\varphi}_{\alpha} - \dot{\varphi}_{\alpha}^2 \phi - V_{,\alpha} \delta \varphi_{\alpha} \right)$$

$$\delta \rho = \sum_{\alpha} \left(\dot{\varphi}_{\alpha} \delta \dot{\varphi}_{\alpha} - \dot{\varphi}_{\alpha}^2 \phi + V_{,\alpha} \delta \varphi_{\alpha} \right)$$

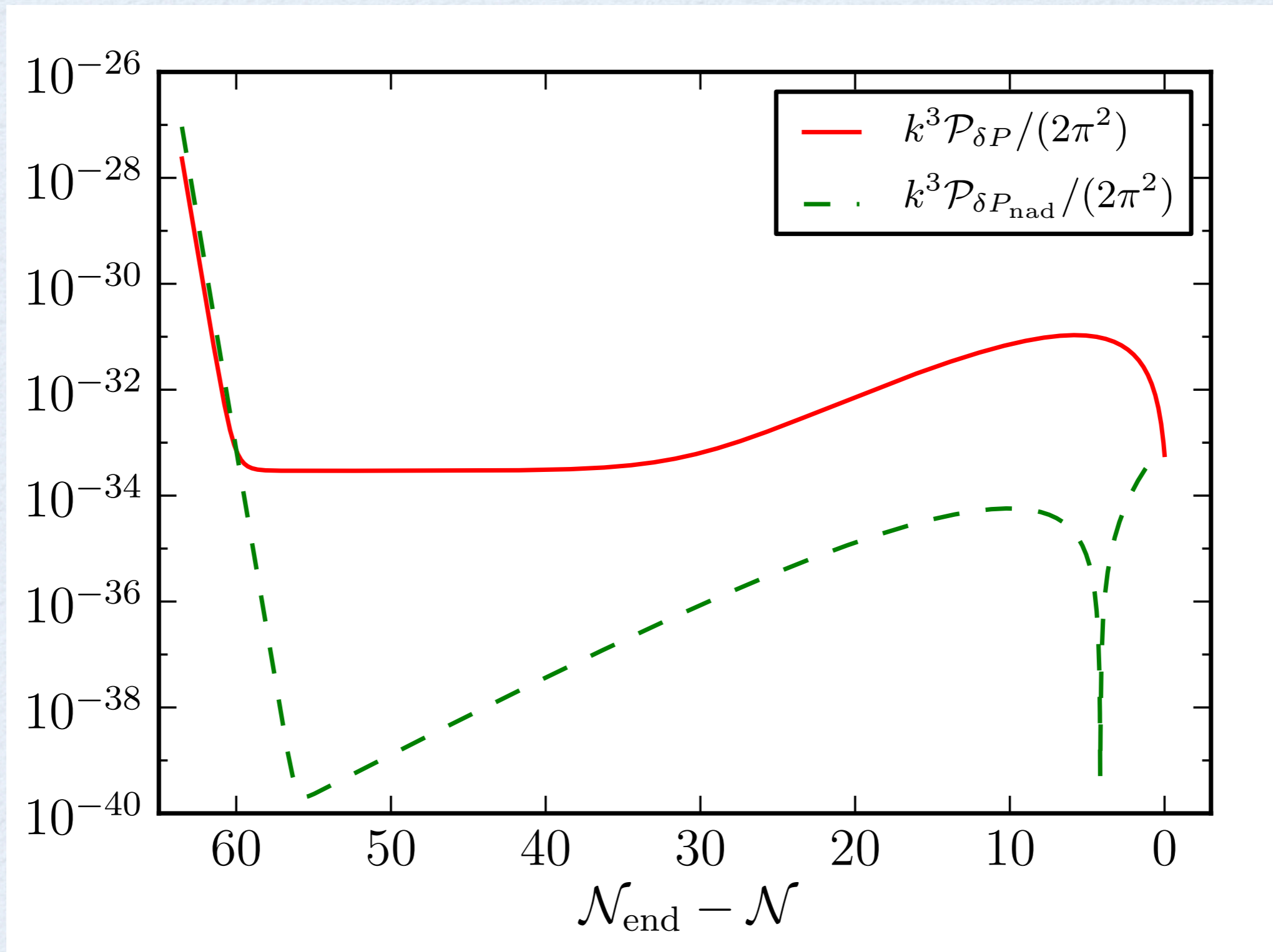
DOUBLE QUADRATIC



DOUBLE QUARTIC



PRODUCT EXPONENTIAL



REHEATING

- How do the scalar fields driving inflation decay into the standard model?
- Include an additional friction term in the scalar field equation $\ddot{\varphi} + (3H + \frac{1}{2}\Gamma)\dot{\varphi} + U_{,\varphi} = 0$
- Usually, one considers the decay of the fields into radiation
- In order to allow isocurvature to survive, consider decay into **both** radiation and matter, subject to constraint $\frac{\Gamma_m}{\Gamma_\gamma} < 10^{-6}$

- Evolution equations from energy-momentum conservation $\nabla_{\mu} T^{\mu}_{(\alpha)\nu} = Q_{\nu}^{(\alpha)}$

- Fluid equations

$$\dot{\rho}_{\gamma} + 4H\rho_{\gamma} = \frac{1}{2} \left(\Gamma_{\gamma}^{\varphi} \dot{\varphi}^2 + \Gamma_{\gamma}^{\chi} \dot{\chi}^2 \right)$$

$$\dot{\rho}_{\text{m}} + 3H\rho_{\text{m}} = \frac{1}{2} \left(\Gamma_{\text{m}}^{\varphi} \dot{\varphi}^2 + \Gamma_{\text{m}}^{\chi} \dot{\chi}^2 \right)$$

- Field equations

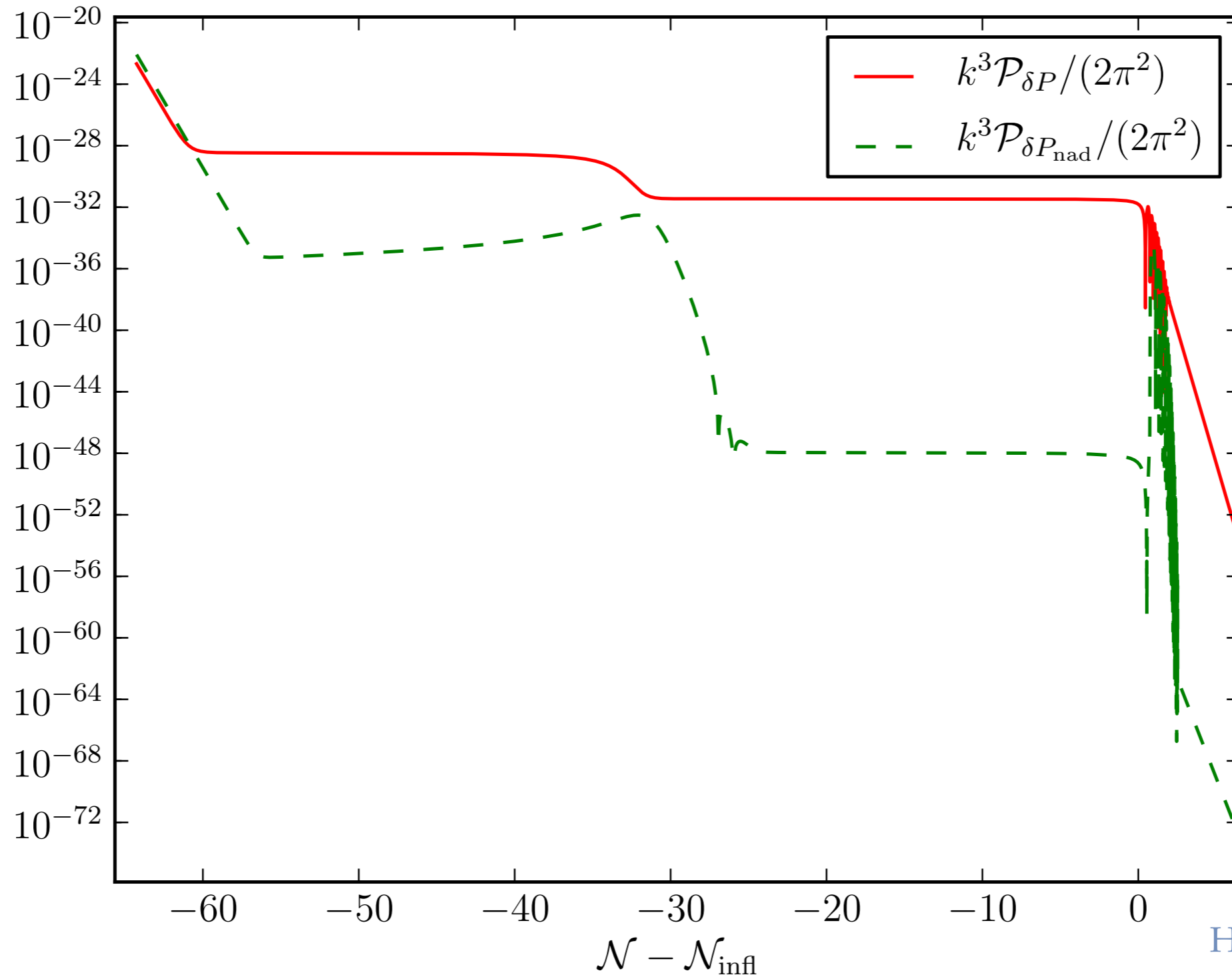
$$\ddot{\varphi} + \left[3H + \frac{1}{2} \left(\Gamma_{\gamma}^{\varphi} + \Gamma_{\text{m}}^{\varphi} \right) \right] \dot{\varphi} + U_{,\varphi} = 0$$

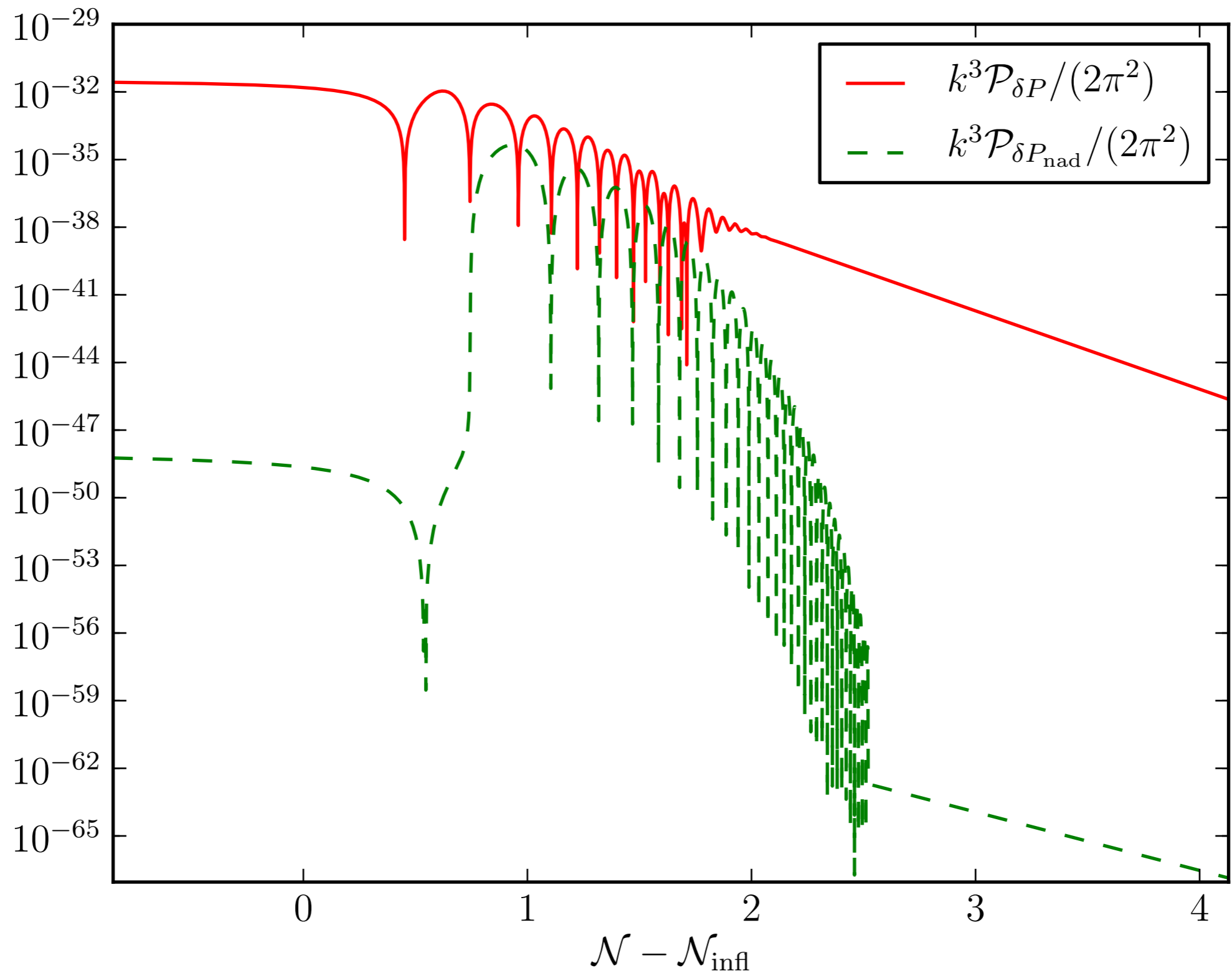
- And similarly (though more complicated) for perturbations

- Solve the evolution equations for the scalar field and matter / radiation fluids
- Again, non-adiabatic pressure perturbation is computed, defined as $\delta P_{\text{nad}} = \delta P - c_s^2 \delta \rho$
- Now, contribution from the scalar fields, but also from the fluids, i.e.

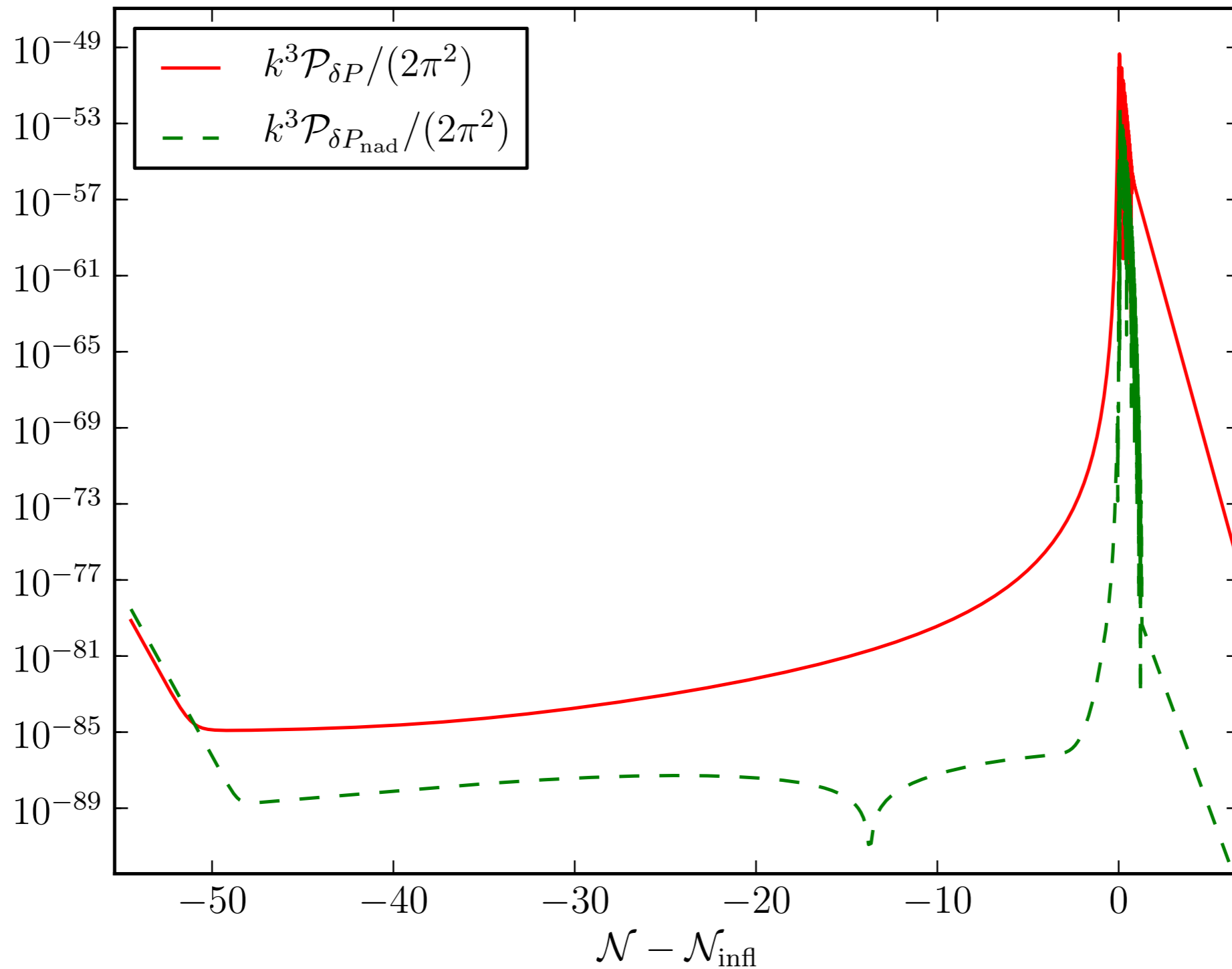
$$\delta P_{\text{nad}} = \frac{1}{3} \delta \rho_\gamma \left[1 - \frac{\dot{\rho}_\gamma}{\dot{\rho}_m + \dot{\rho}_\gamma} \right] - \frac{1}{3} \frac{\dot{\rho}_\gamma \delta \rho_m}{\dot{\rho}_m + \dot{\rho}_\gamma}$$

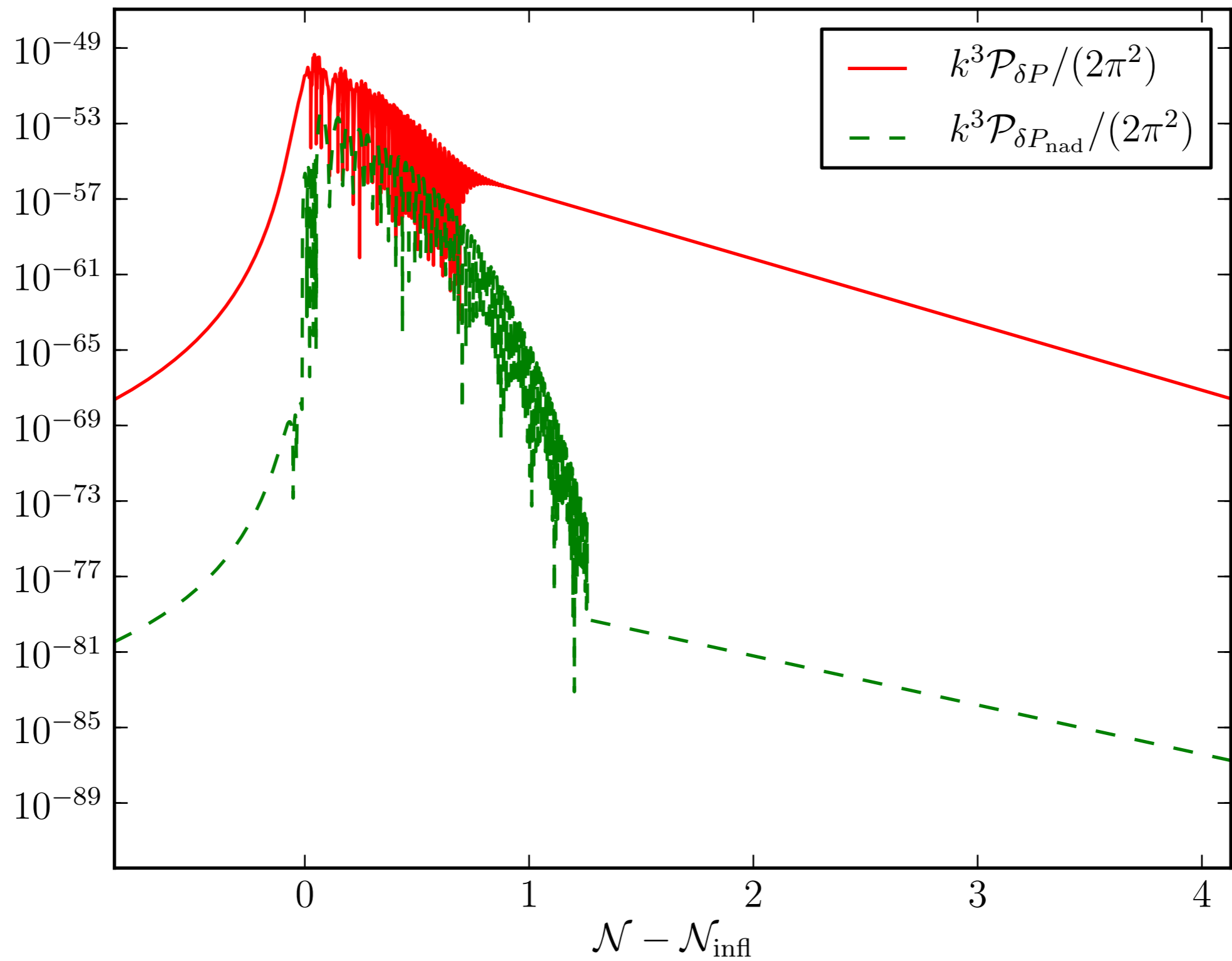
DOUBLE QUADRATIC INFLATION



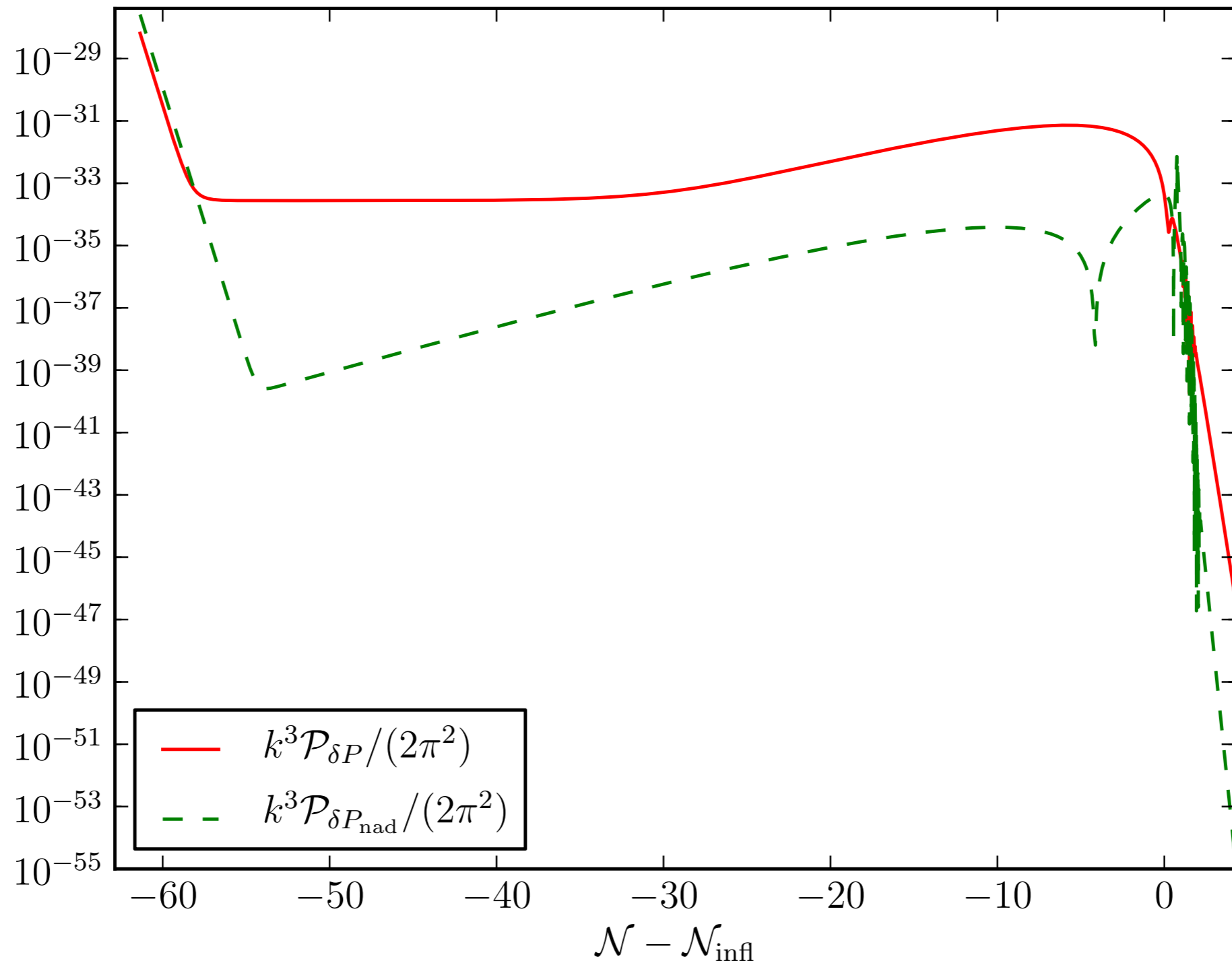


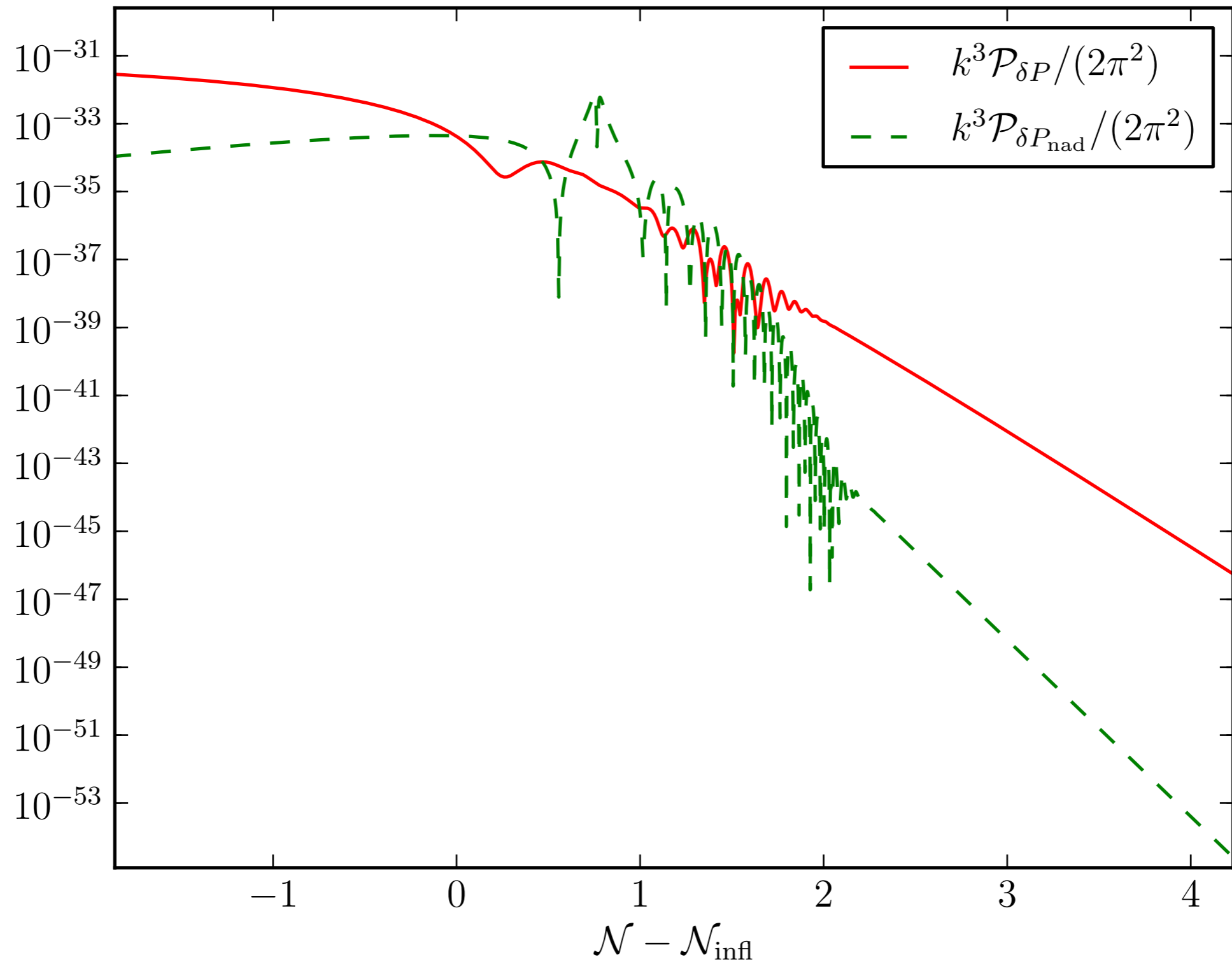
DOUBLE QUARTIC INFLATION





PRODUCT EXPONENTIAL





SUMMARY

- Isocurvature perturbations common in multi-field models; can source source vorticity?
- Reheating should be taken into account when isocurvature perturbations are present
- Non-adiabatic pressure perturbation subdominant in all three models studied

Huston & AJC, PRD 85, 063507 (2012)

Huston & AJC, 1302.4298 (2013)

- BUT parameter choice conservative: could we saturate the bounds to source isocurvature?