“Early Universe”: Relics of Preheating after Inflation

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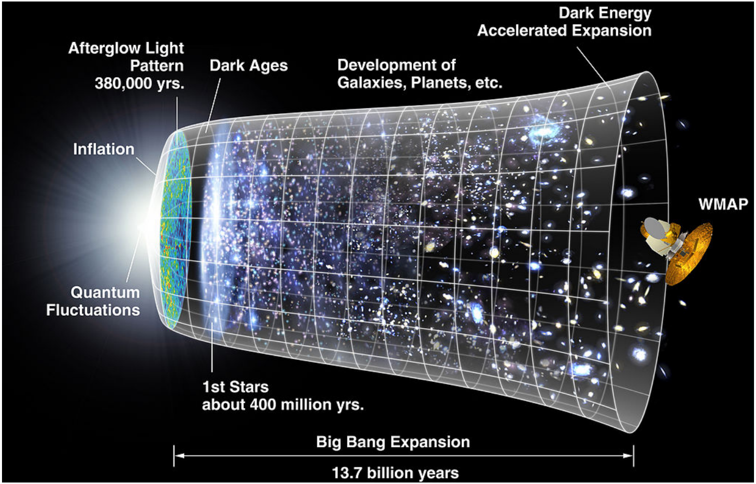
Outline

Cosmology



(graphic by: NASA/WMAP Science Team)

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- bubble nucleation and collisions.
- Primordial black-hole formation

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- For certain models, this nonperturbative phase is necessary to ensure that reheating completes.
- Coupled modes can enter resonance bands which cause resonant amplification.

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- $\Box\phi^i = \frac{\partial V}{\partial \phi^i}$, V is a nonlinear function of all of the $\{\phi^i\}$.

FRW Background

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The field equations of motion become:

$$\ddot{\phi}^i + 3H\dot{\phi}^i - \frac{\Delta}{a^2}\,\phi^i + \frac{\partial V}{\partial \phi^i} = 0 \tag{2}$$

$V(\phi, \psi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\psi^2$

Using the potential:

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In terms of Fourier modes:

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k^2}{a^2} + m^2 + g^2\psi^2\right)\phi_k = 0 \tag{5}$$

H = 0 and the Mathieu Equation

When $H = 0$, substitute:

$$q = \frac{g^2\psi^2}{4m^2},\ A = \frac{k^2}{m^2} + 2q,\ z = mt \tag{6}$$

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where primes denote differentiation with respect to z. All solutions:

$$\phi_k \propto f(z)e^{\pm i\mu z} \tag{8}$$

Solution Stability

$m \geq 0$ implies that $A \geq 2q$. ϕ_k grows exponentially if μ has an imaginary part:

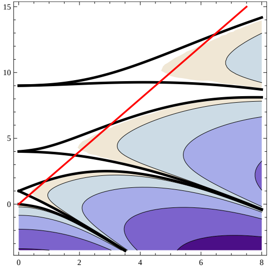

Figure: The imaginary part of the Mathieu critical exponent. Outside the heavy black lines the exponent is real-valued. The diagonal line is $A = 2q$.

Outline

1. Introduction
   - Inflation
   - Nonlinear Processes in the Early Universe
   - Preheating
2. Oscillons
3. Oscillons in Monodromy Inflation
4. Gravitational Radiation
5. Conclusion

Oscillon?

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- A quasi-periodic, localizable feature of a solution to a nonlinear field theory.
- Similar to a soliton, but not protected by a symmetry of the Lagrangian.
- Mustafa Amin (MIT), has done some of the best recent theory work.

Sextic Oscillon Potential

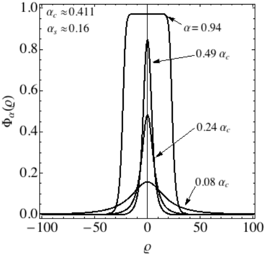
$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \frac{g^2}{6m^2}\varphi^6 \tag{9}$$

with $\lambda > 0$ and $(\lambda/g)^2 \ll 1$. Assuming spherical symmetry and ignoring expansion gives:

$$\partial_t^2\varphi - \partial_r^2\varphi - \frac{2}{r}\partial_r\varphi + m^2\varphi - \lambda\varphi^3 + \frac{g^2}{m^2}\varphi^5 = 0 \tag{10}$$

Sextic-Potential Oscillon Profiles

Assuming a bounded, periodic solution gives an ODE which can be (approximately) solved to yield the radial profile of an oscillon. It is a one-parameter family of curves.



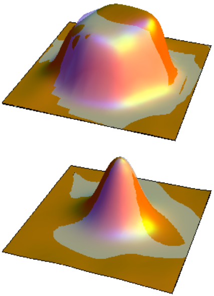


Figure: Oscillon profiles in the sextic potential

Why Do We Care?

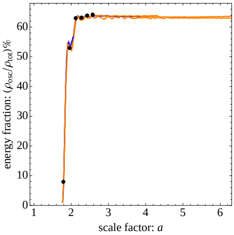
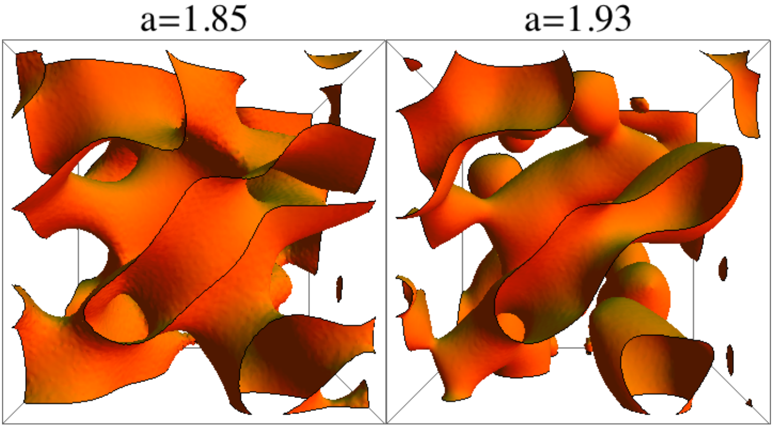


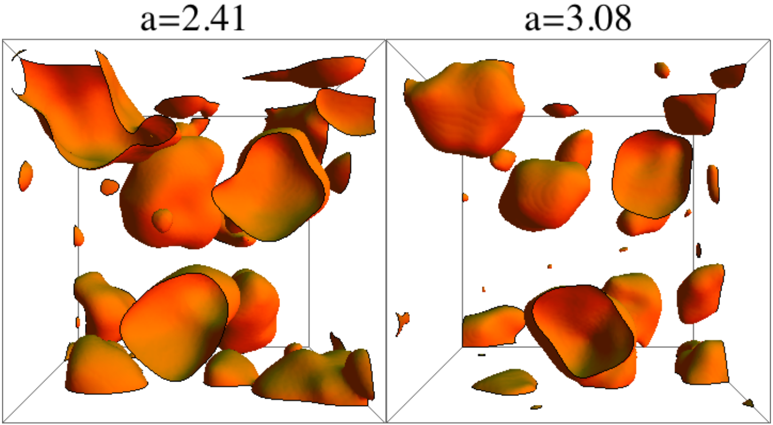
Figure: The fraction of the energy density of the universe after inflation which is in oscillons. The orange and blue curves are from PSpectRe runs (a lot of them) at 256^3 and 384^3 respectively. The black dots are 1024^3 MPI Defrost runs.

A Universe of Oscillons

Simulation using PSpectRe at $L = 200$ and $N = 256$.



A Universe of Oscillons (cont.)



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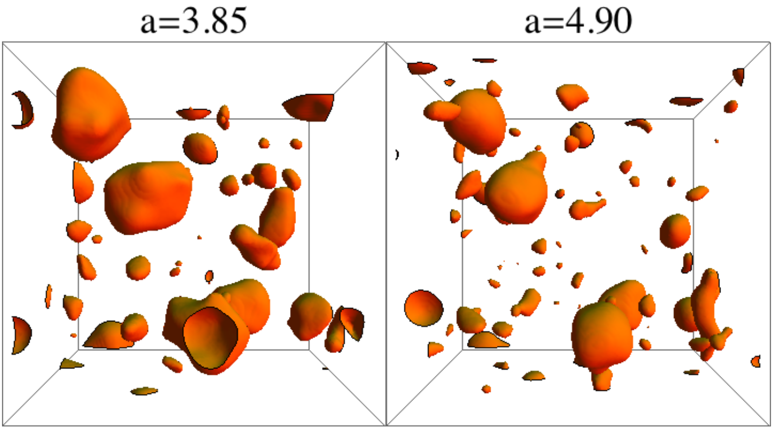

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$$V(\phi) = m^2 M^2 \left[\left(1 + \frac{\phi^2}{M^2} \right)^{\alpha} - 1 \right] \tag{11}$$

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- Potentials for which $V(\phi) \sim \phi^{2\alpha}$ with $\alpha < 1$ at large ϕ arise in a wide variety of string and supergravity scenarios!
- Quartic inflation ($\alpha = 2$) is ruled out, and even quadratic inflation ($\alpha = 1$) is somewhat disfavored, relative to models with $\alpha < 1$.

Oscillons can form in potentials of the form:

$$V(\phi) = \frac{m^2\phi^2}{2} + U(\phi) \tag{12}$$

where $U(\phi) < 0$ for *some interval* of the field ϕ.

Resonance and Oscillons in Monodromy Inflation

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- For our monodromy model this requirement is satisfied if $\alpha < 1$.
- If M is significantly sub-Planckian, $U(\phi)$ is both negative and non-vanishing as the field oscillates about $\phi = 0$. This yields resonance and oscillon production!

Monodromy Oscillons!

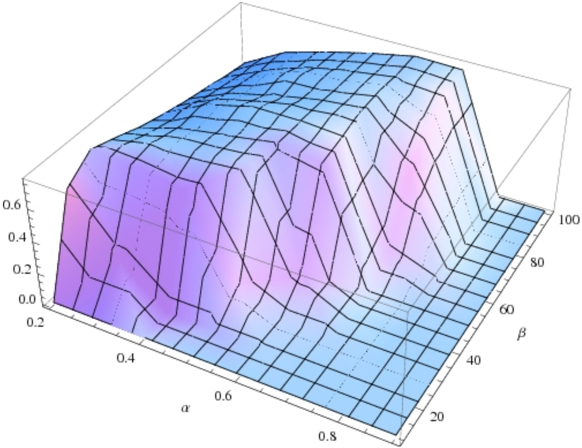

Figure: The fractional energy density in oscillons after a monodromy-inflation preheating phase as a function of α and β.

Monodromy Oscillons!

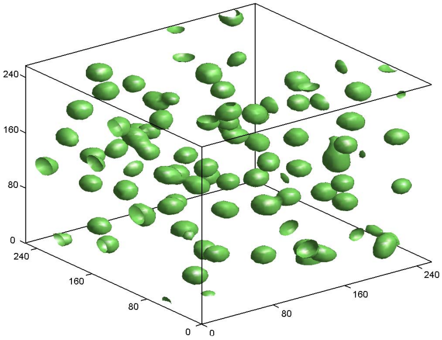

Figure: Plot from Zhou et al. (2013) - with $\alpha = 1/2$ and $M = 0.01 M_P$. The box size is $L = 50/m$ and the energy density isosurface is taken at a value 5 times the average energy density.

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Metric-Perturbation Evolution

Write the metric as:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu} \tag{13}$$

Then metric perturbation obeys:

$$\Box h_{\alpha\beta} - \hat{g}_{\alpha\beta}\Box h + h_{;\alpha\beta} + 2\hat{R}^{\mu\nu}_{\alpha\beta} h_{\mu\nu} - h^{;\mu}_{\alpha\mu;\beta} - h^{;\mu}_{\beta\mu;\alpha} + \hat{g}_{\alpha\beta} h^{;\mu\nu}_{\mu\nu} = -16\pi G \delta T_{\alpha\beta} \tag{14}$$

which simplifies after a gauge is chosen.

Gravitational-Wave $T_{\mu\nu}$

The stress-energy tensor associated with gravitational radiation is given by:

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$$\rho_{gw} = \frac{1}{32\pi G} \left\langle h_{ij,0} h^{ij}_{,0} \right\rangle = \sum_{i,j} \frac{1}{32\pi G} \left\langle h^2_{ij,0} \right\rangle \tag{16}$$

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The fractional contribution to the overall density per logarithmic interval in wave-number:

$$\frac{d\Omega_{gw}}{d\ln k} = \frac{1}{\rho_{crit}} \frac{d\rho}{d\ln k} = \frac{\pi k^3}{3H^2L^2} \sum_{i,j} |h_{ij,0}(k)|^2 \tag{17}$$

Preheating GW Spectrum

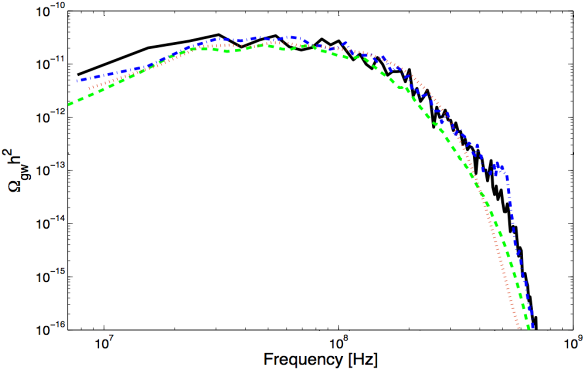

Figure: Plot by Price and Siemens showing their results along with results by: Easther, Giblin and Lim; Dufaux, et al.; and García-Bellido, et, al. - a basic $m^2\phi^2$ model

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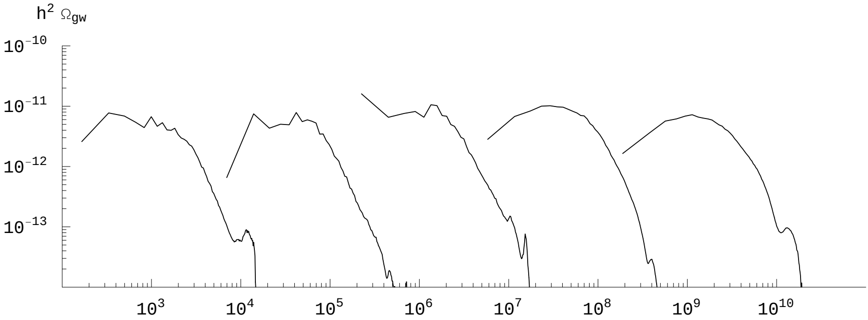

Figure: Plot by Easther, Giblin and Lim - a basic $m^2\phi^2$ model - initial energy densities run from $(4.5 \times 10^9\mathrm{GeV})^4$ to $(4.5 \times 10^{15}\mathrm{GeV})^4$

General Features of Preheating Spectrum

General features of the peak in the gravitational-wave spectrum from preheating:

- GUT-scale inflation: MHz-GHz. Inflation at 10 TeV: 10^{-2} Hz. Peak $\approx 1/(\text{inflation scale})$.

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- Most power occurs in a narrow frequency band, rapid-drop-off k^3 high-frequency tail.
- The higher the inflationary scale the more post-inflation growth takes place and the smaller the wavelength of the resonant modes.
- Maximal production: $\frac{d\Omega_{gw}}{d\ln(k)} \approx 10^{-5}$, 10^{-10} today.

Experiment?

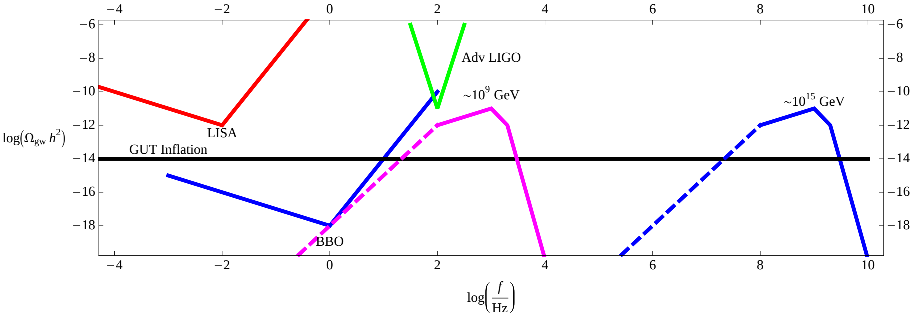

Figure: Plot by Easther, Giblin and Lim

Monodromy Oscillons!

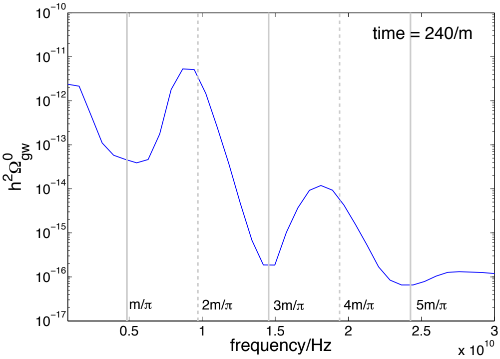

Figure: Plot from Zhou et al. (2013) - with $\alpha = 1/2$ and $M = 0.01M_P$ at $t = 240/m$. The box size is $L = 25/m$. The vertical lines correspond to the gravitational wave frequencies associated with the different harmonics of the oscillon, which are twice the frequencies of the oscillon harmonics. The values indicated are the frequencies before redshifting to today's frequencies. - $f_{\rm gw}^{typical} \sim 10^8$ Hz and $h^2\Omega_{\rm gw}^{typical} \sim 10^{-14}$ today.

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Acknowledgments

I would like to thank:

- Richard Easther, Mustafa Amin, and my other collaborators.
- Tom Giblin and Eugene Lim, and everyone else who has helped over the years.
- DOE CSGF (who paid for most of this work), and DOE/ANL/ALCF, etc.

The End

“Begin at the beginning and go on till you come to the end: then stop.” - Lewis Carroll, Alice’s Adventures in Wonderland.

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- The isotropy (homogeneity) of the entire observable universe
- The extreme flatness of the observable universe
- The very-nearly-scale-free nature of the initial density-perturbation power spectrum

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- Typical regions undergo de Sitter expansion with $H \sim \sqrt{V(\phi_1)}/M_p$ (M_p is the reduced Plank mass, ϕ_1 is the location of the minimum).
- Small regions may tunnel to another minimum ϕ_2, $V(\phi_2) < V(\phi_1)$, forming a "bubble."

Example Potential for Bubble Collision Scenario

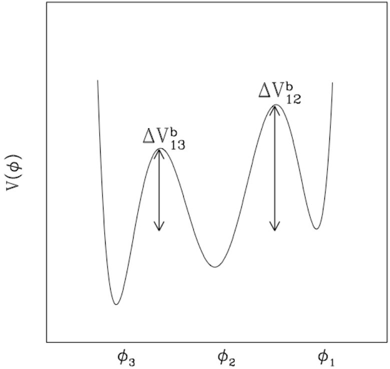

Figure: Diagram of $V(\phi)$ supporting bubble collisions scenarios. Figure from Easther, et al., 2009

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- Produce Hawking radiation as they decay, including gravitational radiation. Could cause a matter-dominated phase.
- For average masses larger than ≈ 1 gram, constrained by nucleosynthesis, x-ray background, dark-matter abundance.

Other Codes

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- LatticeEASY: Felder and Tkachev (2000)
- DEFROST: Frolov (2008)
- PSpectRe: Easther, Finkel and Roth (2010)
- HLattice: Huang (2011)

PSpectRe

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- Parallelized using OpenMP.
- Naturally integrates with Fourier-space $h_{\mu\nu}^{TT}$ evolution.

Energy Conservation?

GR's dynamic metric does not generally allow for a conserved energy. In this case, the FRW background is changing, but homogeneous, and so we have (the averaged Friedmann equation):

$$\frac{\langle \rho \rangle}{3H^2} - 1 \tag{18}$$

And it should be as good as the homogeneity assumption (parts in 10^7).

4^{th} Order vs 2^{nd} Order

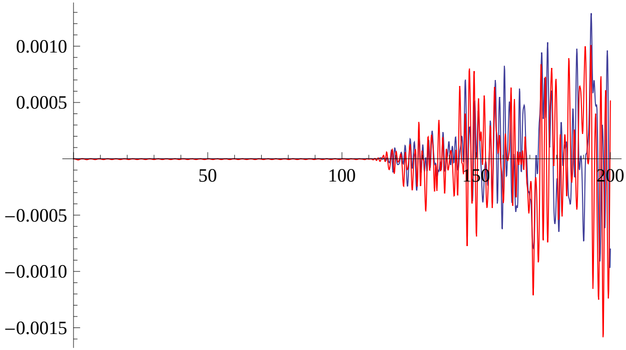

Figure: PSpectRe runs at 32^3 ($L = 2$ and the time step is 0.005). The red line uses the Verlet integrator, blue shows the Runge-Kutta results.

Padding Helps

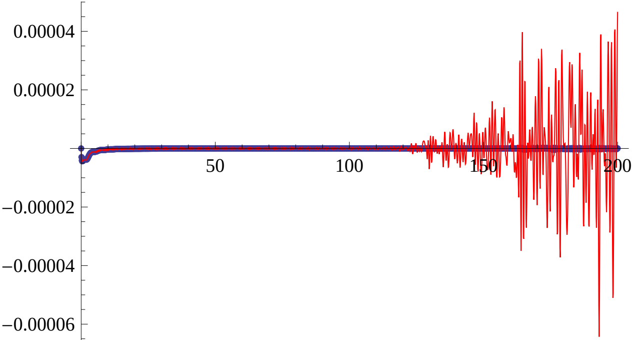

Figure: PSpectRe: Red is unpadded, blue is padded by a factor of 2.

Compare with Defrost: Energy Conservation

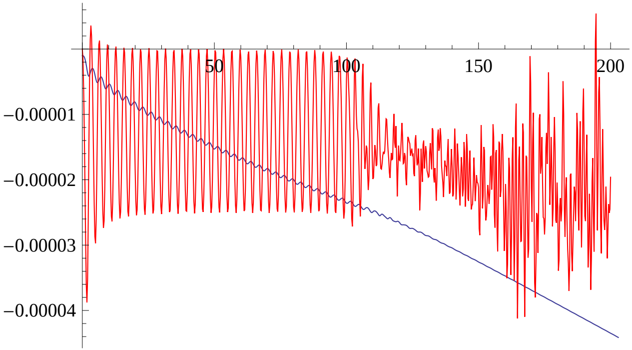

Figure: Runs with 256^3 points and $L = 10$ (for Defrost's default model).

PSpectRe's convergence for equation-of-state observables is better than Defrost's too (see the paper).

FFT

FFT = Fast Fourier Transform (transforms from (discrete) position space to "frequency space")

$$\Phi(\vec{k}) = \sum_{\vec{r}} \phi(\vec{r}) e^{-i\vec{k}\cdot\vec{r}}, \tag{19}$$

$$\phi(\vec{r}) = \frac{1}{N^3} \sum_{\vec{k}} \Phi(\vec{k}) e^{i\vec{k}\cdot\vec{r}}. \tag{20}$$

FFT evaluates these using a recursive decomposition: $O(n \log n)$.
ϕ is real: $\phi(\vec{r}) = \phi(\vec{r})^\star$, so $\Phi(\vec{k}) = \Phi(-\vec{k})$ and the number of free parameters matches in both representations.

Derivatives in Fourier Space

Each derivative operator brings down a factor of $-ik$, so:

$$\nabla^2 \rightarrow \vec{k} \cdot \vec{k}$$

And so (for example):

$$\int_{\text{box}} |\nabla \phi|^2 = \frac{1}{N^3} \sum_{\vec{k}\_\text{space}} |\vec{k}|^2 |\Phi(\vec{k})|^2 \tag{21}$$

A Complication: Mode Aliasing

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- This works if the modes $\frac{\pi(N+2)}{L}$ through $\frac{2\pi(N-1)}{L}$ are negligible, compared to the modes $-\frac{\pi(N-2)}{L}$ through $-\frac{2\pi}{L}$.

Nonlinear Terms

Terms such as $\chi^2\phi$ are implemented as:

- (Optionally) Pad the Fourier-space grid.
- Perform an inverse FFT (transform to position space).
- Compute the nonlinear operation.
- Perform an FFT (transform to Fourier space).
- (Optionally) Unpad the Fourier-space grid.

Padding in Fourier space is equivalent to performing a polynomial fit using all of the available data points and then filling in using interpolation. It is a bit tricky to implement when using a conjugate-symmetry-reduced storage layout; the details are in the paper.

Code by Easther, Giblin and Lim

Easther, Giblin and Lim (2006, 2007, 2008), published work on preheating models. Bubble-collision and Oscillon calculations in progress.

- Evolves $h_{\mu\nu}^{TT}$ in Fourier space given real-space inputs.

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- No scalar or vector pieces, no back-reaction.

What Others Do

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- Estimate by integrating a Green's function for $h^{TT}_{\mu\nu}$ assuming a particular expansion history (matter or radiation dominated, etc.). (Price and Siemens, 2008).
- Assuming Gaussian initial conditions and that (mean) vorticity vanishes, evolve uncoupled h_{ij} (back-reaction included, approx. effect unknown). (García-Bellido, et, al., 2008).

PSpectRe Does Quite Well!

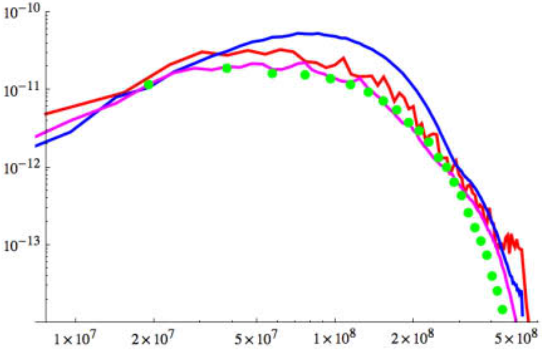

Figure: PSPectRE+h_{ij} at 128^3 (dots) vs. LatticeEasy+h_{ij} at 128^3, 256^3, 512^3. The PSpectRe+h_{ij} run beats the largest LatticeEasy+h_{ij} run (which had been used for publication).

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- Preprint posted on Feb. 1, so I'll have more to say after I've tried it...

MPI Defrost

- Based on Frolov's Defrost code, modified by changing array indexing, adding MPI calls, etc.

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- Used for some 1024^3 oscillon calculations.