Isocurvature Perturbations and Reheating

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Based on work with Ian Huston, arXiv:1111.6919 and arXiv:1302.4298

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11th July 2013

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Inflation: A Very Brief Review

INFLATION

- 'Invented' to solve problems of the big bang:
  - Horizon problem
  - Monopole problem
  - Flatness problem
- Period of rapid accelerated expansion in the early universe

$$\text{Inflation} \iff \ddot{a} > 0 \iff P < -\frac{1}{3}\rho$$

INFLATION

- Accelerated expansion driven by scalar field

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

- Homogeneous field:

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$
$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

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accelerated expansion

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accelerated expansion

Gives primordial spectrum of density perturbations!

WHICH MODEL?

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

'simplest' models, specify potential

Which Model?

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$$\mathcal{L}_\varphi = L\left(\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi\right) - V(\varphi)$$

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$$\mathcal{L}_\varphi = L\Big(\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi, \varphi\Big)$$

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$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} (\partial_\mu \varphi \partial_\nu \varphi + \partial_\mu \chi \partial_\nu \chi) - V(\varphi, \chi)$$

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$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \Big(G^{IJ} \partial_\mu \varphi_I \partial_\nu \varphi_J\Big) - V(\varphi^I)$$

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increasing complexity

etc...

Encyclopædia Inflationaris Page 1/326!!

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Only single field models!

Isocurvature/Entropy/Non-adiabatic Pressure Perturbations

Isocurvature/Entropy/Non-adiabatic pressure perturbations

- Adiabatic system $\frac{\delta P}{\dot{P}} = \frac{\delta \rho}{\dot{\rho}}$

- Breaking requirement of adiabaticity allows the pressure perturbation to be expanded as

$$\frac{\delta P}{\dot{P}} \neq \frac{\delta \rho}{\dot{\rho}} \quad \Rightarrow \quad \delta P = \frac{\dot{P}}{\dot{\rho}} \delta \rho + \cdots$$

- The non-adiabatic pressure perturbation can then be defined as $\delta P_{\rm nad} = \delta P - c_{\rm s}^2 \delta \rho$

Entropy perturbations

- Thus require a fluid with equation of state depending on two independent variables, i.e.

$$P \equiv P(\rho, S)$$

- Inflationary fields can be written as fluids:
  - single field has barotropic equation of state and so, on large scales, no isocurvature
  - multiple fields, entropy perturbations can be non-zero, and may be important

Why isocurvature? 1: vorticity

- In classical fluid dynamics, vorticity: $\vec{\omega} = \vec{\nabla} \times \vec{u}$
- Evolves according to

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \frac{1}{\rho^2} \nabla\rho \times \nabla P$$

- 'Source term' zero if $\vec{\nabla} P$ and $\vec{\nabla}\rho$ are parallel
  - i.e. barotropic fluid, no source term
  - including entropy allows for a source term

Crocco (1937)

Vorticity in cosmology

- Relativistic generalisation, consider vorticity tensor in a perturbed FLRW universe
- Linear perturbations have no source term and decay with expansion

$$\omega'_{1ij} - 3\mathcal{H}c_{\mathrm{s}}^2 w_{1ij} = 0$$

Kodama & Sasaki (1984)

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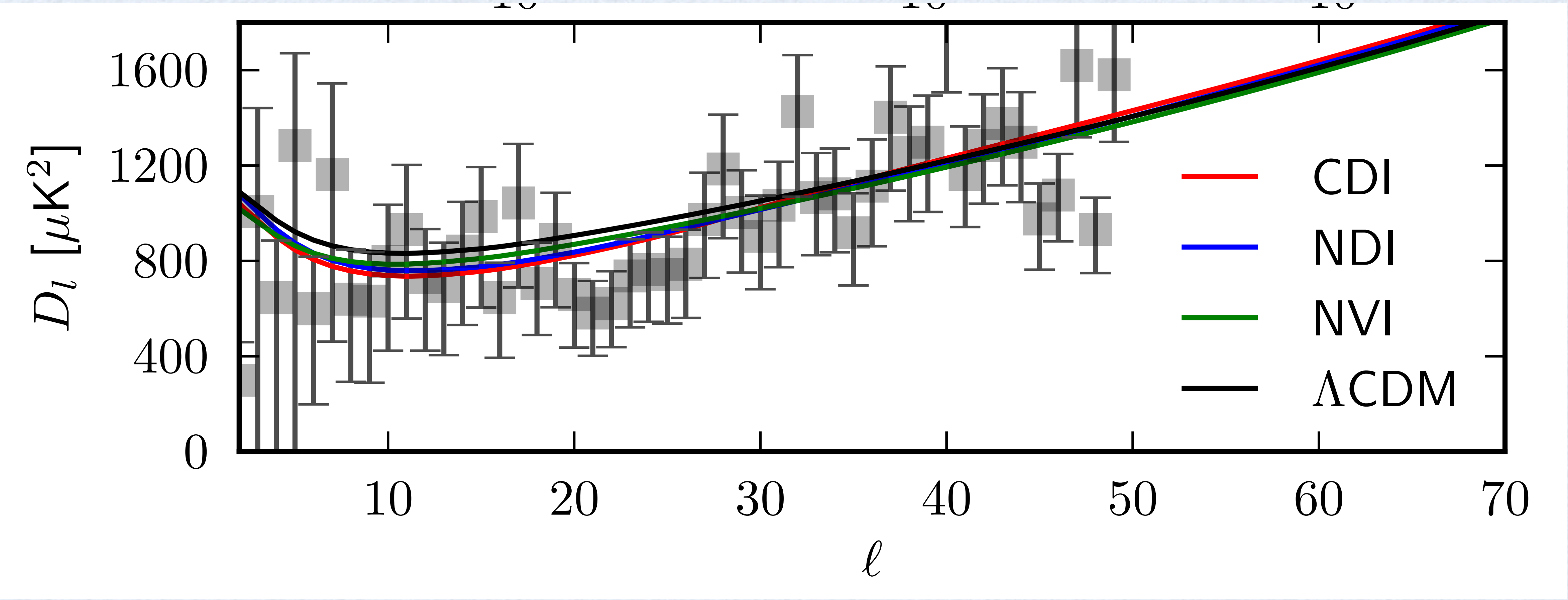
Kodama & Sasaki (1984)

- Beyond linear theory source term depends upon the entropy perturbations

$$\omega'_{2ij} - 3\mathcal{H}c_{\mathrm{s}}^2\omega_{2ij} = \frac{2a}{\rho_0 + P_0}\left\{3\mathcal{H}V_{1[i}\delta P_{\mathrm{nad}1,j]} + \frac{\delta\rho_{1,[j}\delta P_{\mathrm{nad}1,i]}}{\rho_0 + P_0}\right\}$$

AJC, Malik & Matravers (2009)

Why isocurvature? 2: CMB?



Isocurvature perturbations

- We consider multi-field inflationary models with Lagrangian

$$\mathcal{L}(\varphi, \chi) = \frac{1}{2}(\dot{\varphi}^2 + \dot{\chi}^2) + U(\varphi, \chi)$$

  with double quartic, quadratic and product exponential potentials

- Study isocurvature perturbations

  - after inflation

  - after period of perturbative reheating

MULTI-FIELD INFLATION

- Double quadratic

$$V(\varphi,\chi) = \frac{1}{2}m_\varphi^2\varphi^2 + \frac{1}{2}m_\chi^2\chi^2$$

- Double quartic

$$V(\varphi,\chi) = \Lambda^4\left[\left(1-\frac{\chi_0^2}{v^2}\right)^2 + \frac{\varphi^2}{\mu^2} + \frac{2\varphi^2\chi^2}{\varphi_c^2 v^2}\right]$$

Kodama et al. (2011)
Avgoustidis et. al. (2011)

- Product exponential

Byrnes, Choi & Hall (2008)

$$V(\varphi,\chi) = V_0\varphi^2 e^{-\lambda\chi^2}$$

- Solve background field dynamics and linear perturbations numerically $\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0$

Huston & AJC, PRD 85, 063507 (2012)

http://pyflation.ianhuston.net/

- Compute non-adiabatic pressure perturbation

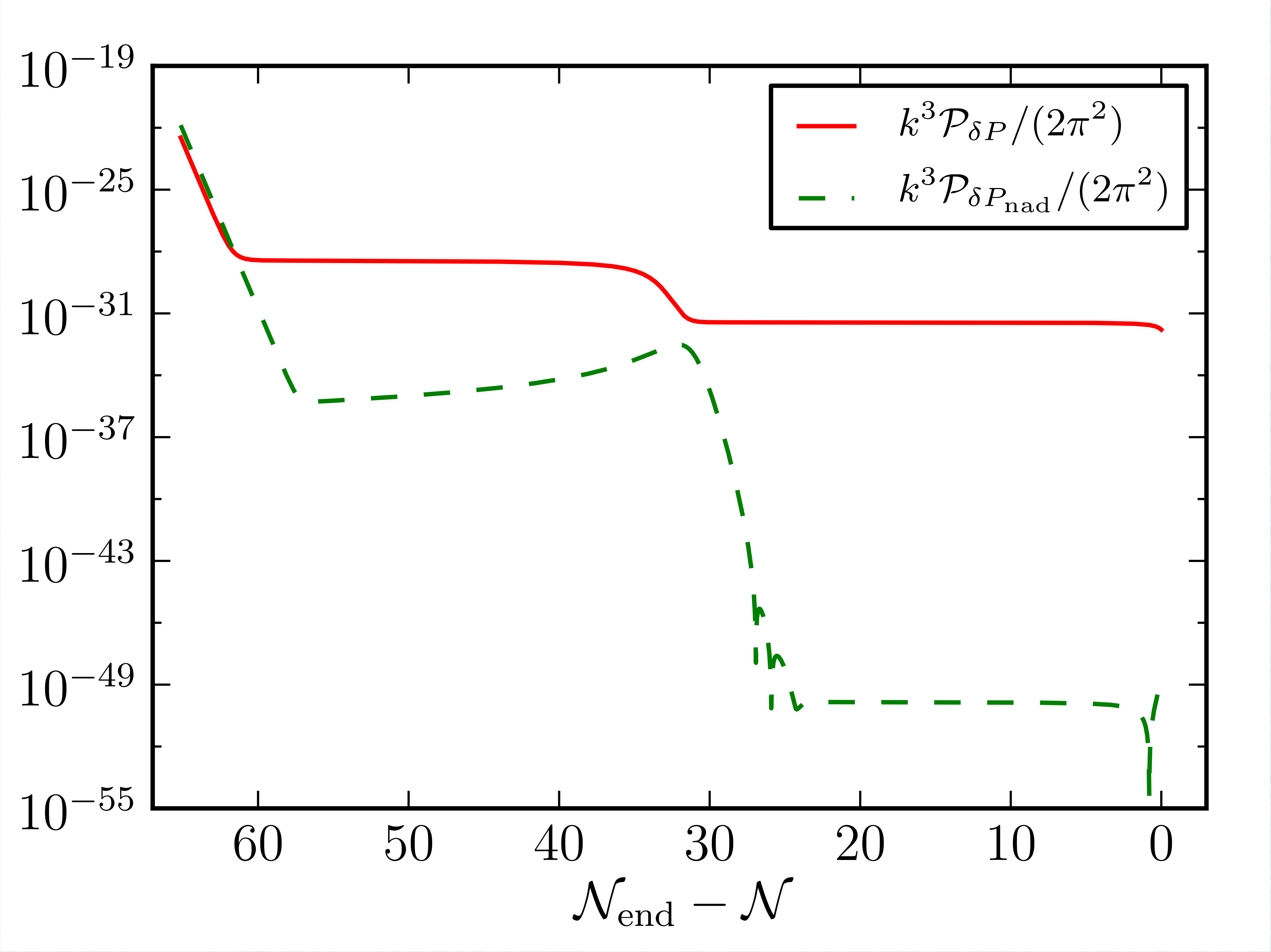
$$\delta P_{\mathrm{nad}} = \delta P - c_{\mathrm{s}}^2 \delta\rho$$

by defining the pressure and energy density perturbation of the scalar fields as

$$\delta P = \sum_{\alpha} \left(\dot{\varphi_\alpha} \delta\dot{\varphi}_\alpha - \dot{\varphi_\alpha}^2 \phi - V_{,\alpha} \delta\varphi_\alpha \right)$$

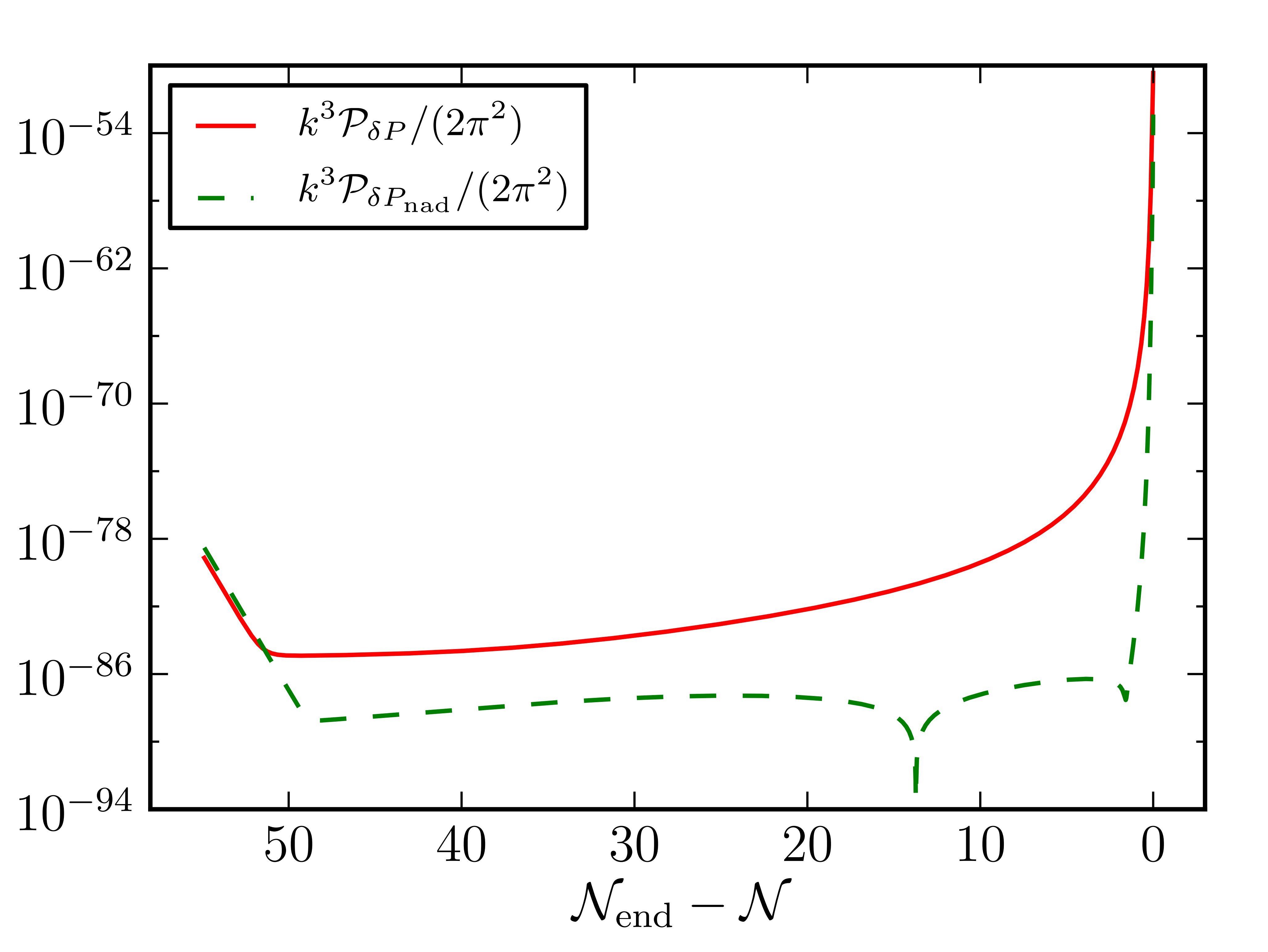
$$\delta \rho = \sum_{\alpha} \left(\dot{\varphi_\alpha} \delta\dot{\varphi}_\alpha - \dot{\varphi_\alpha}^2 \phi + V_{,\alpha} \delta\varphi_\alpha \right)$$

Double Quadratic

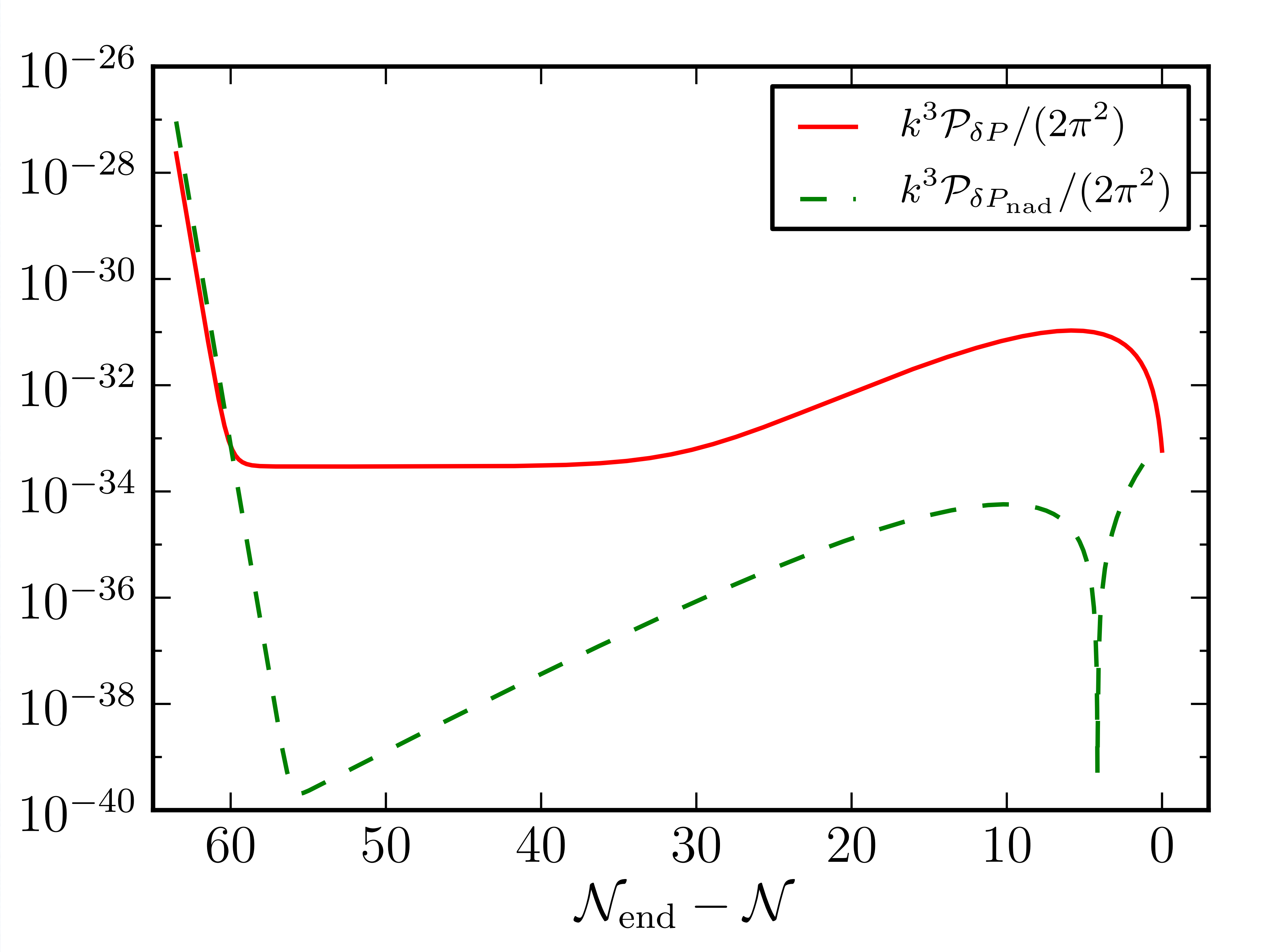


Huston & AJC (2012)

Double Quartic



Product exponential



Reheating

- How do the scalar fields driving inflation decay into the standard model?
- Include an additional friction term in the scalar field equation $$\ddot{\varphi} + (3H + \frac{1}{2}\Gamma)\dot{\varphi} + U_{,\varphi} = 0$$
- Usually, one considers the decay of the fields into radiation
- In order to allow isocurvature to survive, consider decay into **both** radiation and matter, subject to constraint $$\frac{\Gamma_{\rm m}}{\Gamma_\gamma} < 10^{-6}$$

- Evolution equations from energy-momentum conservation $\nabla_\mu T^\mu_{(\alpha)\nu} = Q^{(\alpha)}_\nu$

- Fluid equations

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \frac{1}{2}\left(\Gamma^\varphi_\gamma \dot{\varphi}^2 + \Gamma^\chi_\gamma \dot{\chi}^2\right)$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = \frac{1}{2}\left(\Gamma^\varphi_{\rm m} \dot{\varphi}^2 + \Gamma^\chi_{\rm m} \dot{\chi}^2\right)$$

- Field equations

$$\ddot{\varphi} + \left[3H + \frac{1}{2}\left(\Gamma^\varphi_\gamma + \Gamma^\varphi_{\rm m}\right)\right]\dot{\varphi} + U_{,\varphi} = 0$$

- And similarly (though more complicated) for perturbations

- Solve the evolution equations for the scalar field and matter/radiation fluids

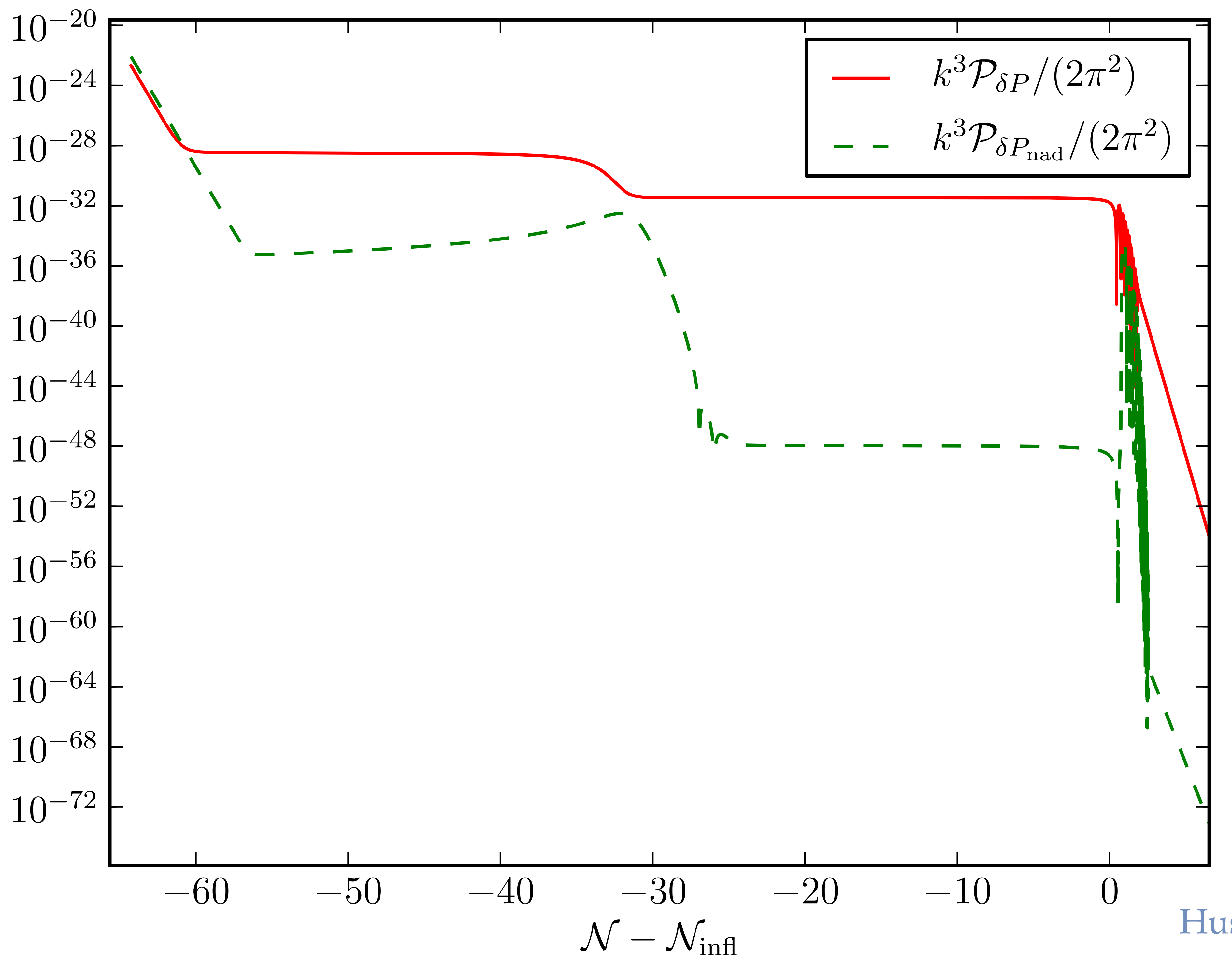
- Again, non-adiabatic pressure perturbation is computed, defined as $\delta P_{\rm nad} = \delta P - c_{\rm s}^2 \delta\rho$

- Now, contribution from the scalar fields, but also from the fluids, i.e.

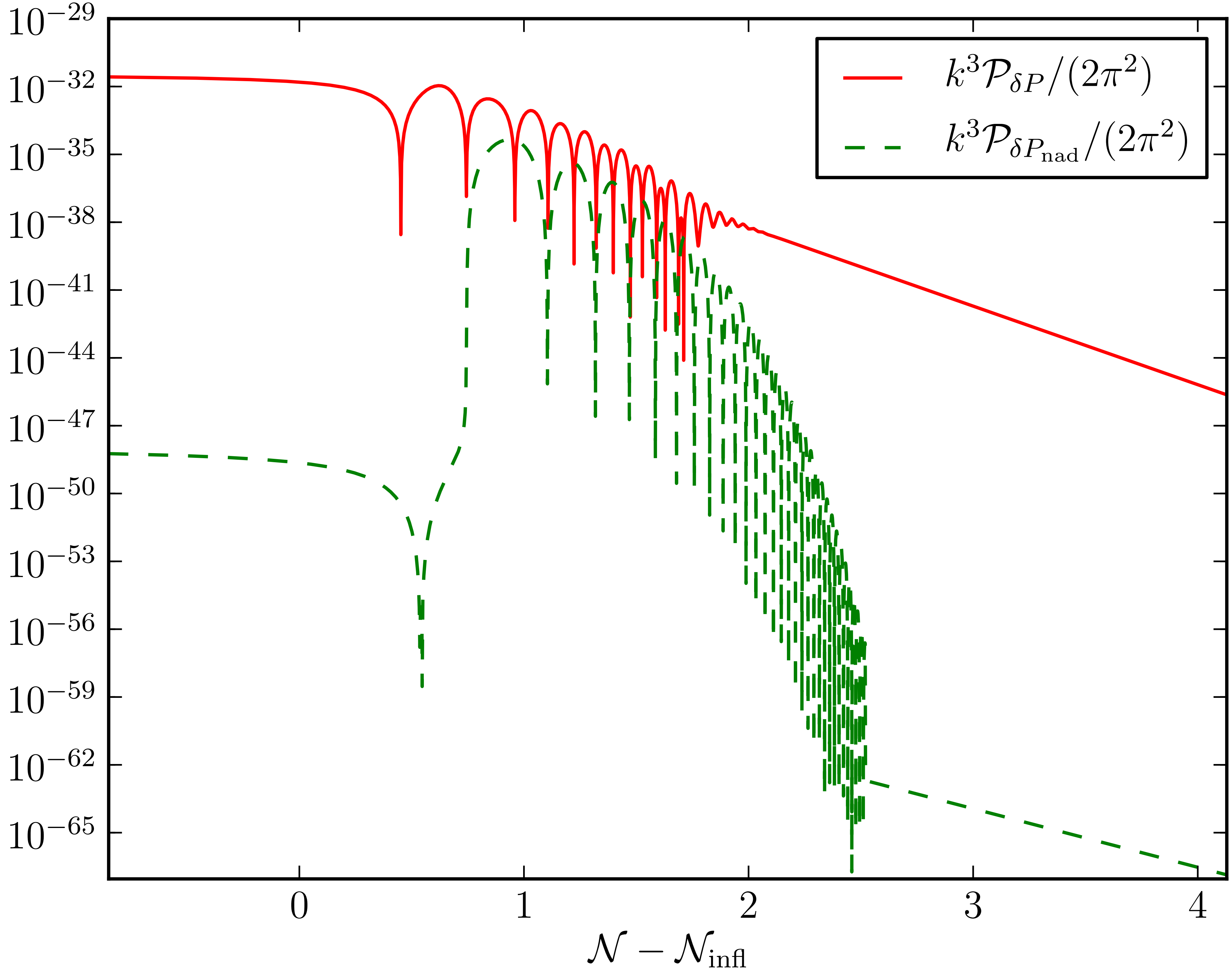
$$\delta P_{\rm nad} = \frac{1}{3}\delta\rho_\gamma \left[1 - \frac{\dot{\rho_\gamma}}{\dot{\rho_{\rm m}} + \dot{\rho_\gamma}}\right] - \frac{1}{3}\frac{\dot{\rho_\gamma}\delta\rho_{\rm m}}{\dot{\rho_{\rm m}} + \dot{\rho_\gamma}}$$

Huston & AJC, arXiv:1302.4298

Double Quadratic Inflation



Huston & AJC (2013)



10^{-29}
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10^{-35}
10^{-38}
10^{-41}
10^{-44}
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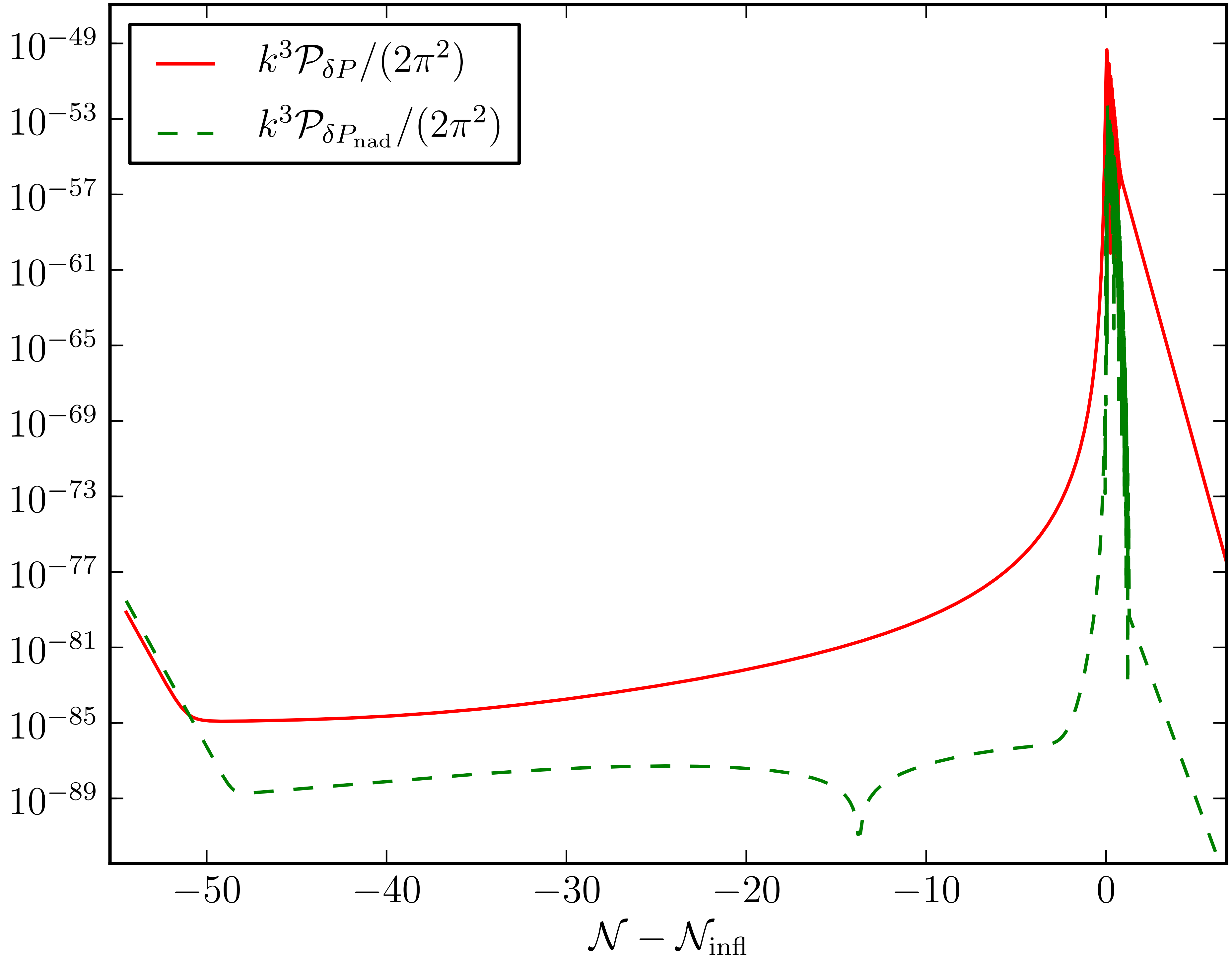
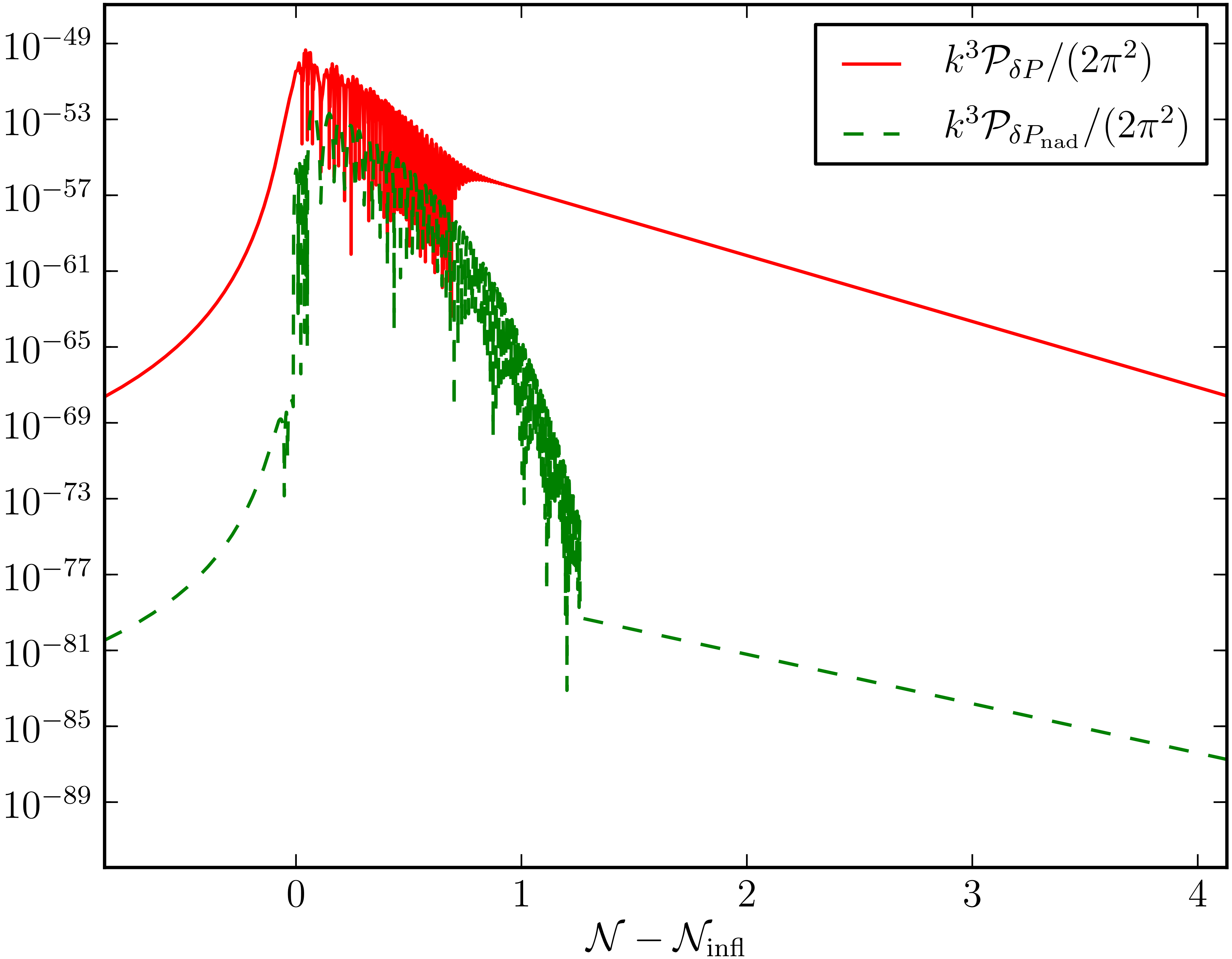
10^{-56}
10^{-59}
10^{-62}
10^{-65}
0
1
2
3
4
$\mathcal{N} - \mathcal{N}_{\rm infl}$
$k^3 \mathcal{P}_{\delta P}/(2\pi^2)$
$k^3 \mathcal{P}_{\delta P_{\rm nad}}/(2\pi^2)$

Double Quartic Inflation




10^{-49}
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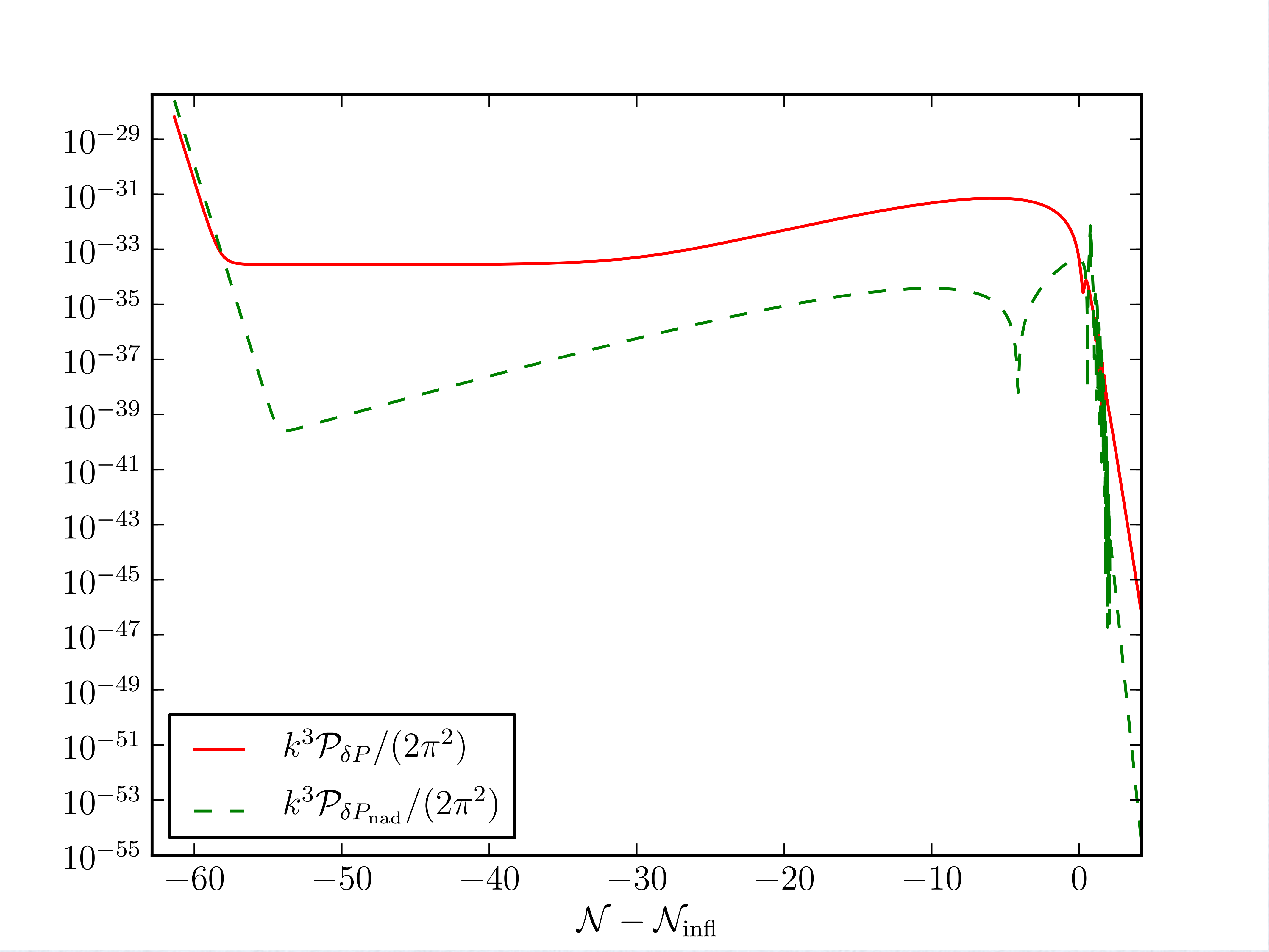
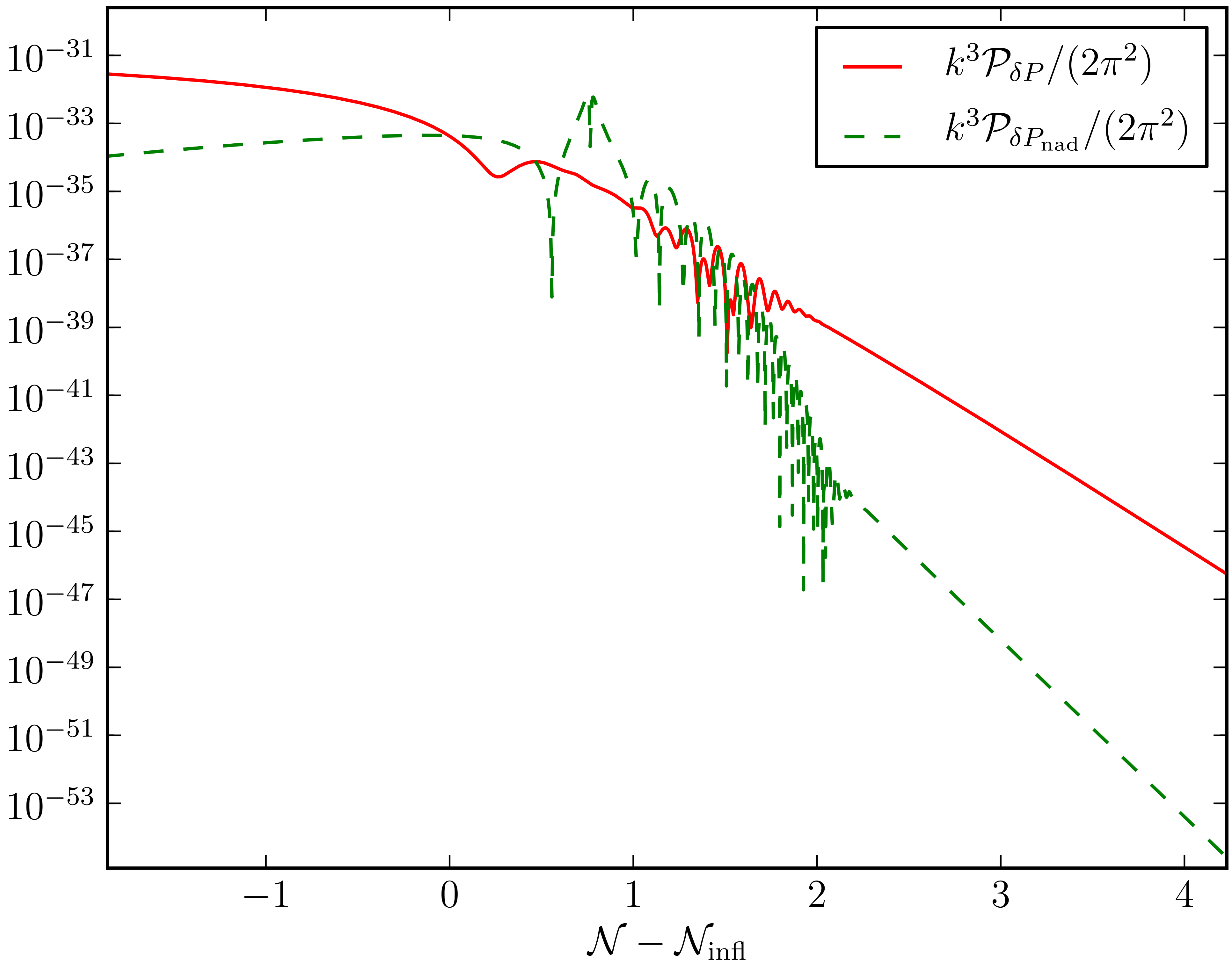
$\mathcal{N} - \mathcal{N}_{\rm infl}$
$k^3\mathcal{P}_{\delta P}/(2\pi^2)$
$k^3\mathcal{P}_{\delta P_{\rm nad}}/(2\pi^2)$

PRODUCT EXPONENTIAL





10^{-31}
10^{-33}
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10^{-43}
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10^{-51}
10^{-53}
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4
$\mathcal{N} - \mathcal{N}_{\rm infl}$
$k^3\mathcal{P}_{\delta P}/(2\pi^2)$
$k^3\mathcal{P}_{\delta P_{\rm nad}}/(2\pi^2)$

Summary

- Isocurvature perturbations common in multi-field models; can source source vorticity?
- Reheating should be taken into account when isocurvature perturbations are present
- Non-adiabatic pressure perturbation subdominant in all three models studied

Huston & AJC, PRD 85, 063507 (2012)

Huston & AJC, 1302.4298 (2013)

- BUT parameter choice conservative: could we saturate the bounds to source isocurvature?