Primordial Non-Gaussianity from the CMB Minkowski Functionals and Bispectrum

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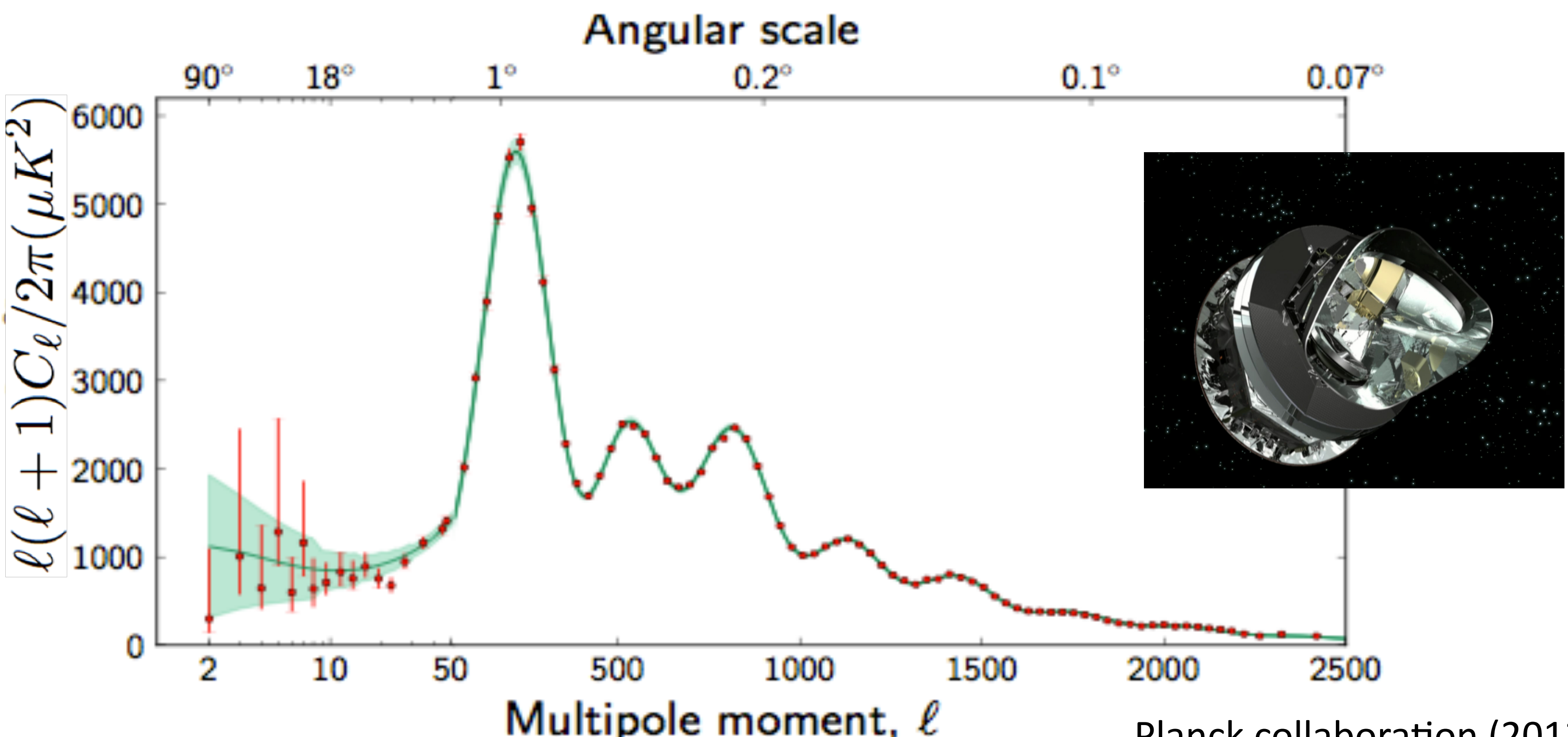
Outline

- What is non-Gaussianity?
- Why is it interesting?
- How to constrain NG using CMB statistics: bispectrum, Minkowski functionals…, and what are these?
- Earlier work.
- This work -- joint constraints.

Inflation supported by observations

- Solves flatness problem, horizon problem, generate initial conditions.
- Planck: flat ΛCDM with nearly scale-invariant, Gaussian, adiabatic initial conditions.



Planck collaboration (2013)

Further test of Inflation

- Tensor Modes: probes the energy scale of inflation

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\mathrm{GeV}$$

r < 0.11 (Planck collaboration2013)

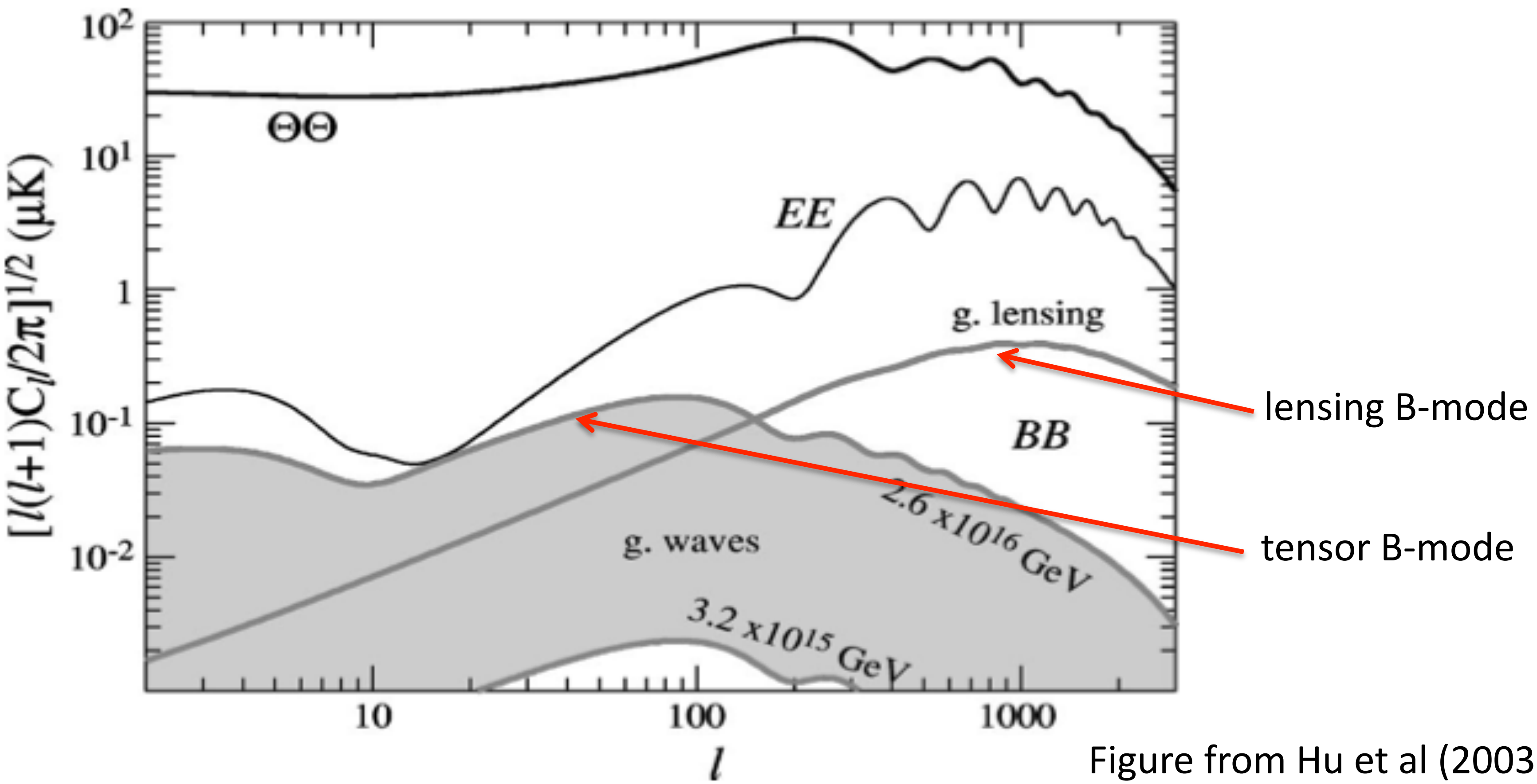
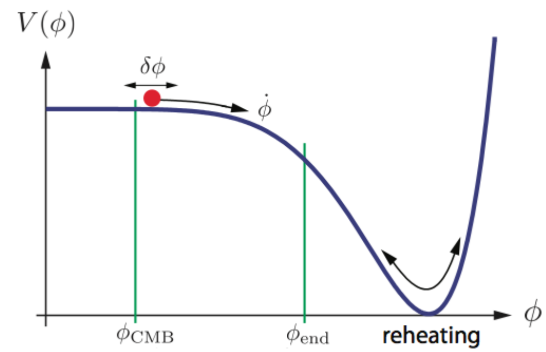


Figure from Hu et al (2003)

Further test of Inflation

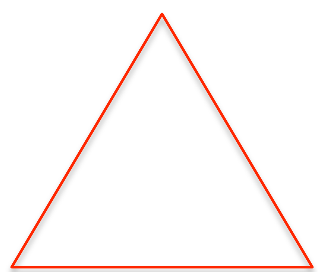
- Primordial Non-Gaussianity — beyond the 2pt statistics
  - Canonical single-field slow-roll: small NG
  - Deviations from SFSR: can have detectable NG
- Bispectrum: different amplitudes and shapes



$$\langle \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\Phi(k_1,k_2,k_3)$$

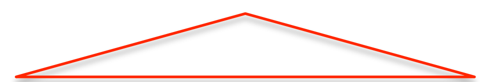
local (squeezed)

$$k_1 \approx k_2 \gg k_3$$



equilateral

$$k_1 = k_2 = k_3$$



folded

$$k_1 = 2k_2 = 2k_3$$

powerful to discriminate different mechanisms for generating primordial perturbations

Local-type PNG

- Generated by SFSR, multifield, curvaton etc, parameterized by

$$\Phi(\mathbf{x}) = \underbrace{\Phi_L(\mathbf{x})}_{\text{Gaussian}} + \underbrace{f_{NL}[\Phi_L^2(\mathbf{x}) - \langle\Phi_L^2(\mathbf{x})\rangle]}_{\text{Non-Gaussian} \sim o(10^{-5} f_{NL})}$$

Φ: minus gravitational potential

SFSR: $f_{NL} \sim (1 - n_s) \sim 0.01$

Detection: rule out standard SFSR.
Non detection: rule out various extensions to SFSR.

Planck constraints: $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$

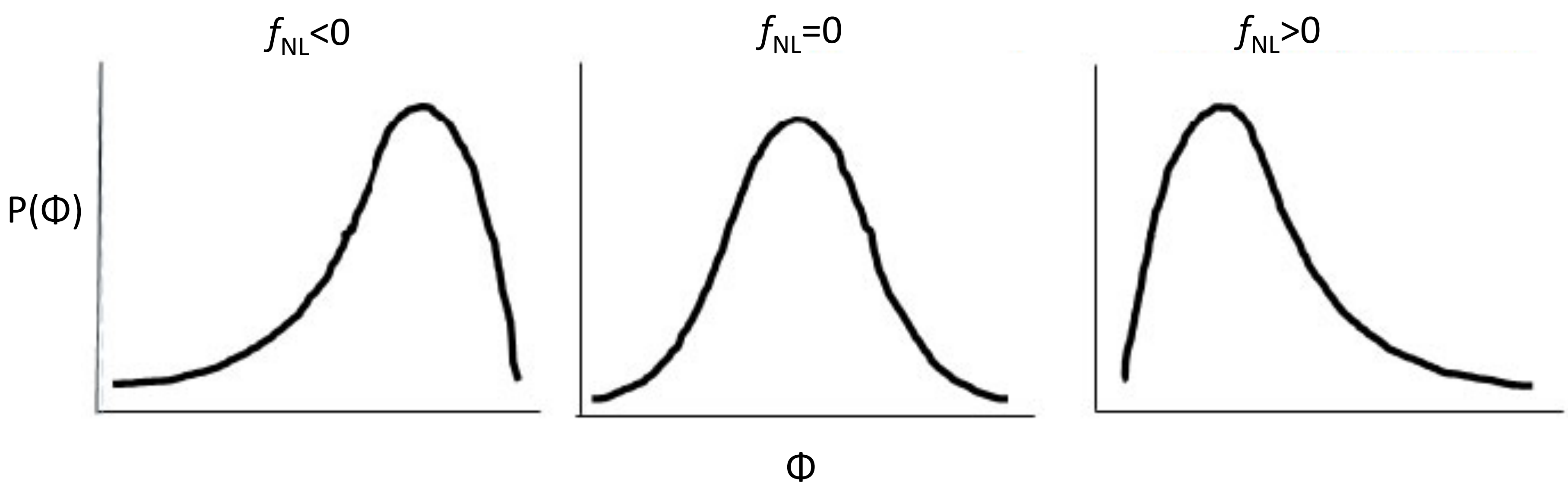
Local-type PNG

- Generated by SFSR, multifield, curvaton etc, parameterized by

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}[\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle]$$

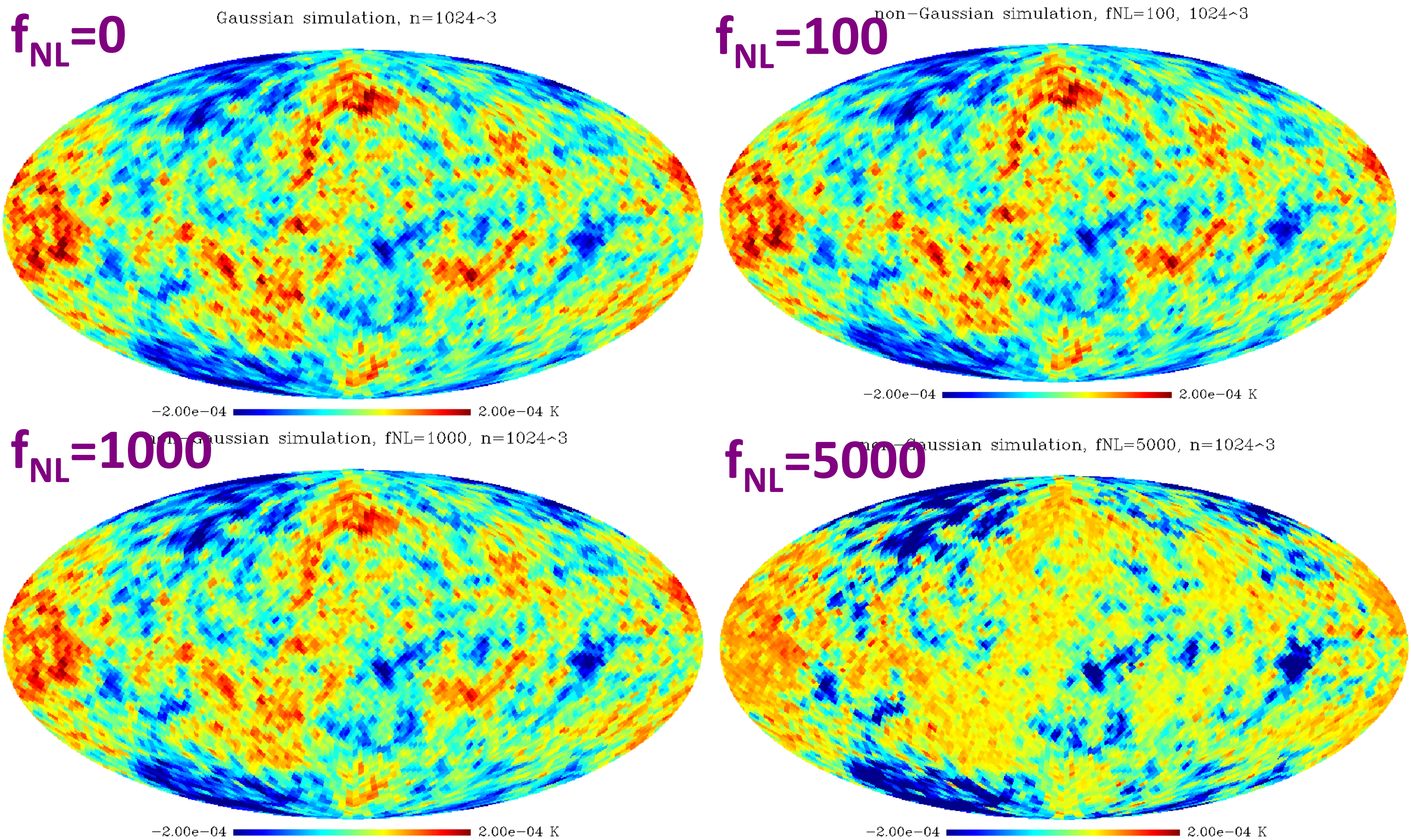
$$B_\Phi(k_1, k_2, k_3) = 2f_{\rm NL}\left[P_\Phi(k_1)P_\Phi(k_2) + \text{ perm}\right]$$

Skewness: $\langle \Phi^3 \rangle = 2f_{\rm NL} \langle \Phi^2 \rangle^2$

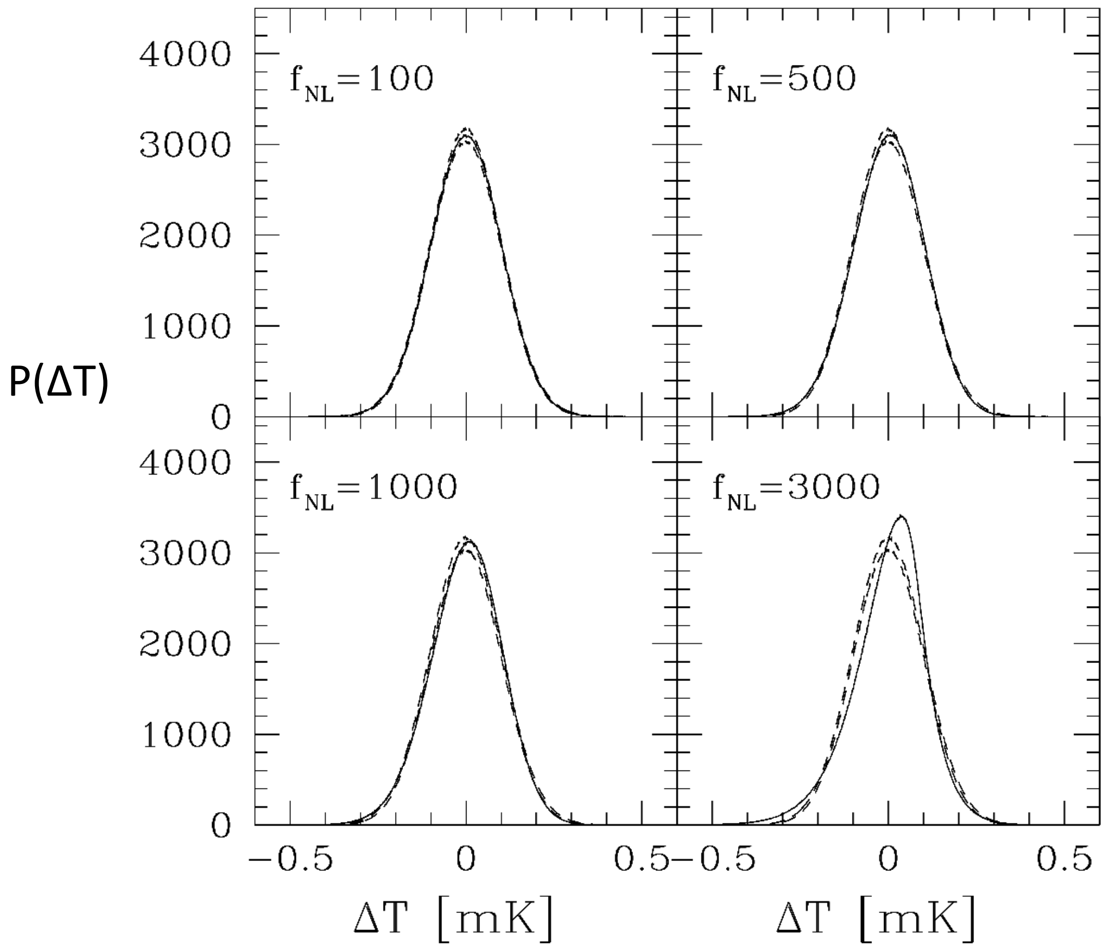


PNG and CMB

Figure from E. Komatsu



PNG and CMB



Positive skew in Φ causes negative skew in ΔT.

Komatsu++ 2003

CMB Bispectrum

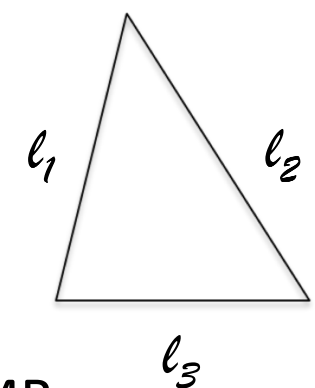
- Angle-averaged bispectrum

$$B_{\ell_1\ell_2\ell_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \underline{\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle} \equiv B^{m_1 m_2 m_3}_{\ell_1,\ell_2,\ell_3}$$

vanishes unless:

$$\ell_1 + \ell_2 + \ell_3 = \text{even}$$

$$|\ell_i - \ell_j| \le \ell_k \le \ell_i + \ell_j$$

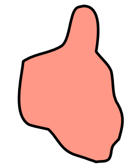


linear evolution from primordial perturbation to CMB:

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Phi(\mathbf{k}) g_{Tl}(k) Y^*_{lm}(\hat{\mathbf{k}})$$

$$B_{\ell_1\ell_2\ell_3} \propto f_{NL}$$

CMB Minkowski Functionals



$\Delta T \geq \nu\sigma$

- Morphological descriptors of excursion sets of CMB anisotropies--regions of the CMB sky above a temperature threshold.

Three MFs to define on a 2D sphere.

Ensember averages sensitive to statistical properties of the CMB.

For Gaussian:

Area

Contour length

Genus
(# hot spots - # cold spots)

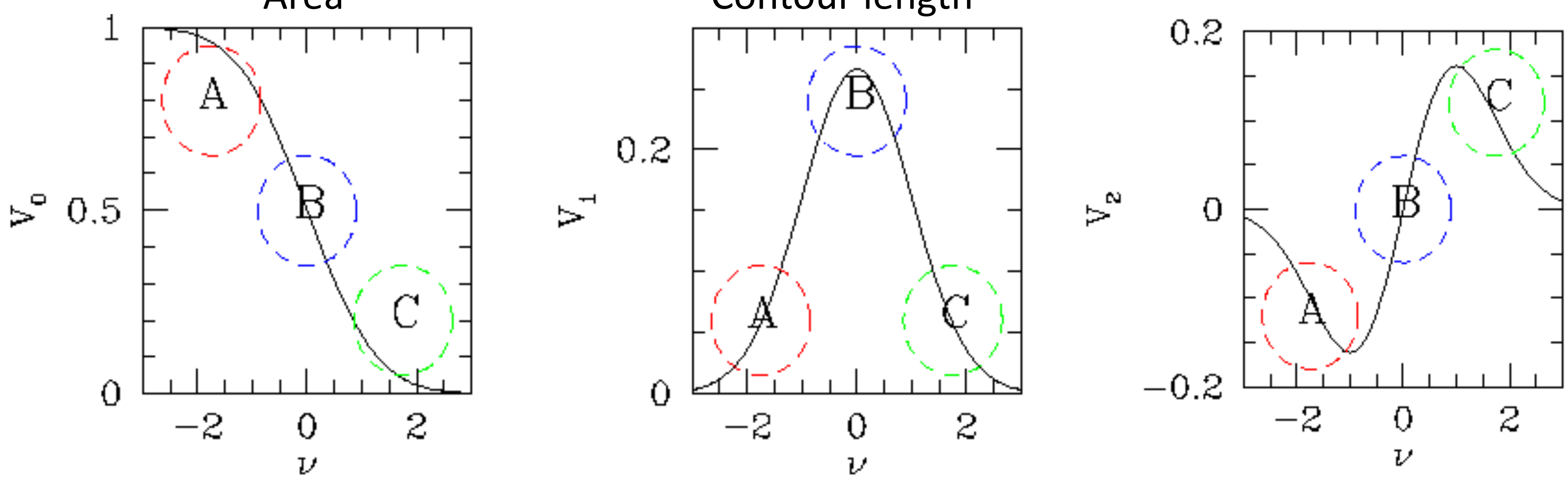
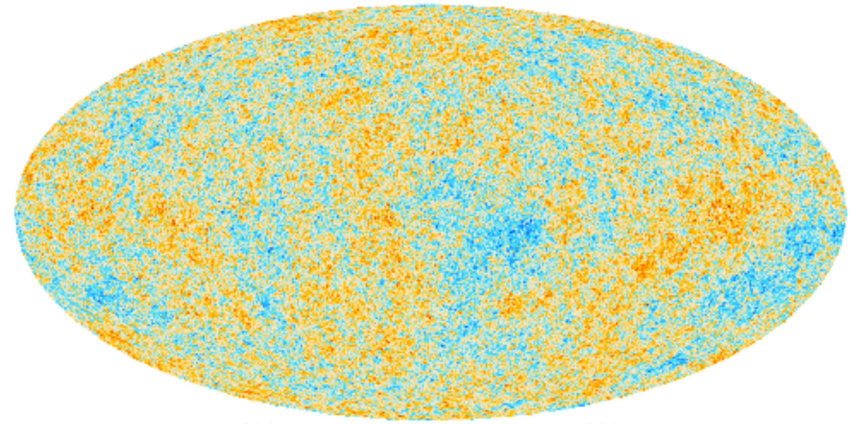


Figure from E. Komatsu

Calculate the MFs



- Numerically for a given map

$$V_0 = \frac{1}{4\pi}\int_{Q_\nu} d\Omega = \frac{1}{4\pi}\int d\Omega\ \Theta(u-\nu)$$

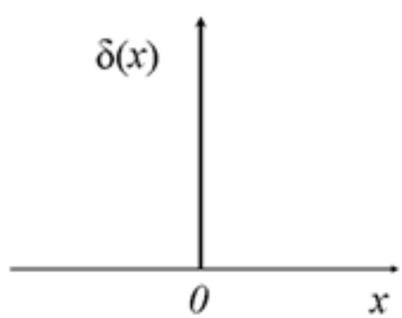
$$V_1 = \frac{1}{16\pi}\int_{\partial Q_\nu} dl = \frac{1}{16\pi}\int d\Omega\ \delta(u-\nu)|\nabla u|$$

$$V_2 = \frac{1}{8\pi^2}\int_{\partial Q_\nu} dl\ \kappa = \frac{1}{8\pi^2}\int d\Omega\ \delta(u-\nu)|\nabla u|\kappa$$

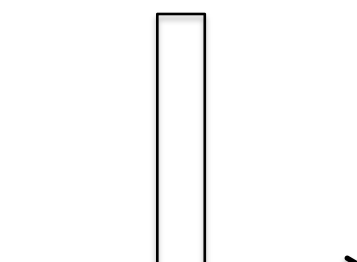
1. Change line integrals to surface integrals

2. Replace surface integrals by summing over all pixels

3. Replace delta function



by



Calculate the MFs

- Theoretically for the ensemble average
  - Gaussian: exactly known, determined by σ and σ_1 (or by power spectrum)
  - Non-Gaussian: expand around Gaussian, for hierarchical $\langle f^n \rangle_c \sim \mathcal{O}(\sigma^{2n-2})$

$$V_k(\nu) = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_2}{\omega_{2-k}\omega_k} \left(\frac{\sigma_1}{\sqrt{2}\sigma}\right)^k e^{-\nu^2/2} \Big(\underbrace{v_k^{(0)}}_{\text{Gaussian}} + \underbrace{v_k^{(1)}\sigma + v_k^{(2)}\sigma^2 + \cdots}_{\text{nonGaussain}} \Big)$$

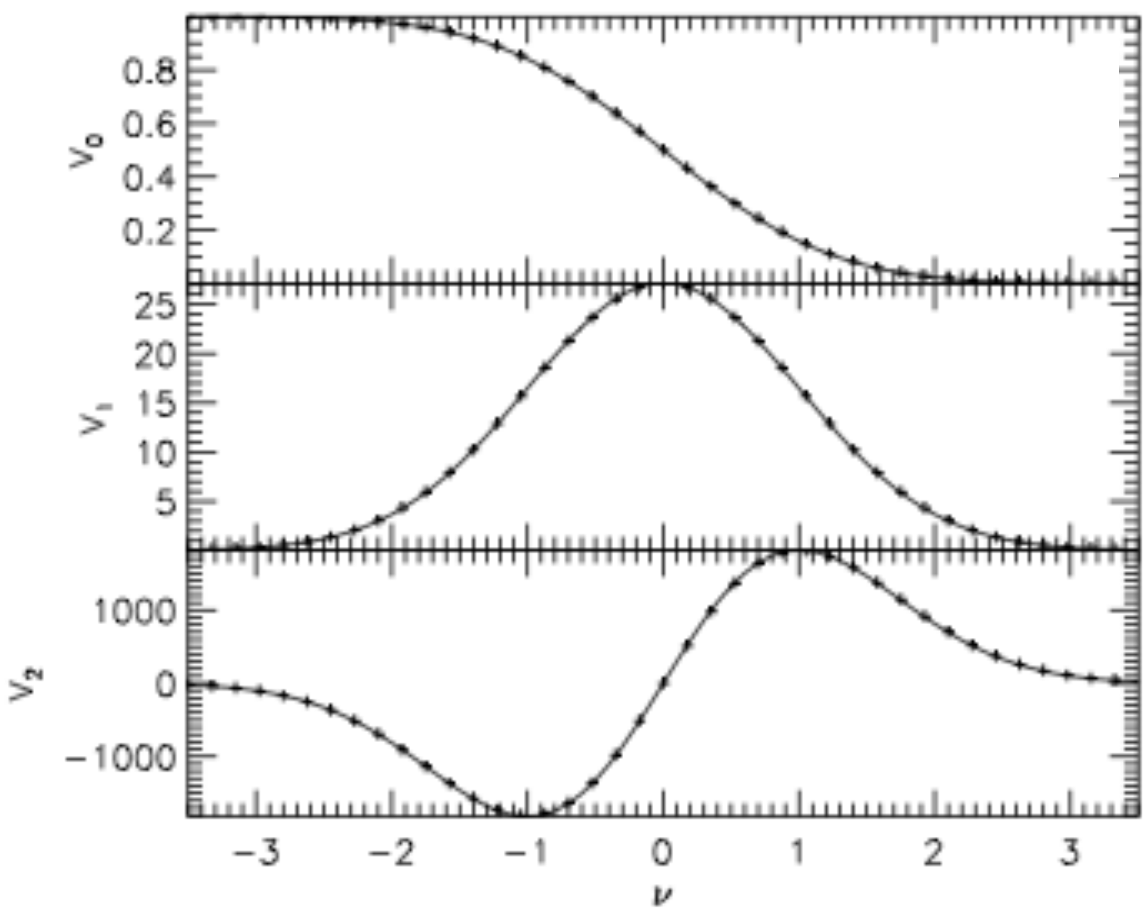
for k = 0, 1, 2

leading non-Gaussian term determined by 3 skewness parameters, which are weighted sums of the bispectrum.

$$v_k^{(1)}(\nu) = \frac{S}{6} H_{k+2}(\nu) - \frac{kS_{\mathrm{I}}}{4} H_k(\nu) - \frac{k(k-1)S_{\mathrm{II}}}{4} H_{k-2}(\nu)$$

$$S = \frac{\langle f^3 \rangle}{\sigma^4}, \qquad S_{\mathrm{I}} = \frac{\langle f^2 \nabla^2 f \rangle}{\sigma^2 \sigma_1^2}, \qquad S_{\mathrm{II}} = \frac{2\langle |\nabla f|^2 \nabla^2 f \rangle}{\sigma_1^4} \quad \boxed{\propto f_{NL}}$$

Numerical results agree well with theoretical predictions



Points: averages over 1000 simulated Gaussian maps.

Lines: theoretical prediction for the Gaussian averages.

MF vs Bispectrum Constraints

- MF and Bispectrum: complementary NG probes
  - Real space (MF) vs harmonic space (Bispectrum)
  - Sensitive to different systematics
  - MF can probe all order of statistics, Bispectrum probe the 3^{rd}.
- Constraints in the literature

	Bispectrum (KSW)	MFs
WMAP5	58 ± 36	-60 ± 60
Planck	2.7 ± 5.8	4.2 ± 20.5

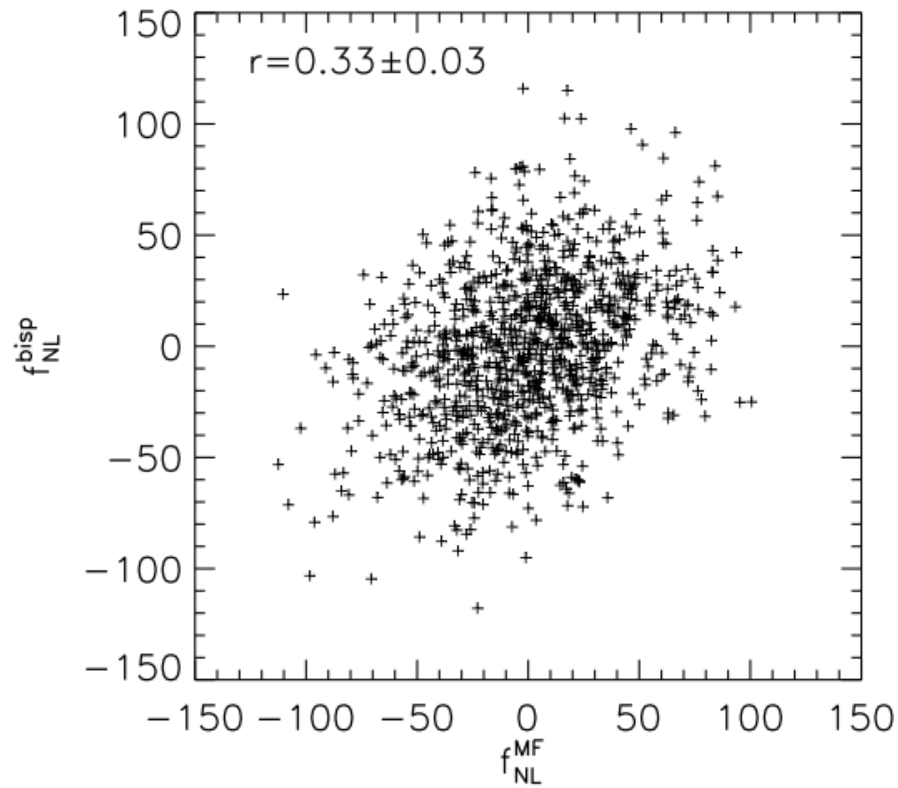
→ differ by 3σ

- Correlation between the two? Combined constraints?

Correlations b/w the two estimators

- Simulate CMB (Gaussian) maps including observational effects typical to the WMAP 7yr V+W band data
  - Beam smearing
  - Instrumental noises
  - Poisson Point sources
- MF analysis
  - Smooth maps at 3 different angular scales
  - Combine results at all scales
- Find positive correlation

  r=0.33



WF, Becker, Huterer & Lim 2013

Tighter Combined Constraints:

Combine the two estimators of f_{NL}- assumming bi-variate Gaussian:

$$\mathcal{L} \propto |\mathbf{C}|^{-1/2} \exp\left[-\frac{1}{2}(\mathbf{f}_{\rm NL} - \bar{\mathbf{f}}_{\rm NL})\mathbf{C}^{-1}(\mathbf{f}_{\rm NL} - \bar{\mathbf{f}}_{\rm NL})^T\right]$$

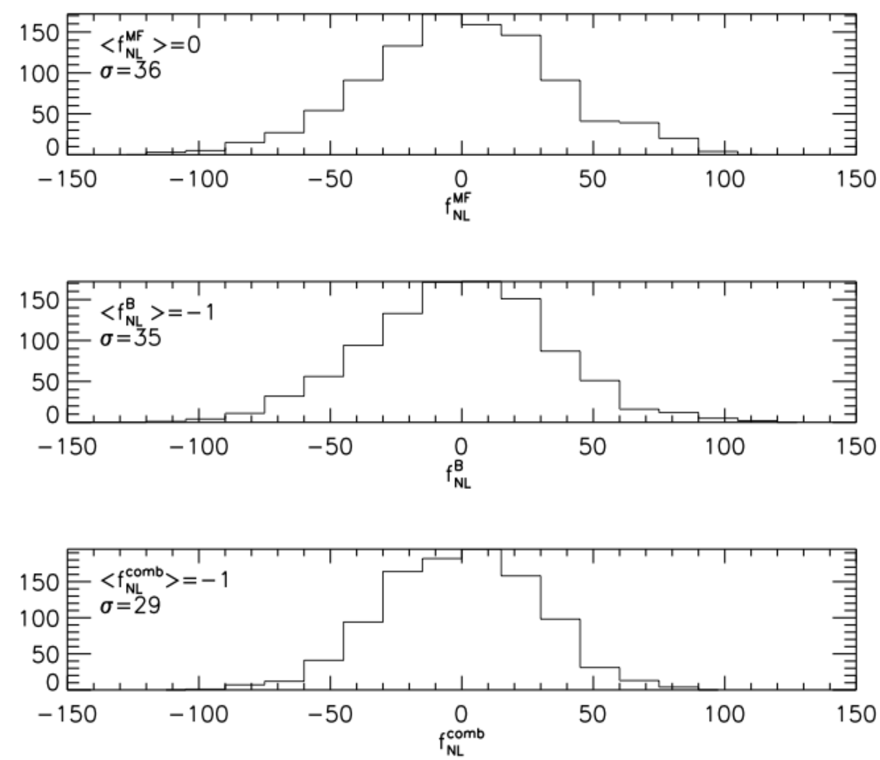
Given a pair of measurement

$$\mathbf{f}_{\rm NL} \equiv [f_{\rm NL}^{\rm MF},\, f_{\rm NL}^{\rm bisp}]$$

find the best estimate for $\bar{f}_{\rm NL}$

- Assuming C independent on $\bar{f}_{\rm NL}$
- Maximize the likelihood w. r. t $\bar{f}_{\rm NL}$

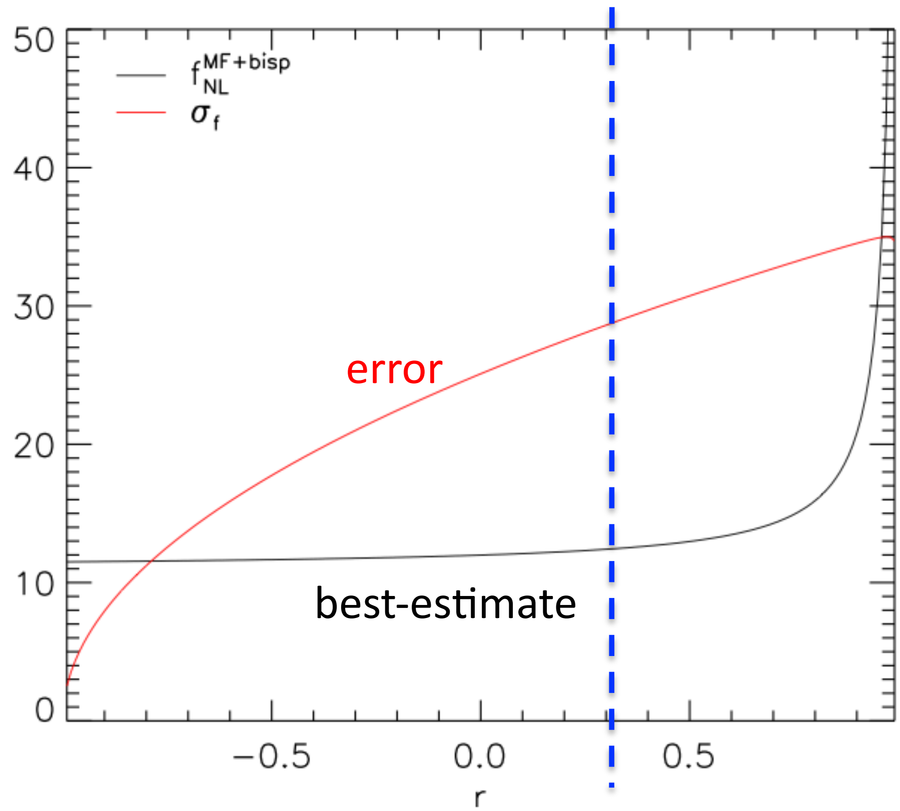
$$\bar{f}_{\rm NL} = \frac{\mathbb{I}\,\mathbf{C}^{-1}\,\mathbf{f}_{\rm NL}{}^T}{\mathbb{I}\,\mathbf{C}^{-1}\,\mathbb{I}^T}, \quad \sigma^2_{\bar{f}_{\rm NL}} = \frac{1}{\mathbb{I}\,\mathbf{C}^{-1}\,\mathbb{I}^T}$$



WF, Becker, Huterer & Lim 2013

Apply to WMAP:

Dependence on r



r=0.33

7yr co-added V+W band

	$\theta(')$	f_{NL}
Minkowski Functionals	5	−33±110
	20	−27± 52
	80	−8 ± 70
	all	−24 ± 36
bispectrum		46 ± 35
MF + bisp		**12 ± 29**

Combination improves error by ~20%.

WF, Becker, Huterer & Lim 2013

Conclusions

- We find positive correlation between the estimators of $f_{\rm NL}$ from MF and from the bispectrum.
- Combination of the two could improve constraints by ~20%, hence shrink more of the allowed model space.
- SFSR consistent with our results from WMAP7.