Disentangling the complexity of the cosmic web

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OUTLINE

- Introduction
- Density or particles (particles → density)
- Zeldovich Approximation
  - Lagrangian Submanifold, Phase space
  - 1D, 2D, 3D, 4D, 6D
  - Caustics, Skeleton of Cosmic Web
- Parity, Number of Streams field, volume void fraction
- Number of flip-flops field
- Summary

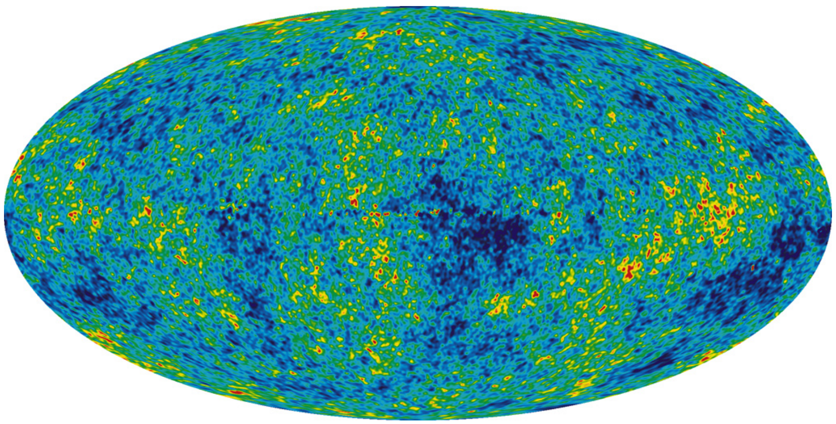
It is a profound and necessary truth that the deep things in science are not found because they are useful;

they are found because it was possible to find them.

Robert Oppenheimer

Structure in the Universe

13.7 billion years ago



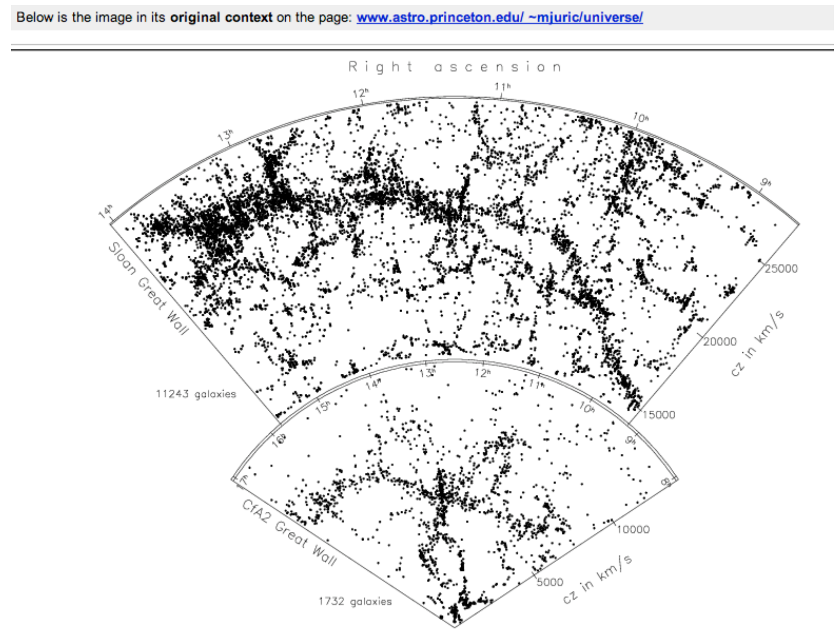
WMAP

Temperature fluctuations of Cosmic Microwave Background

T=2.73 K

(fluctuations: of the order of 1/100,000)

NOW



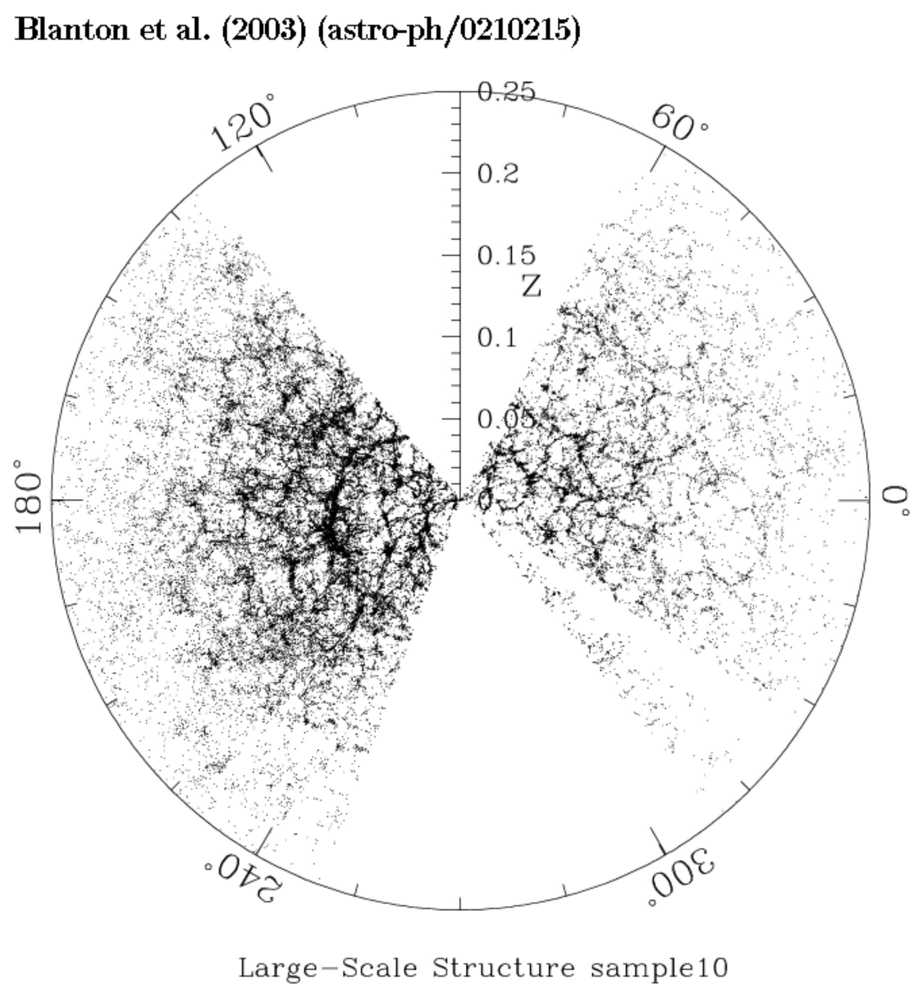
SDSS

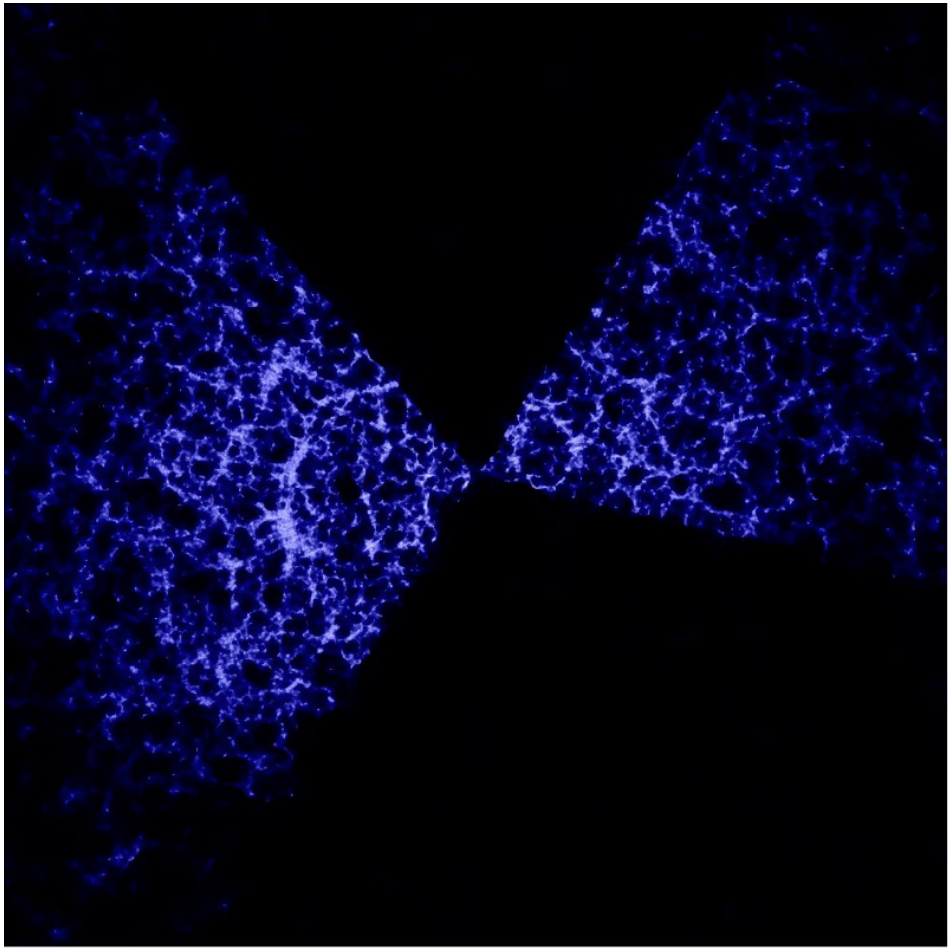
Galaxy distribution in a thin slice

The structure in the Universe

Sloan Digital Sky Survey

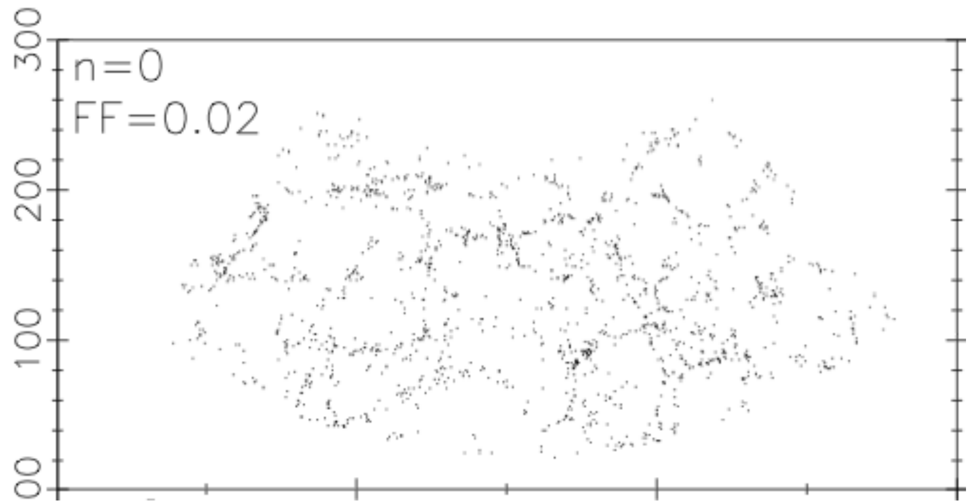
Dots OR Density Contours?

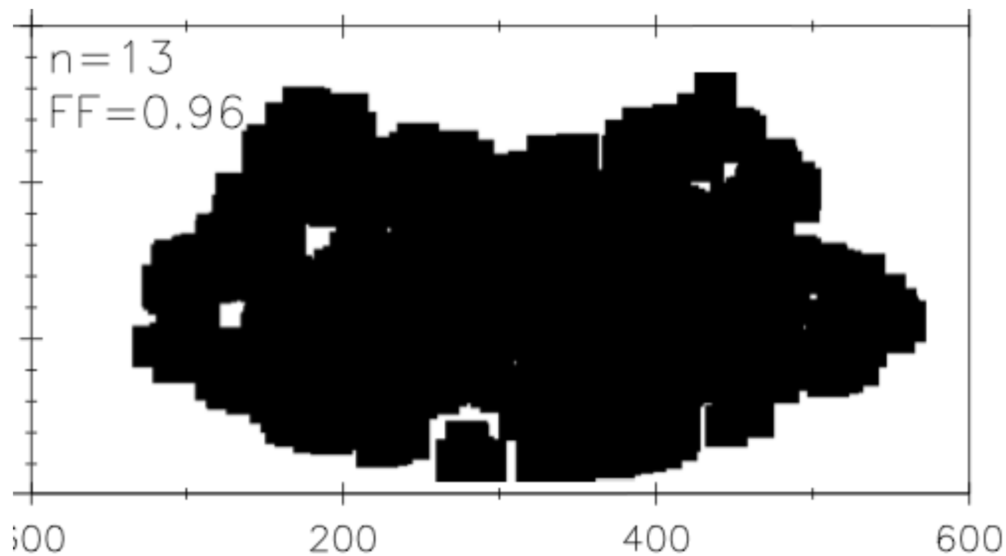




Delaunay Tessellation Field Estimator
www.astro.rug.nl/~weygaert/**dtfesdss**.html

What is common between two plots?





Bharadwaj et al 2000
ApJ 528, 21

"Evidence for Filamentarity in the Las Campanas Redshift Survey"

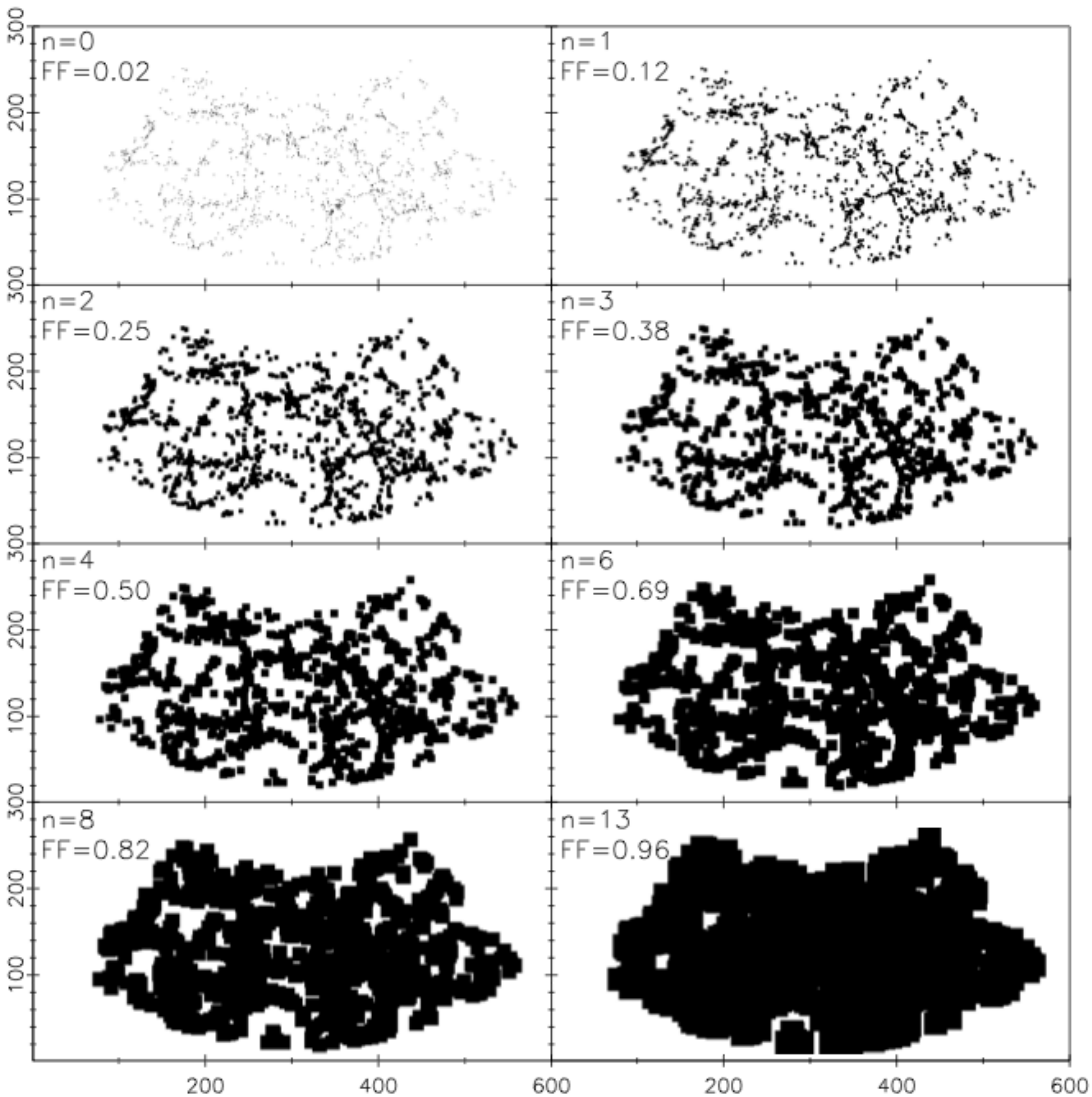


FIG. 1.—First northern LCRS slice, shown at different values of coarse-graining (n). The value of the filling factor (FF) at each level of coarse-graining is also shown. Here $n = 0$ shows galaxies in the original N1 slice without coarse graining. The axis units are h^{-1} Mpc.

Klypin & Shandarin 1983: Dot plots (preprint Inst. Appl. Math Moscow, 1981)

3D numerical model of the Universe 899

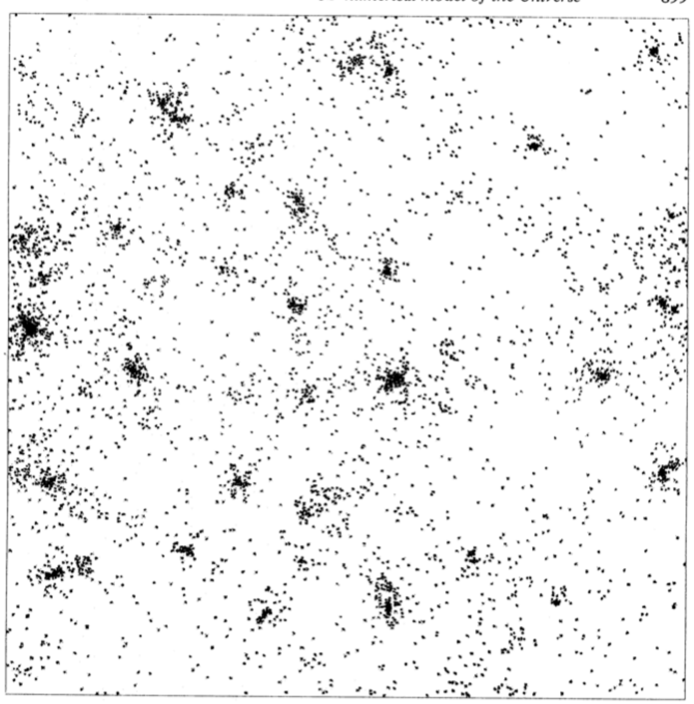
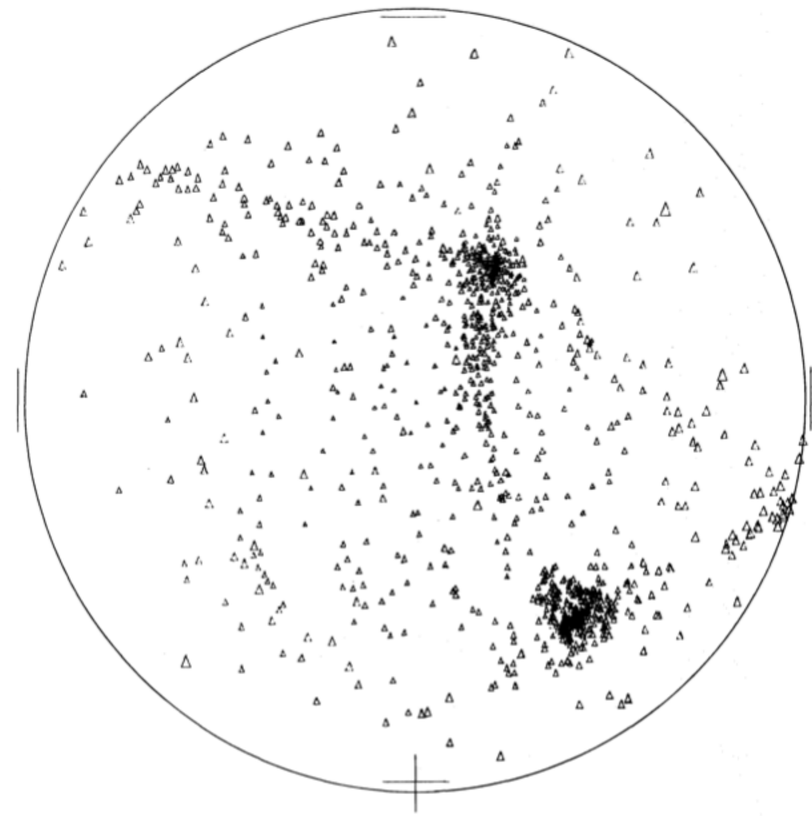


Figure 2 (c)

900 A. A. Klypin and S. F. Shandarin



(a)

Figure 3. A small fragment of the particle distribution plotted in Fig. 2(b), when $a = 13.6\,a_{\mathrm{start}}$. As opposed to Fig. 2 here all particles in the sphere with radius $R = 6r_0 = 30h^{-1}$ Mpc are plotted. Every particle is depicted as a triangle whose size is inversely proportional to distance from an observer. The observer is situated at a distance 1.5 R from the centre of the sphere.

Klypin & Shandarin 1983; Shandarin & Klypin 1984

First demonstration of filaments in 3D N-body by plotting density contours

3D numerical model of the Universe 903

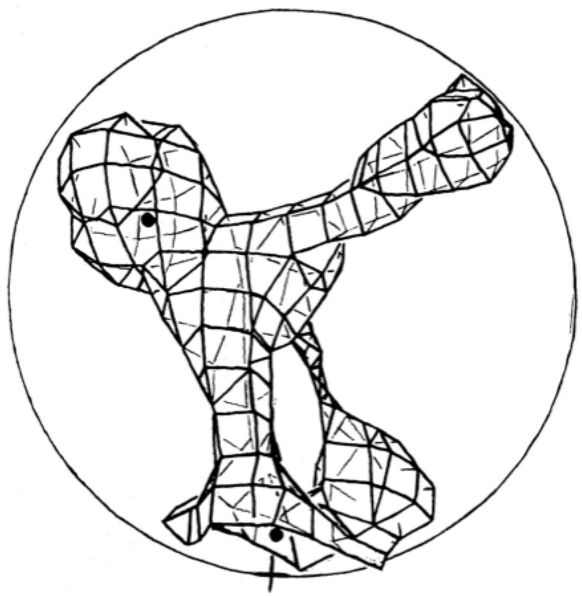


Figure 4. A surface of constant density level is plotted for the same region as that in Fig. 3.

'Cosmic chicken'
(according to C. Frenk)

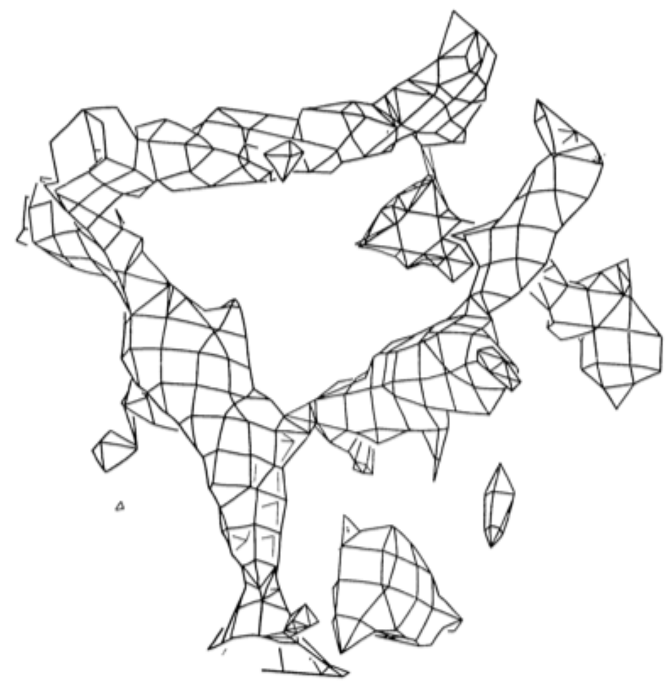


FIG. 1. A typical model isodensity surface, $\rho = 2.5\ \bar{\rho}$, within a randomly selected sphere of radius 45 h^{-1} MPc.

Frenk, White, Davis 1983

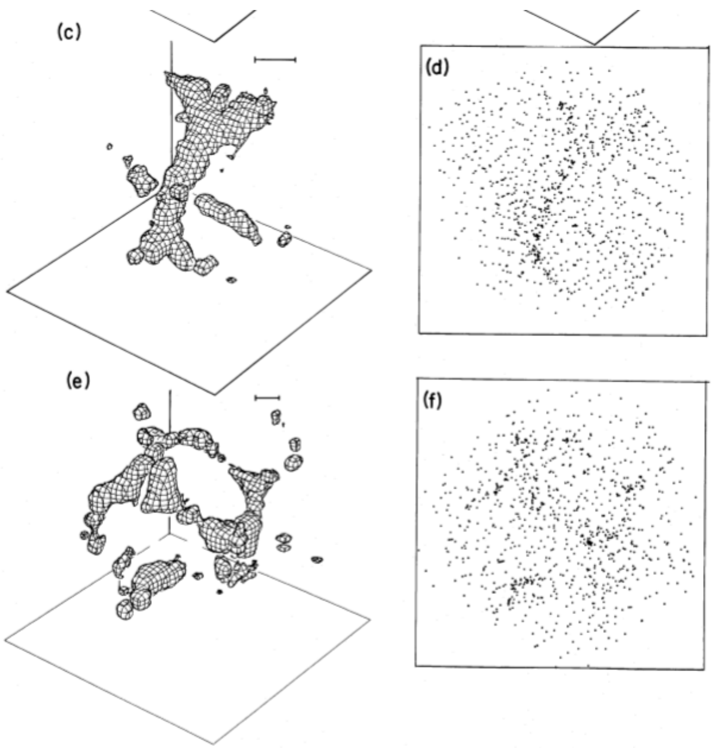
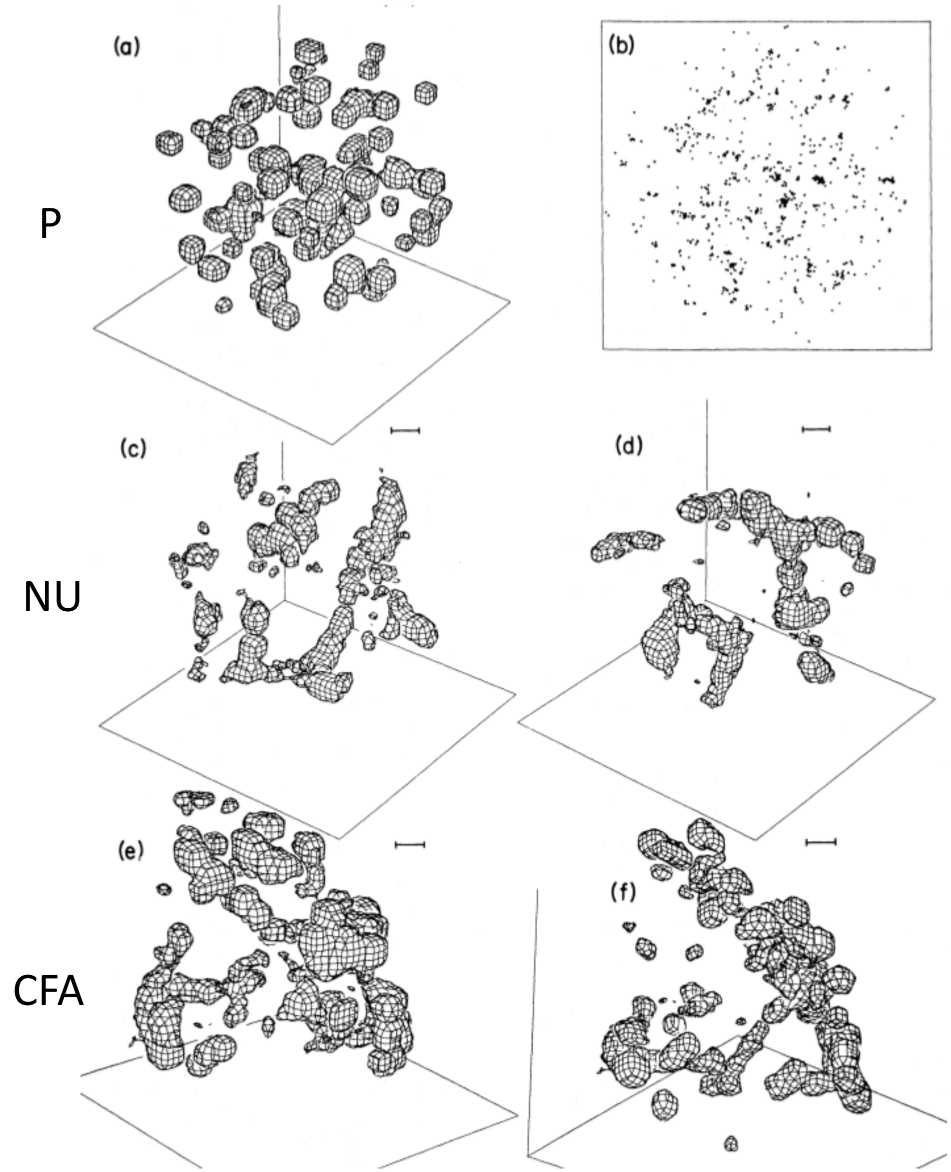


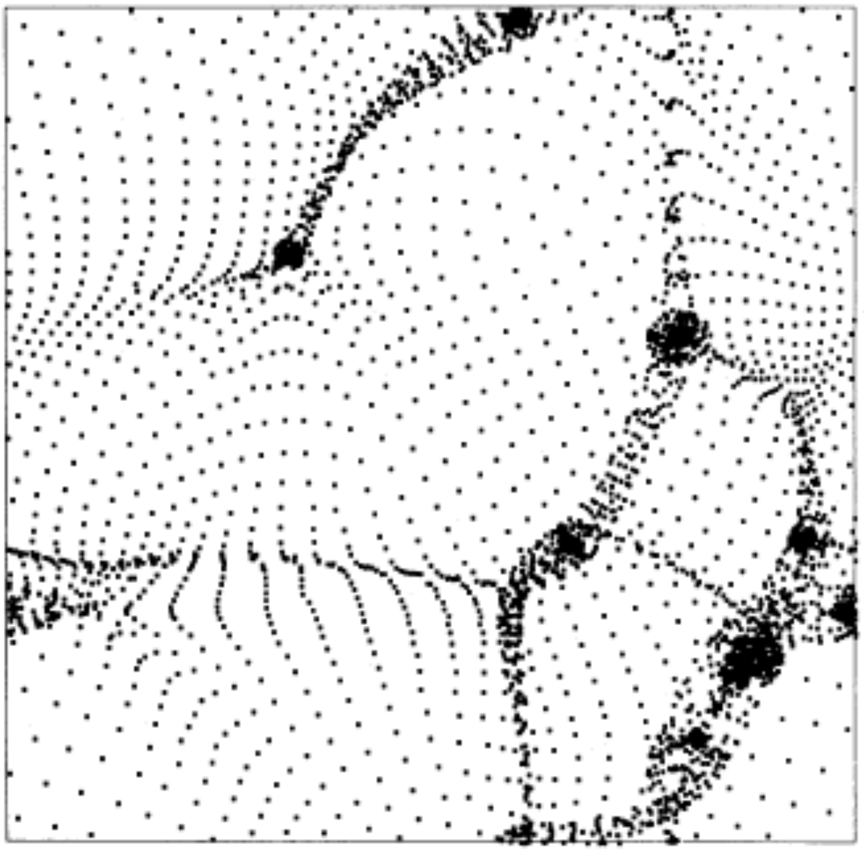
FIG. 5.—Isodensity surfaces and projected point distributions. (*a*), (*b*), (*c*), and (*d*) Pancake models with $\lambda_{min}/R_i = 1$; (*e*) and (*f*) pancake model with $\lambda_{min}/R_i = 0.6$. Isodensity surfaces are contoured at 4 times the mean density; the bar on the upper right-hand corner represents 10 Mpc with the scalings described in the text. The point distributions are projections along the same line of sight and on the same scale as the surfaces depicted to the left of each panel.

428

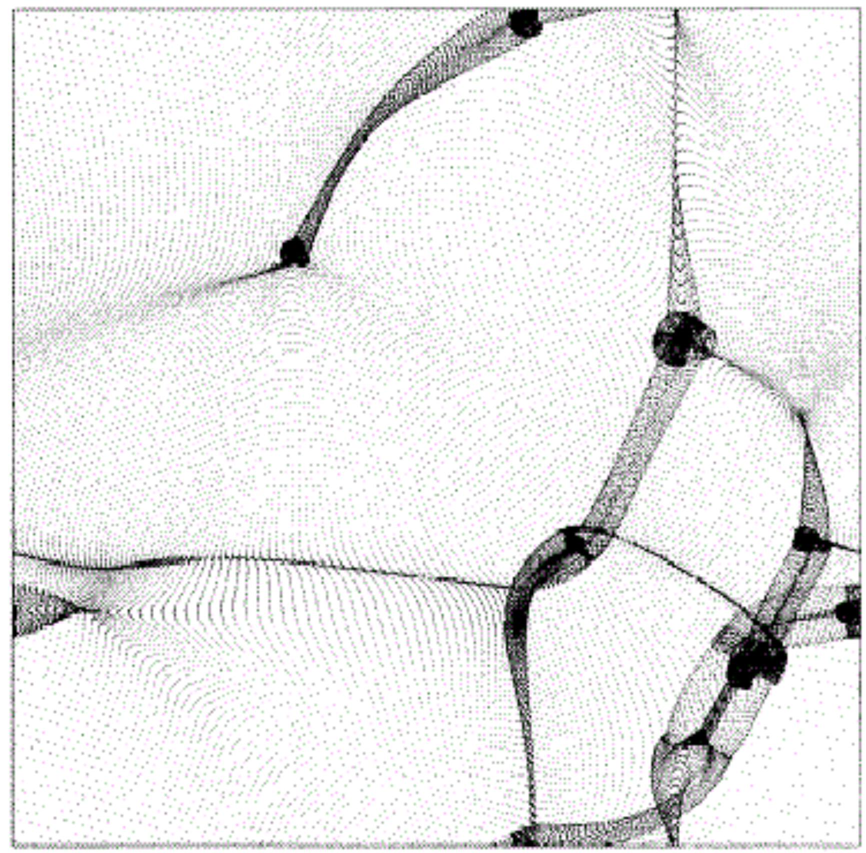


2D N-body simulations
(brute force approach)

64^2 particles,   64^2 mesh



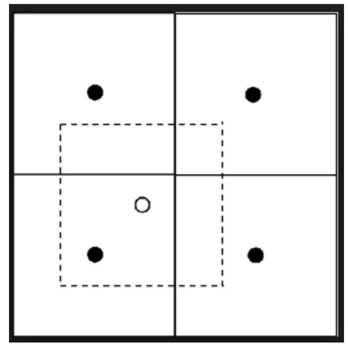
512^2 particles,   512^2 mesh



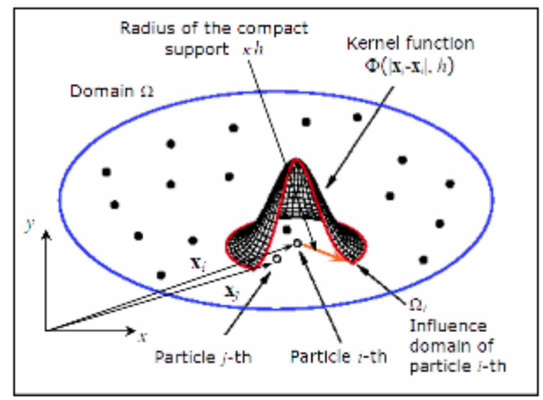
It works, but computationally very expensive.

From particles to density

CIC = Cloud in Cell



SPH = Smooth Particle Hydrodynamics



Voronoi tessellation and Delaunay triangulation

Tessellation of Lagrangian Submanifold

Georgy Voronoi 1868-1908

Zhuravka, Ukraine, Russian Empire

By 1913, he became one of the top three Russian mountain climbers.
One of the mountains (4300 m) is named after him.

Boris Delaunay 1890-1980
St. Petersburg – Moscow

Delaunay triangulation and Voronoi tessellation

2 Rien van de Weygaert & Willem Schaap

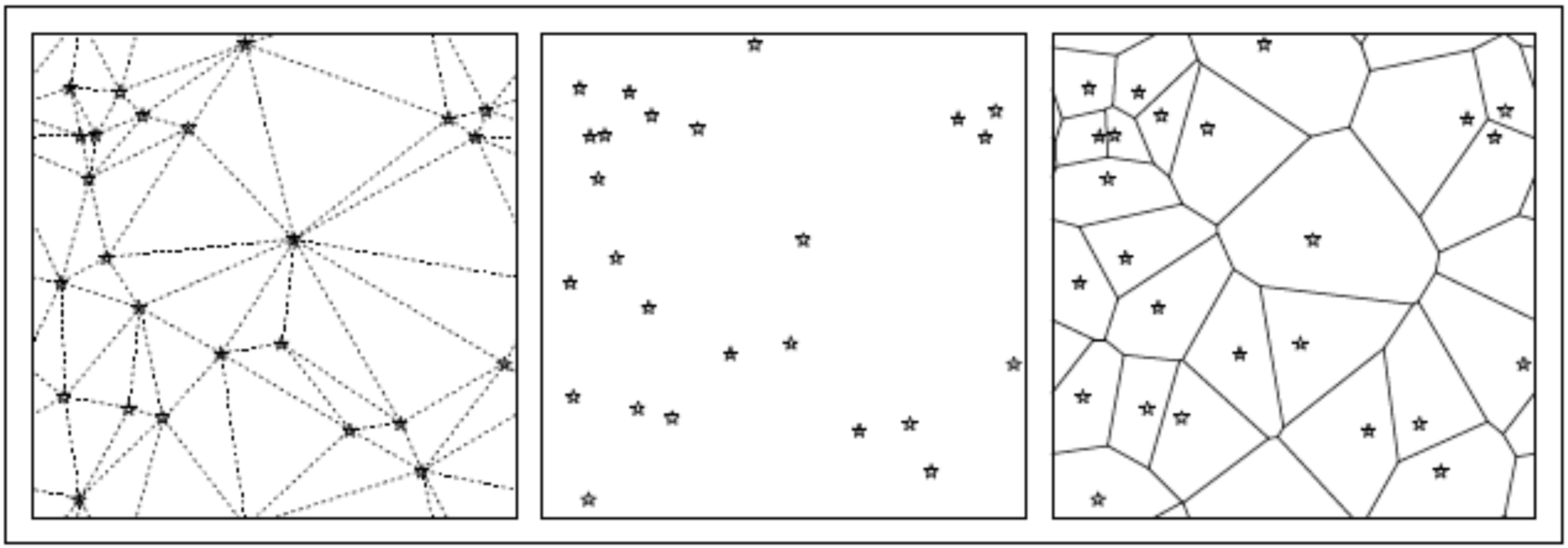


Fig. 1. The Delaunay triangulation (left frame) and Voronoi tessellation (right frame) of a distribution of 25 nuclei (stars) in a square (central panel). Periodic boundary conditions are assumed.

Computational Geometry Algorithm Library

Projects Using CGAL (http://www.cgal.org)

Molecular Modeling

Particle Physics, Fluid Dynamics, Microstructures

Medical Modeling and Biophysics

Geographic Information Systems

Geology and Geophysics

Games

Motion Planning

Sensor Networks

Architecture, Buildings Modeling, Urban Modeling

Astronomy

2D and 3D Modelers

Mesh Generation and Surface Reconstruction

Geometry Processing

Computer Graphics

Computer Vision, Image Processing, Photogrammetry

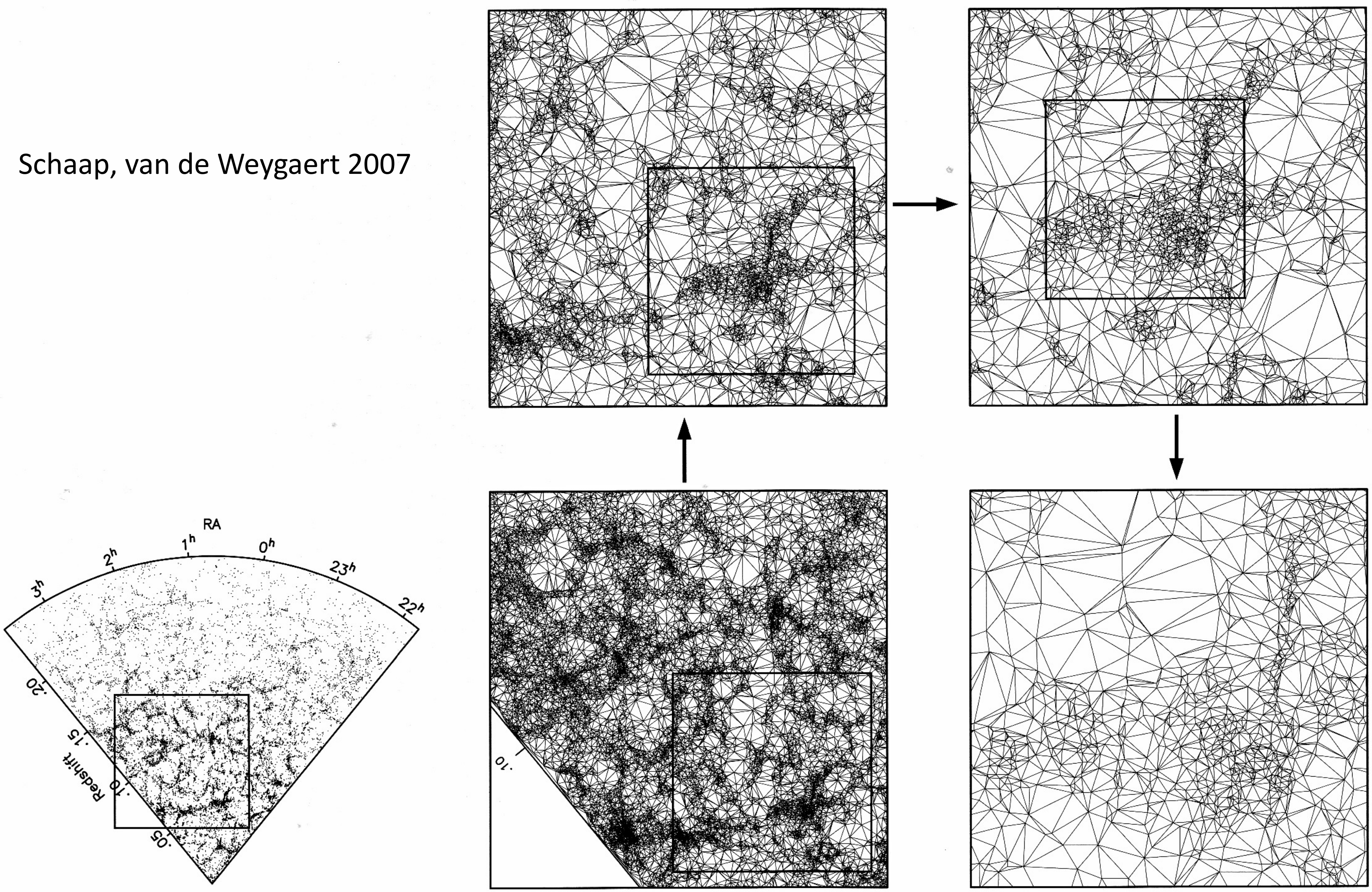
Computational Topology and Shape Matching

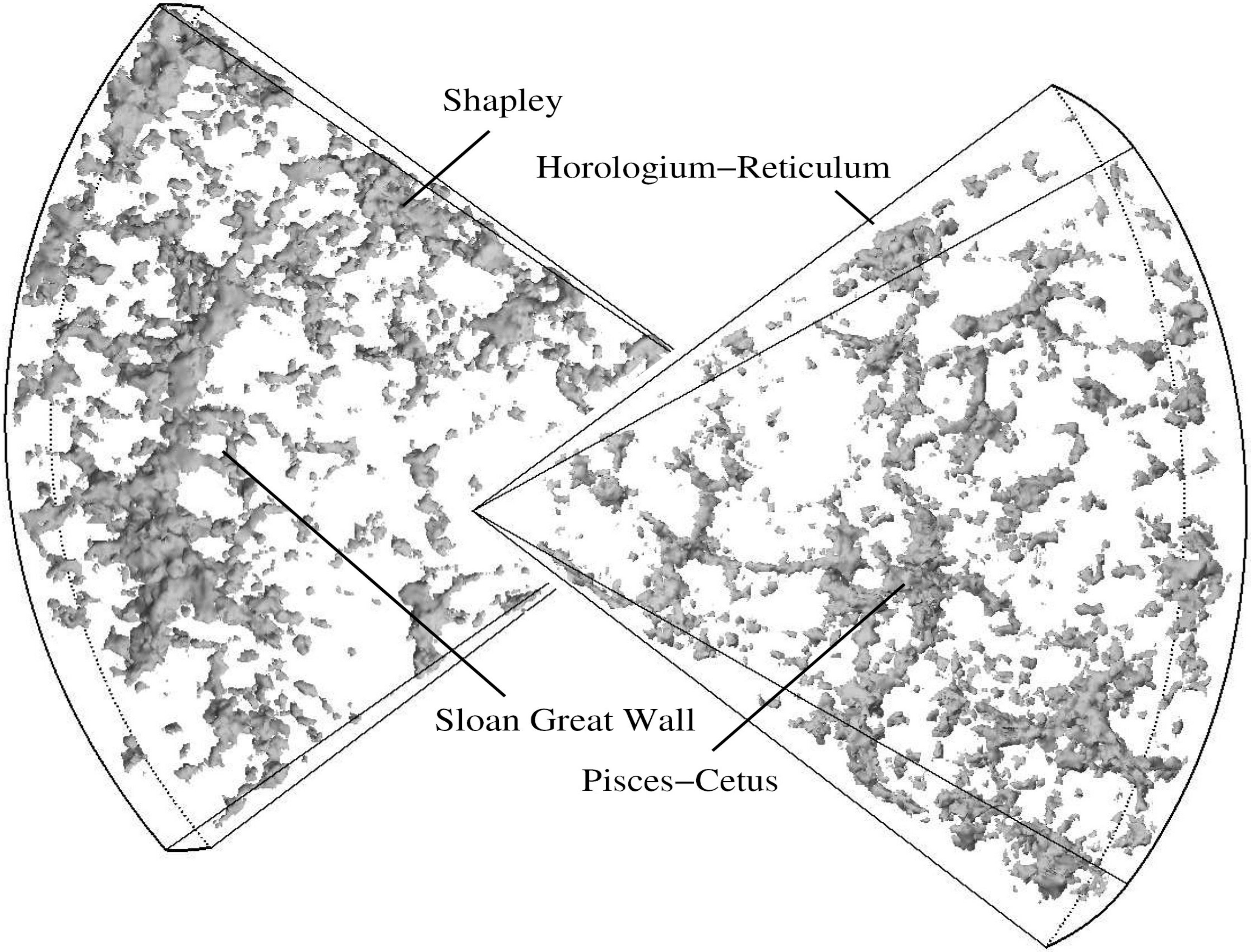
Computational Geometry and Geometric Computing

Interfaces of CGAL in other languages and platforms

Spatial resolution of DTFE

Schaap, van de Weygaert 2007

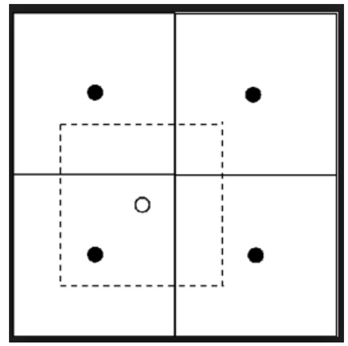




Courtesy of van de Weygaert

From particles to density

CIC = Cloud in Cell



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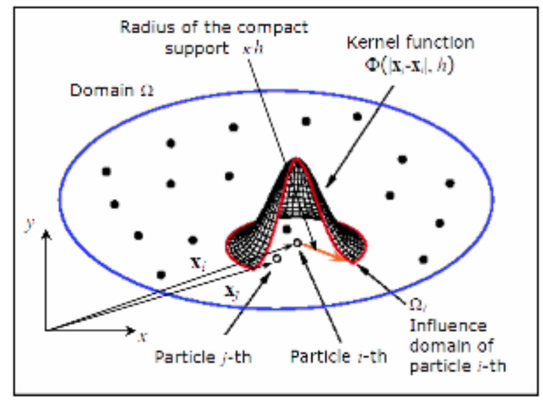
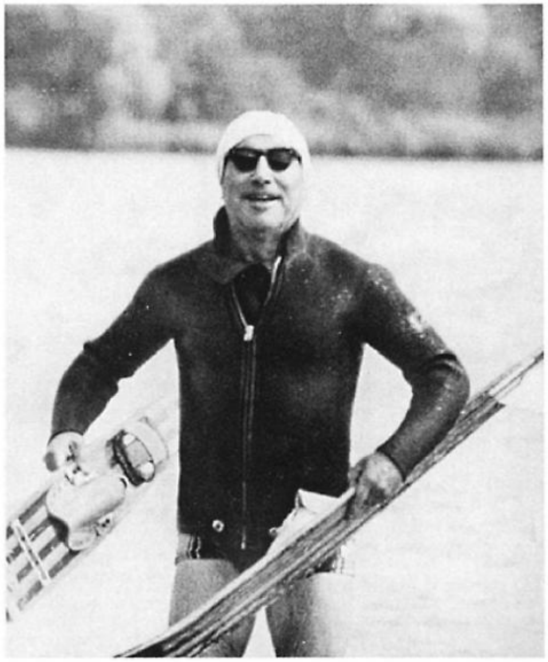


Voronoi tessellation and Delaunay triangulation

Tessellation of Lagrangian Submanifold

Yakov Borisovich
Zel'dovich
1914 - 1987


Late 1970s


'Hero of Socialist Labour'
golden star


Sarov also 'Los Arzamas'
= 'Russian Los Alamos'

Minsk
Belorussia

Stephen W. Hawking said to Zel'dovich, when he met him:

“Before I met you here, I believed you to be a 'collective author', like Bourbaki."

Zel'dovich approximation (1970)

Comoving coordinates: r_i,

Zel'dovich approximation is a map: $r_i(\mathbf{q}, t) = q_i + D(t)s_i(\mathbf{q})$

If $\Phi(\mathbf{q})$ is the linear perturbation of grav. potential then $s_i(\mathbf{q}) = -\partial\Phi/\partial q_i$

Density can be found from the conservation of mass

$$\rho(\mathbf{q}, t) = \bar{\rho}(t)\left|\frac{\partial r_i}{\partial q_k}\right|^{-1} = \bar{\rho}\left|[(1 - D(t)\alpha(\mathbf{q})]^{-1}[(1 - D(t)\beta(\mathbf{q})]^{-1}[(1 - D(t)\gamma(\mathbf{q})]^{-1}\right.$$

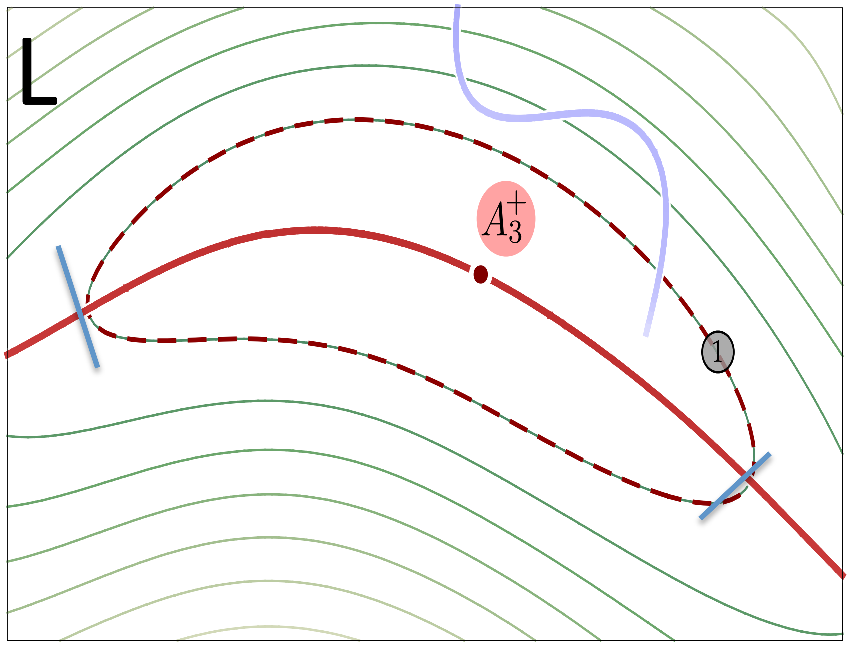
$\alpha(\mathbf{q}) \geq \beta(\mathbf{q})$ and $\beta(\mathbf{q}) \geq \gamma(\mathbf{q})$ are the eigen values of the deformation tensor

$$d_{ik}(\mathbf{q}) = \frac{\partial s_i}{\partial q_k} = -\frac{\partial^2\Phi}{\partial q_i \partial q_k}$$

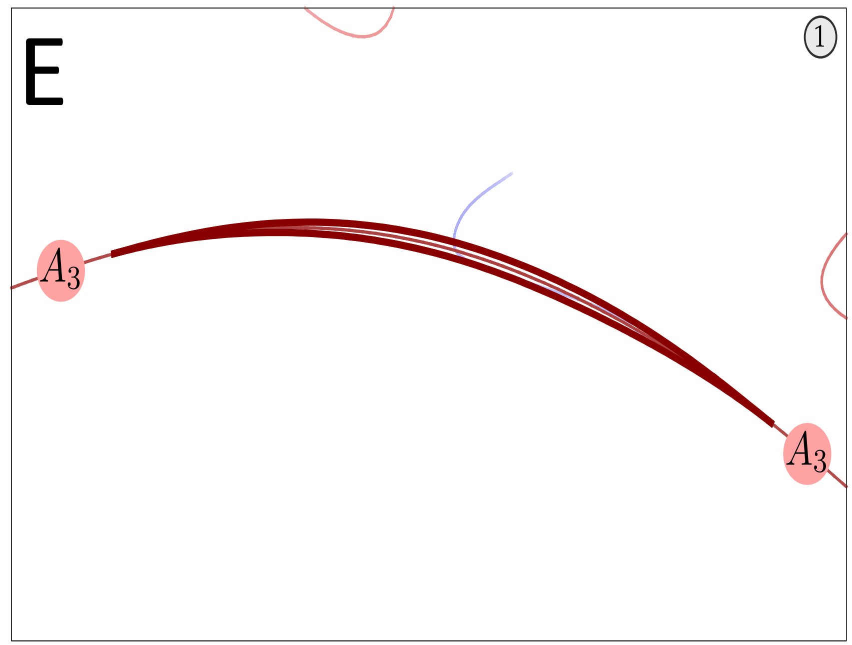
Linear density fluctuations: $\delta\rho/\rho = D(t)(\alpha + \beta + \gamma)$.

The Zel'dovich approximation describes anisotropic collapse and motion.

Zeldovich's pancake (2D)

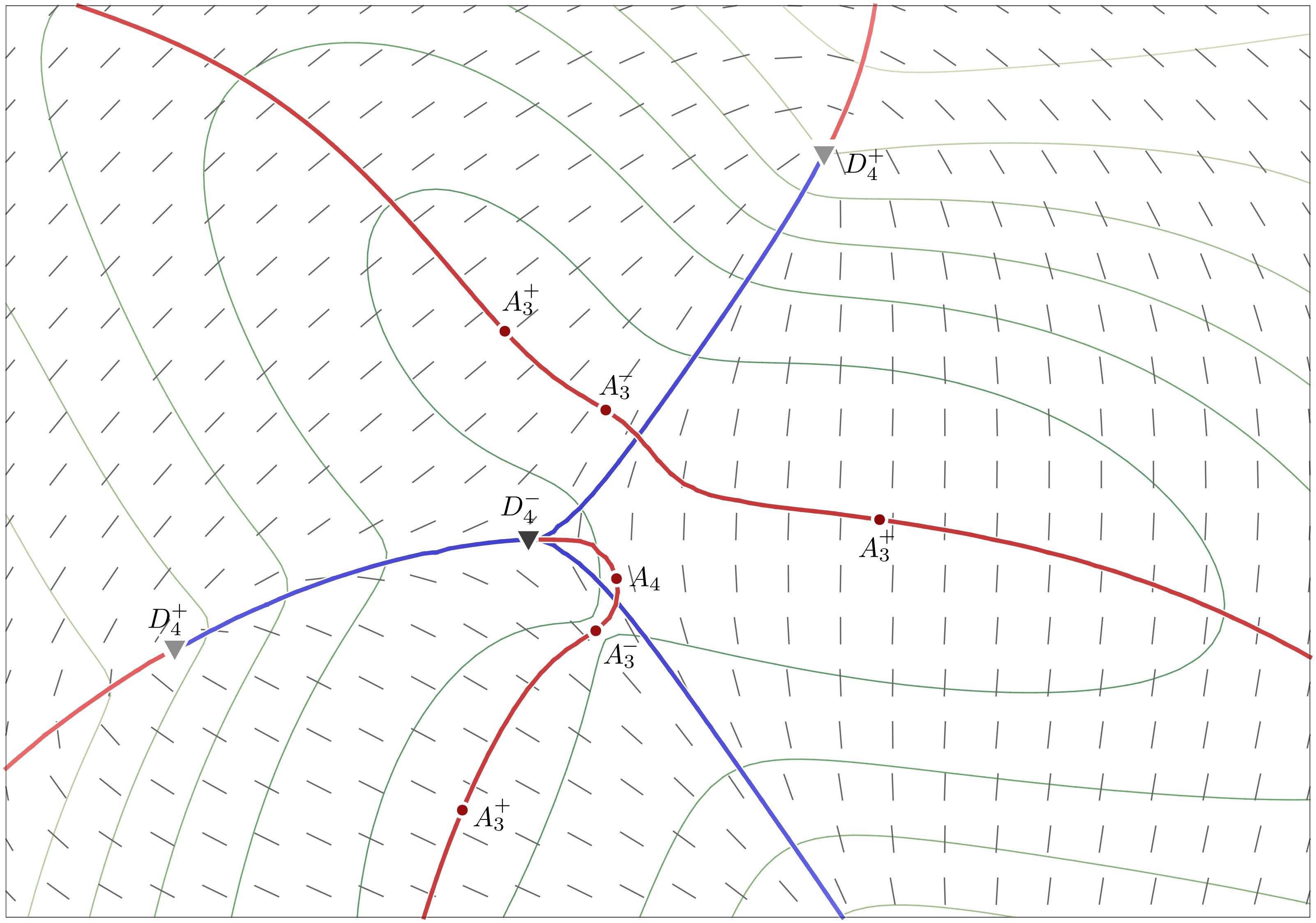


\alpha – contours in Lagrangian space



The \alpha – contour in Eulerian space

1 - D(t)$\times$\alpha(q) = 0



D_4^+
A_3^+
A_3^-
D_4^-
A_3^+
A_4
D_4^+
A_3^-
A_3^+

Lagrangian Submanifold (LS) is N-dim surface in 2N-dim space

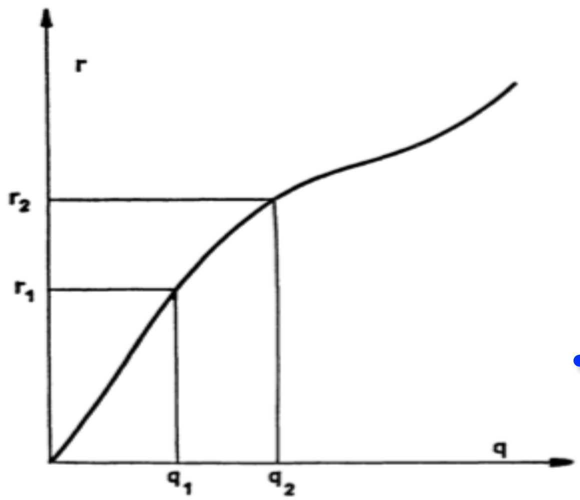
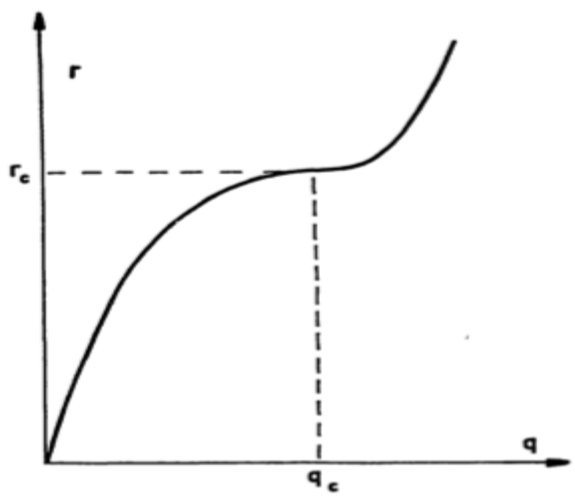


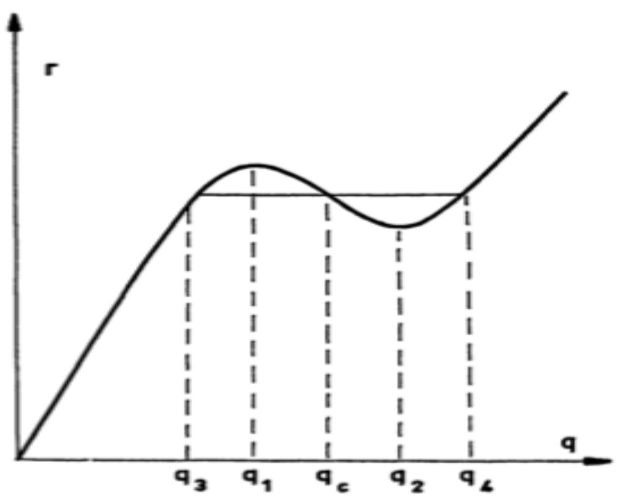
Fig. 1

r = r(q,t), r and q are vectors

OR

x = x(q_1, q_2, q_3; t)
y = y(q_1, q_2, q_3; t)
z = z(q_1, q_2, q_3; t)





Lagrangian Submanifold (LS) is N-dim surface in 2N-dim space

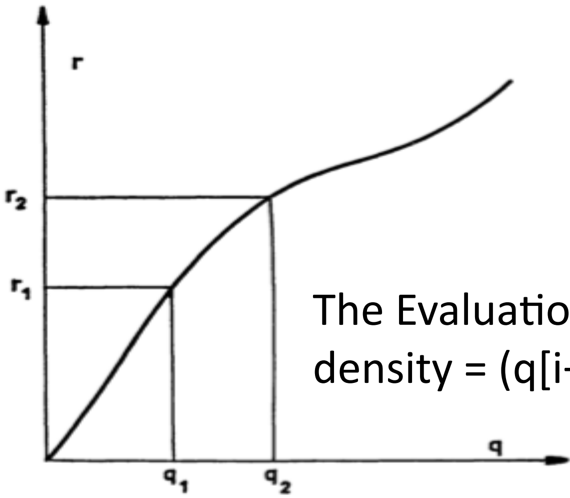
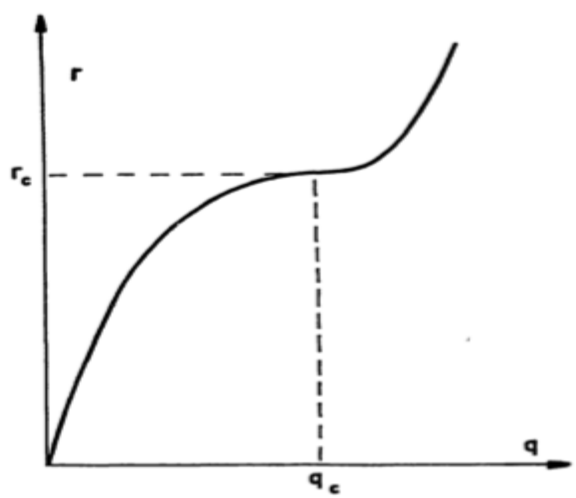
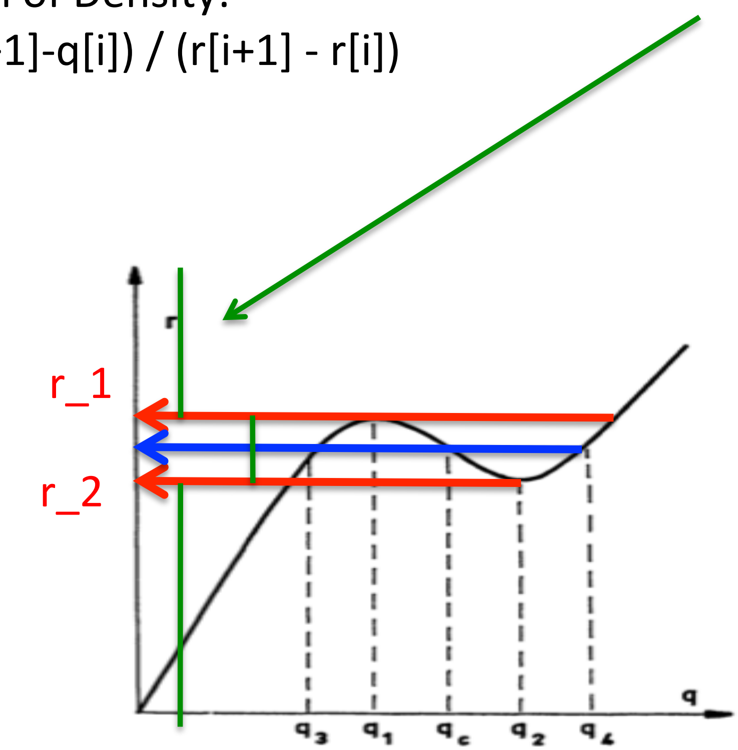


Fig. 1

The Evaluation of Density:
density = (q[i+1]-q[i]) / (r[i+1] - r[i])

Number of streams as a function of r





Phase Space Evolution

Phase space:

Velocity vs. Position

Density:

$$\rho(\vec{x},t) = \int f\left(\vec{x},\vec{v},t\right)d\vec{v}$$

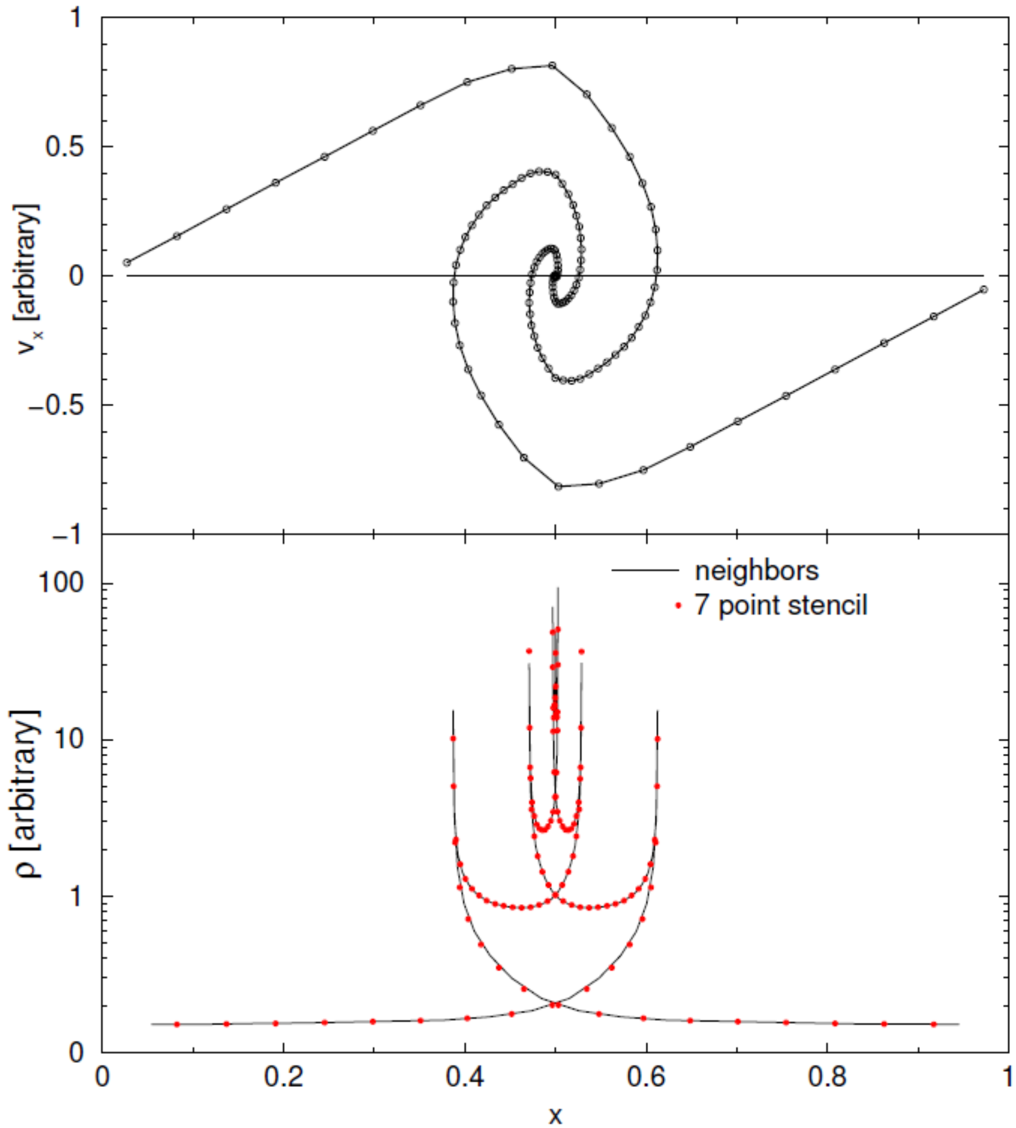


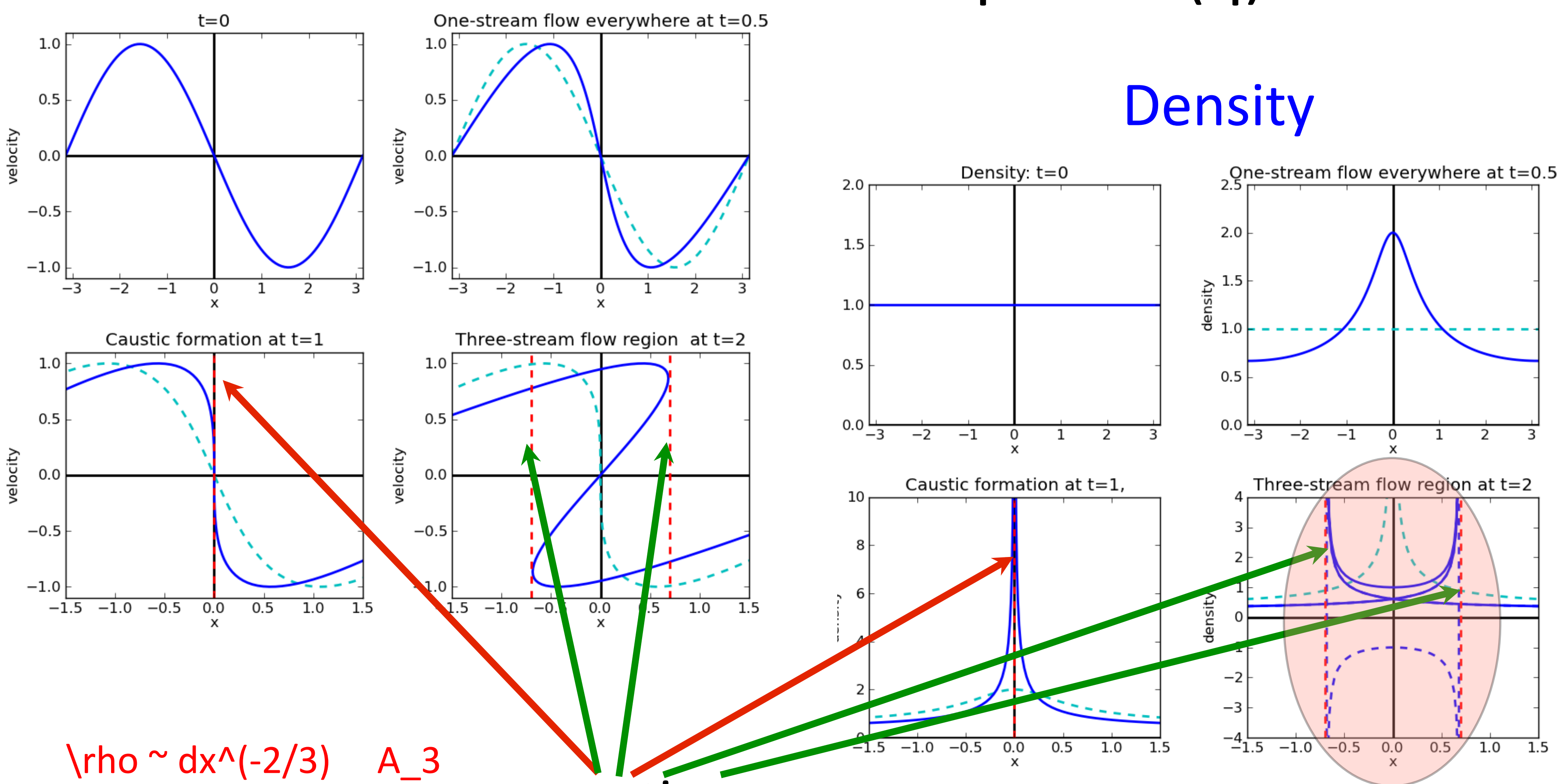
Fig. from Abel et al 2012

Multi-stream flows and caustics in collisionless Dark Matter

(one dimensional example)

Phase space

$$x = q - t\sin(q)$$

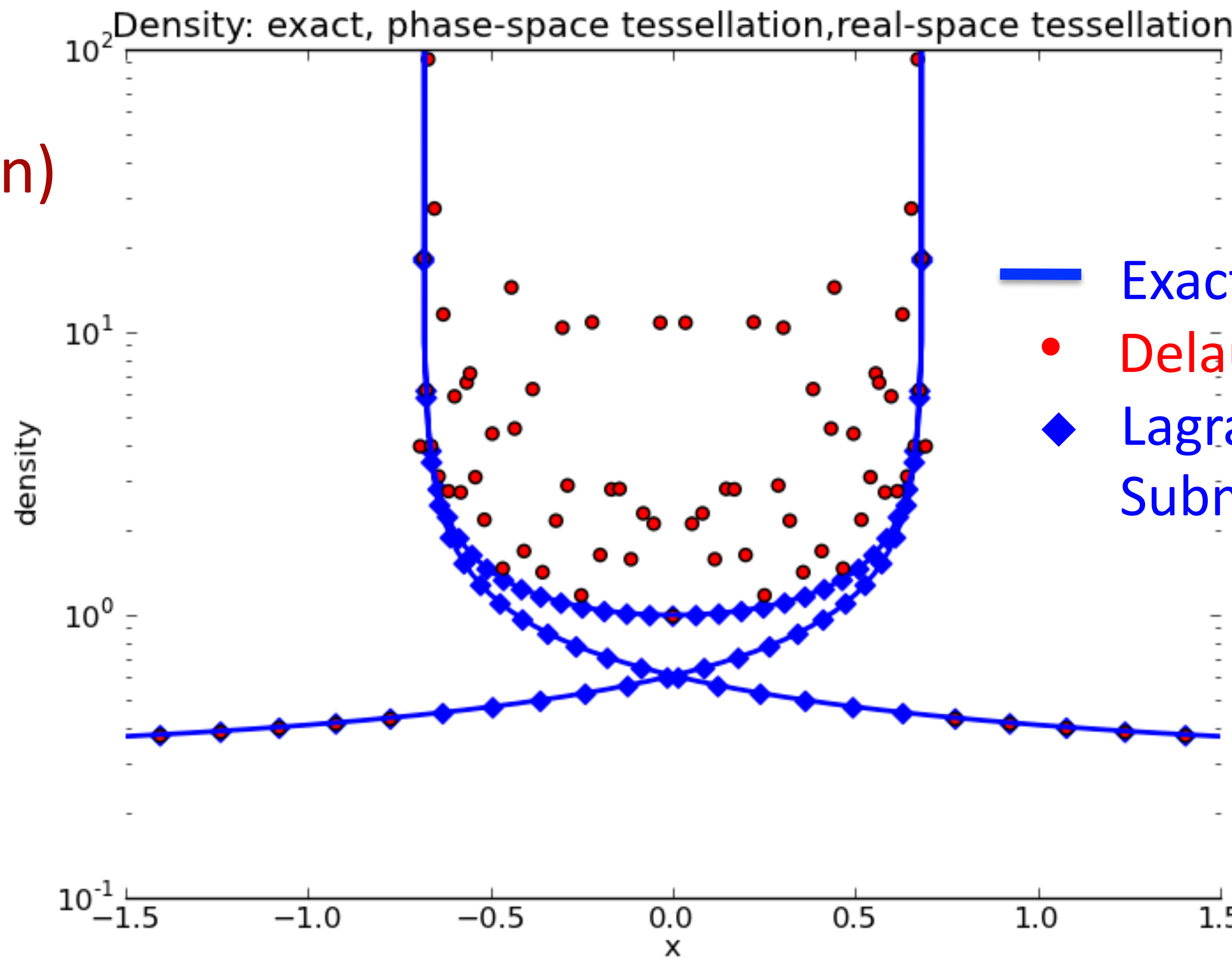


\rho ~ dx^(-2/3)   A_3

\rho ~ dx^(-1/2)   A_2

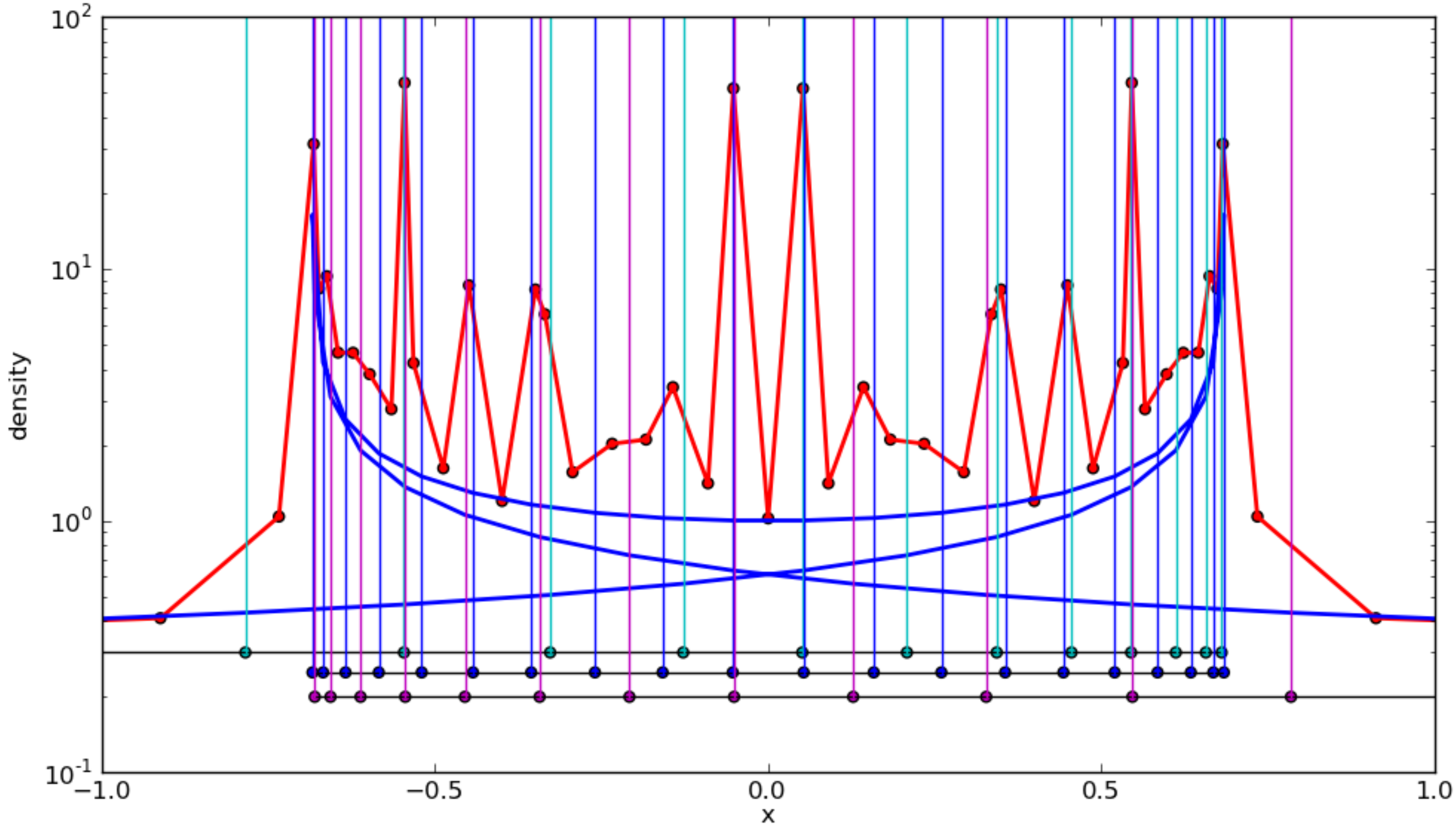
Evaluation of Density: den = 1/(x[i+1] - x[i])
(red dots: configuration space tessellation (1D Delaunay))

log(den)

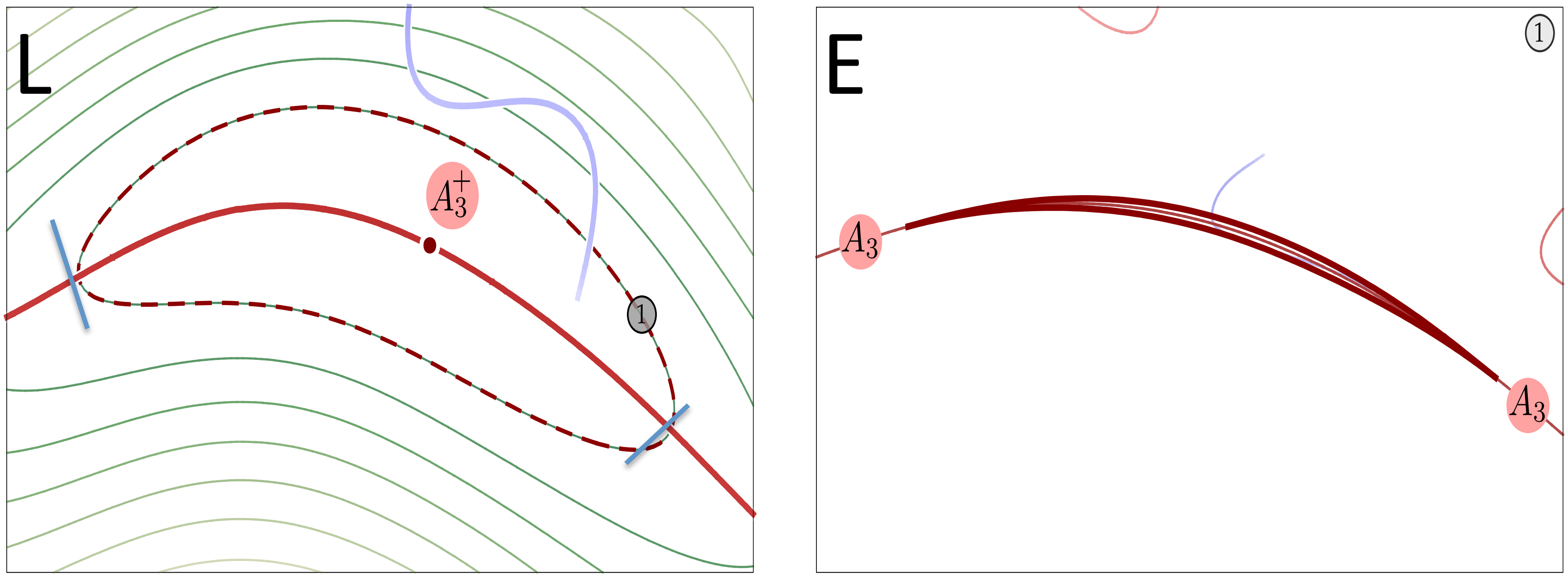


Exact
Delaunay
Lagrangian Submanifold

The Evaluation of Density: density = 1/(x[i+1] - x[i])



Zeldovich's pancake (2D)



\alpha – contours in Lagrangian space

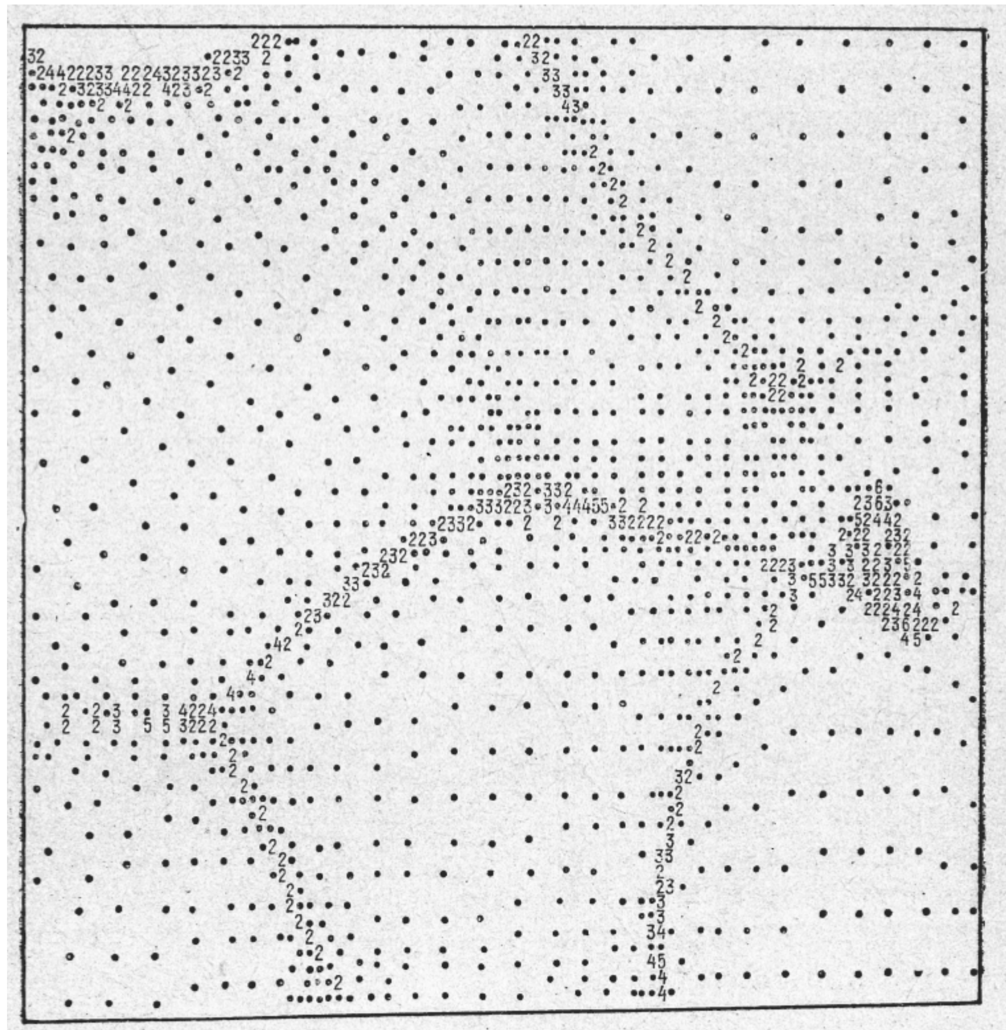
The \alpha – contour in Eulerian space

1 - D(t)\alpha(q) = 0

Zeldovich's pancakes

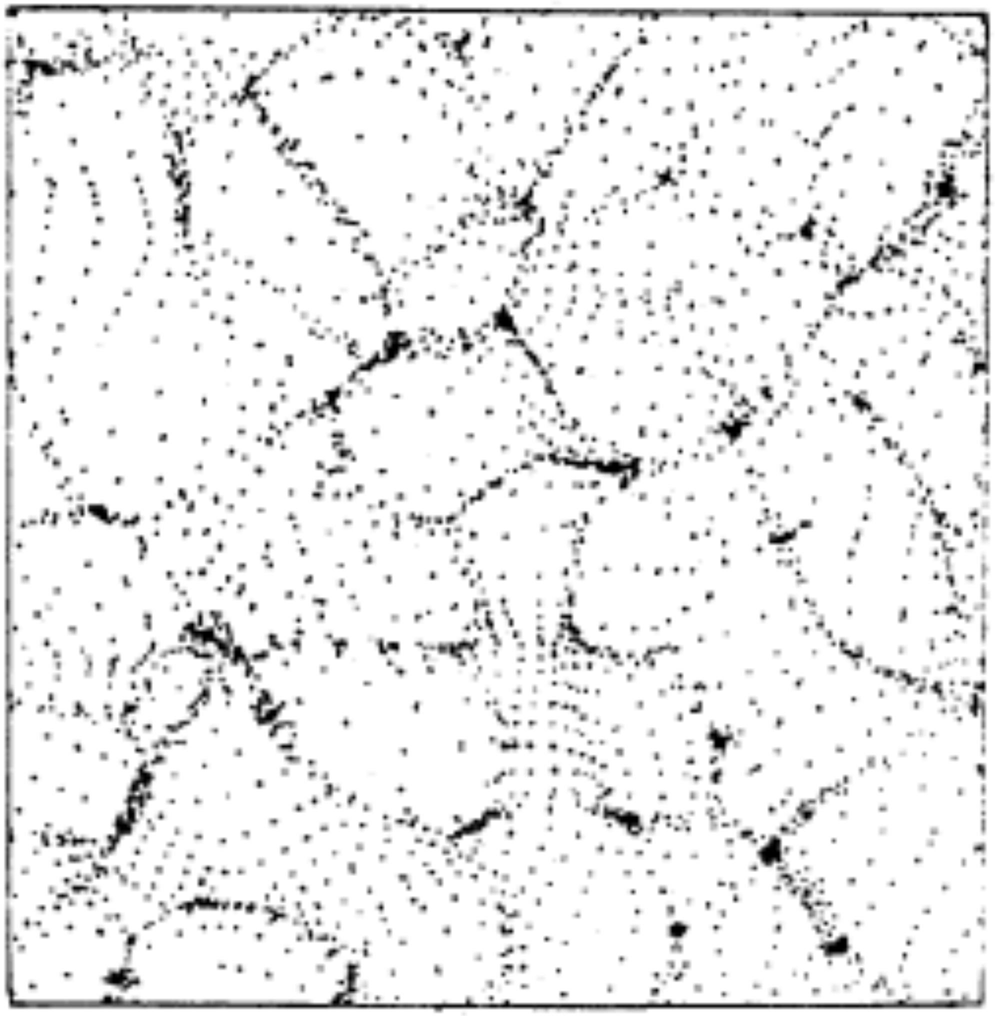
Hogan 2001

First hints of Cosmic Web: Zeldovich Approximation in 2D



Shandarin 1976

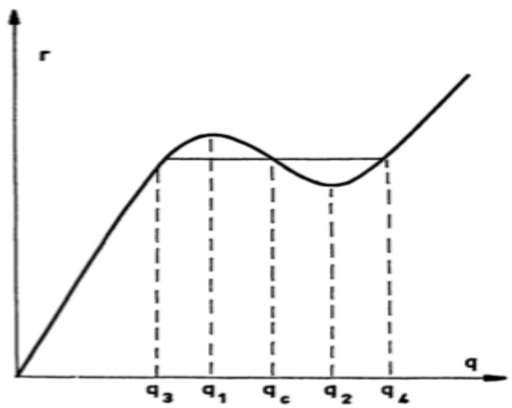
First hints of Cosmic Web: N-body simulation in 2D

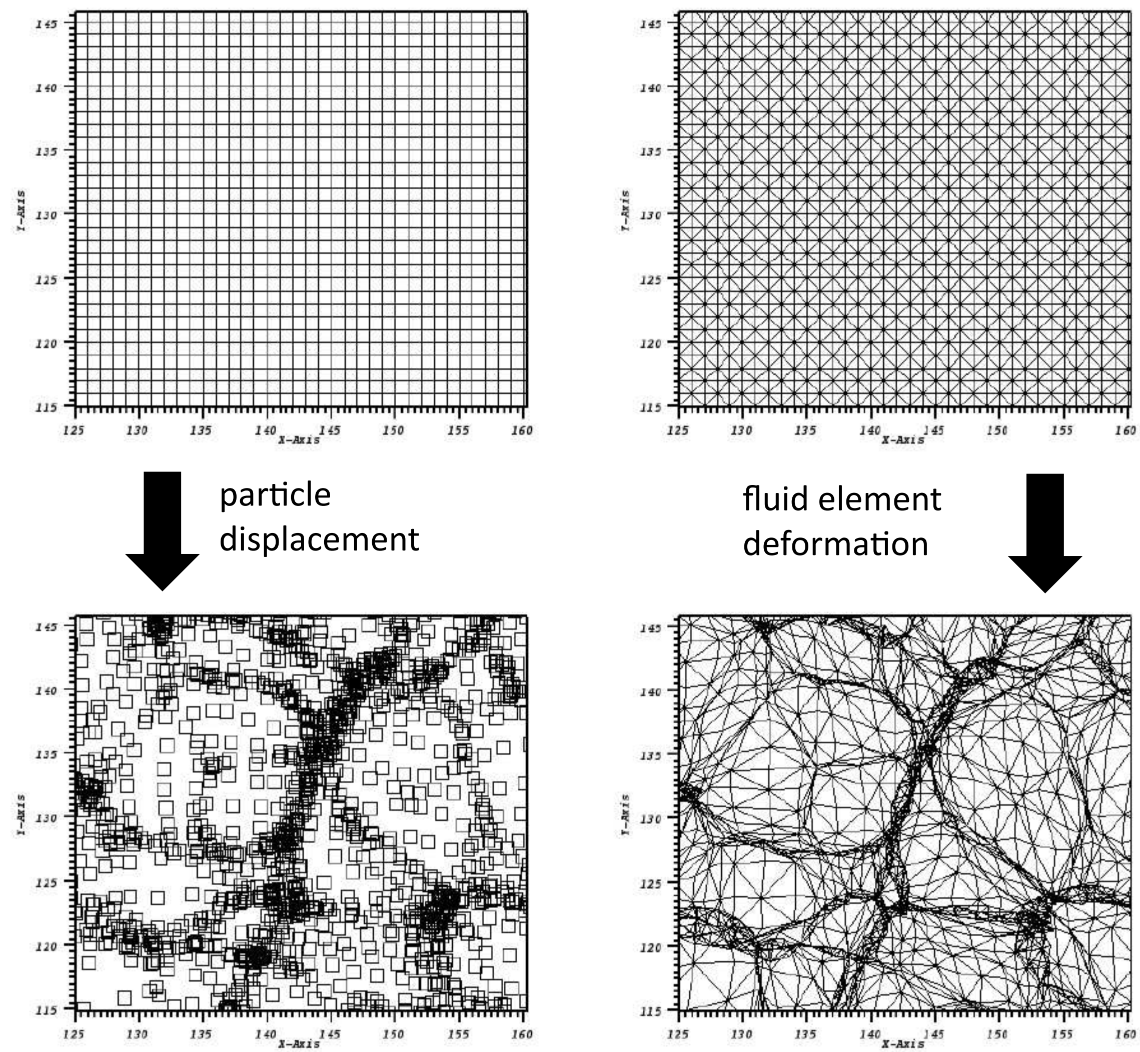


(b)

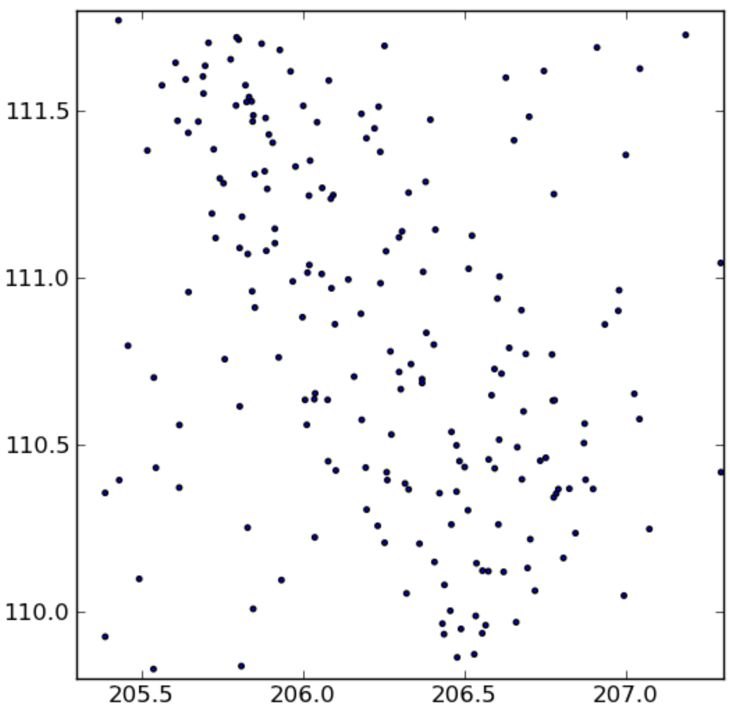
Doroshkevich, Kotok, Novikov, Polyudov, Shandarin, Sigov 1980, MNRAS

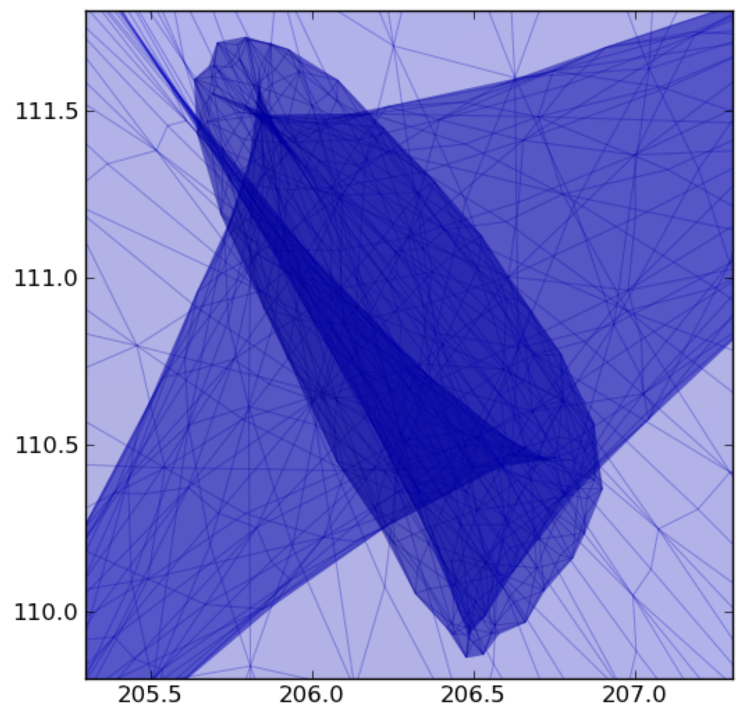
Projection of the Tessellation of LS





Particles VS. Tessellation of LS (2D example)





Decomposition of a cube into tetrahedra in **3D**

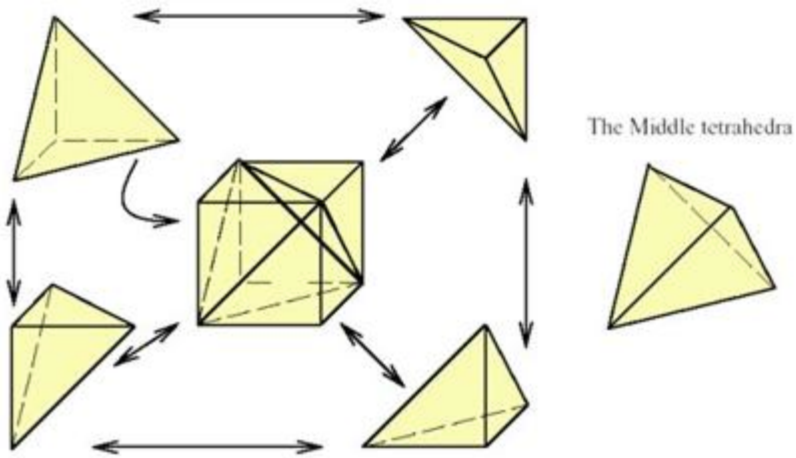
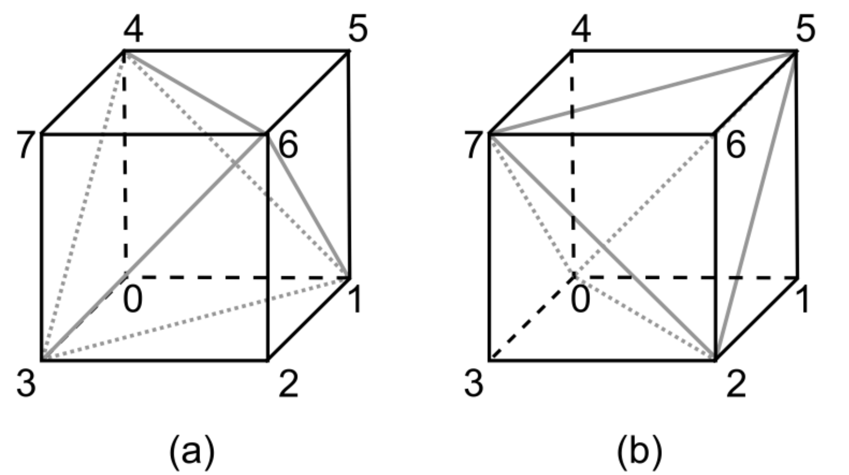


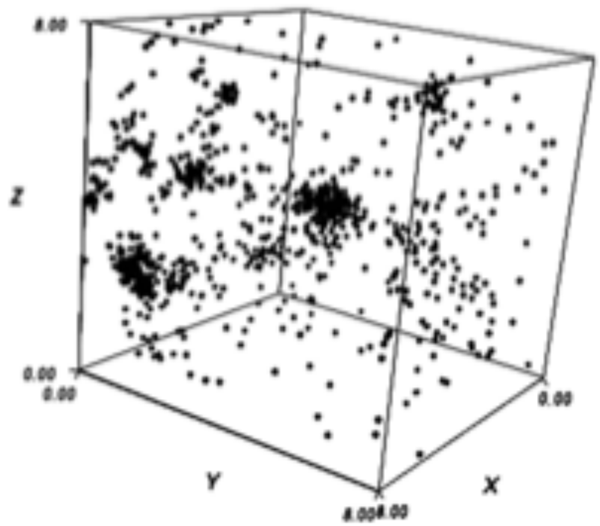
Figure 1.9: The Tetrahedra orientation within a cube



Shandarin,Habib,Heitmann 2012, Phys.Rev.D

Structure: particle representation

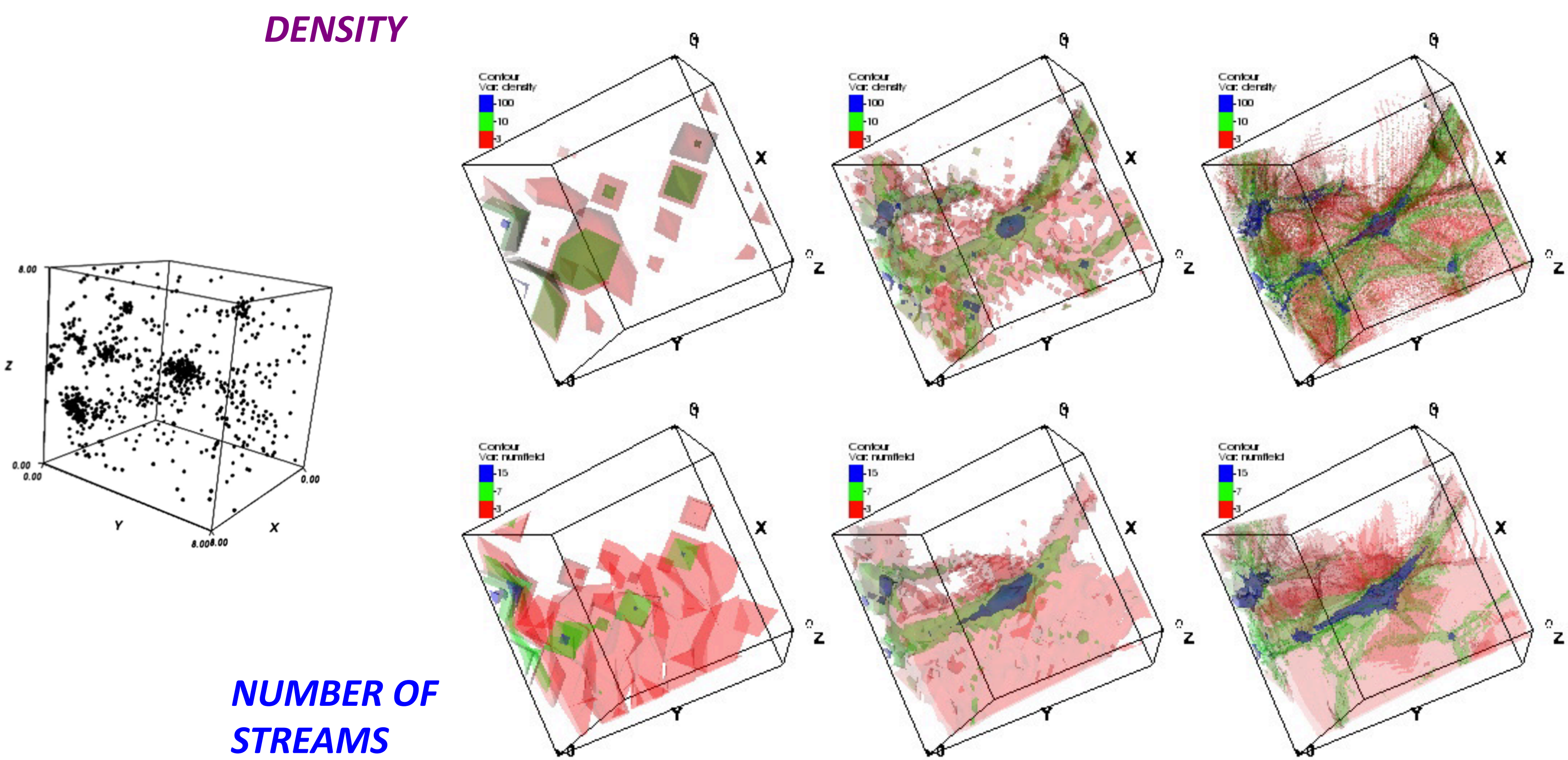
small box
8/h Mpc



N-BODY SIMULATION (PM) - ‘Standard’ LCDM model:
h = 0.72, Omega_t = 0.25, Omega_b=0.043, n=0.97, sig_8=0.8

Full box: 512/h Mpc, N_p=512^3, Force solver 1024^3

Probing complexity of Cosmic Web using Lagrangian Submanifold in 3D N-body simulation



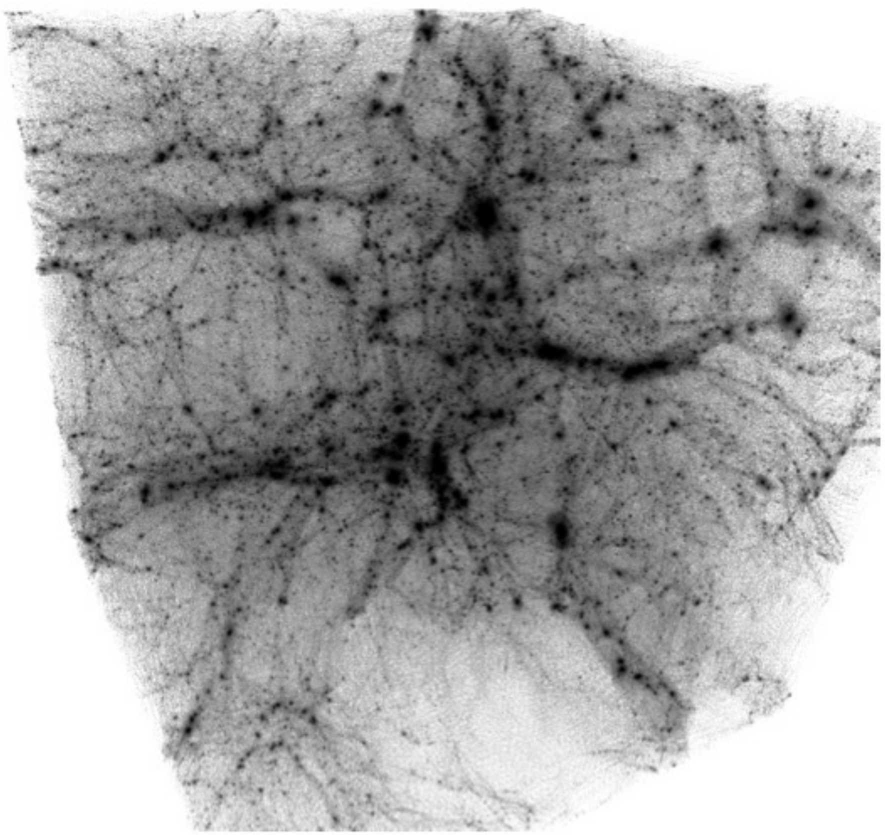
1/h Mpc, 0.25/h Mpc, 0.062/h Mpc

Mesh Resolution →

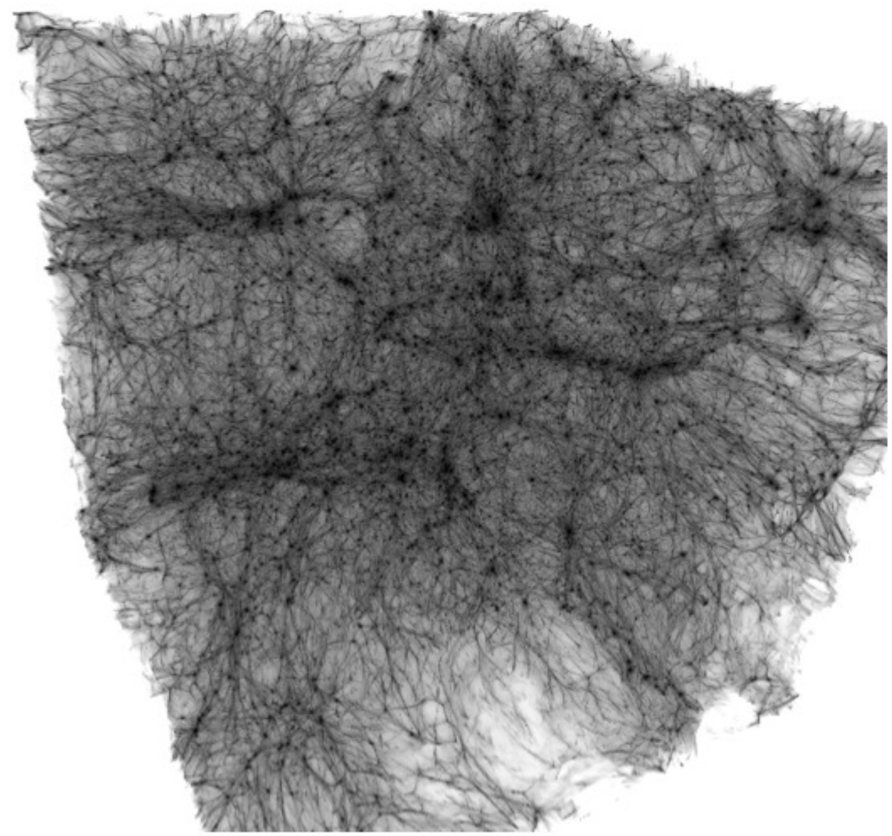
Shandarin, Habib, Heitmann 2012, April 18

Density   Estimate using LS

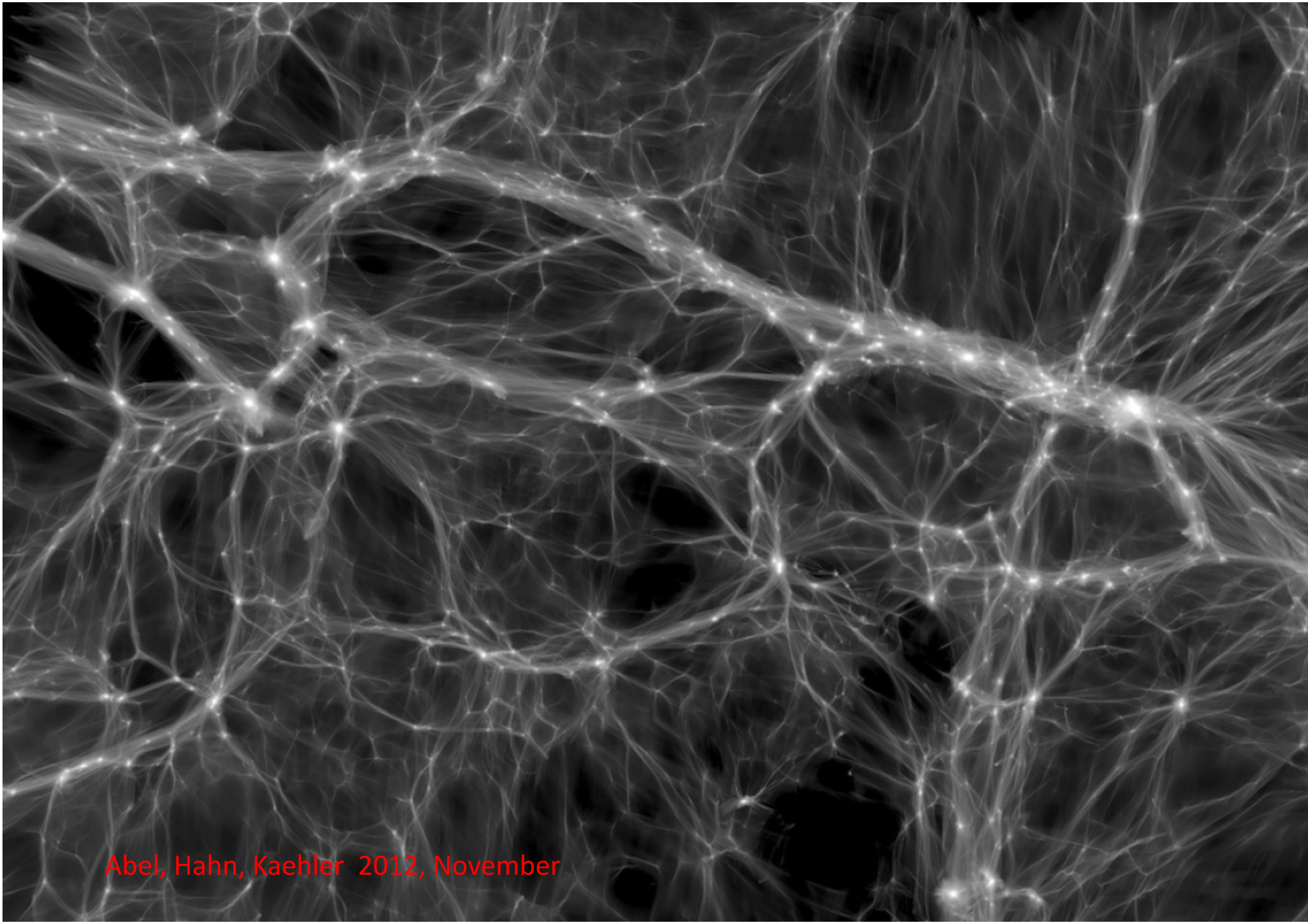
log(N_p per pixel)



Progection of the tesselation of LS



Abel, Hahn, Kaehler 2012, November


Abel, Hahn, Kaehler 2012, November

Adhesion approximation

Particles follow the Zel'dovich approximation

$$x_i(\mathbf{q}, t) = q_i + D(t) s_i(\mathbf{q})$$

until they run into caustic regions, where viscosity causes them to stick together. Burgers' equation describes this stage

$$\frac{\partial \mathbf{v}}{\partial D} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu \nabla^2 \mathbf{v}$$

$$\text{where } \mathbf{v} = \frac{d\mathbf{x}}{dD} \quad \text{and} \quad \nu \to 0$$

Gurbatov, Saichev, Shandarin 1985,1989

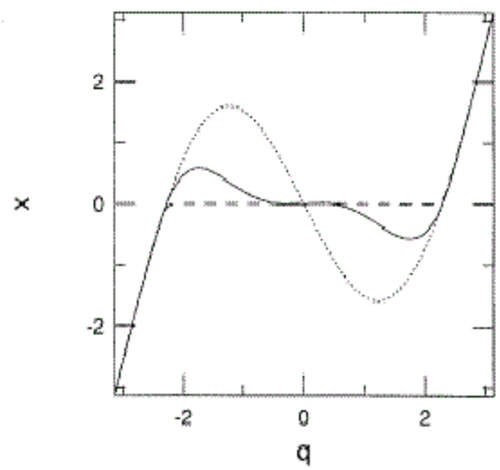


FIG. 1a

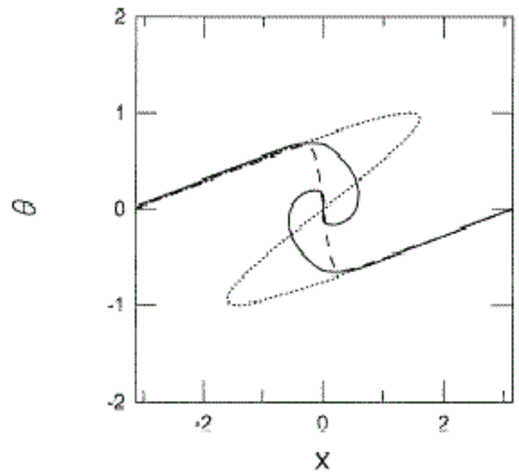
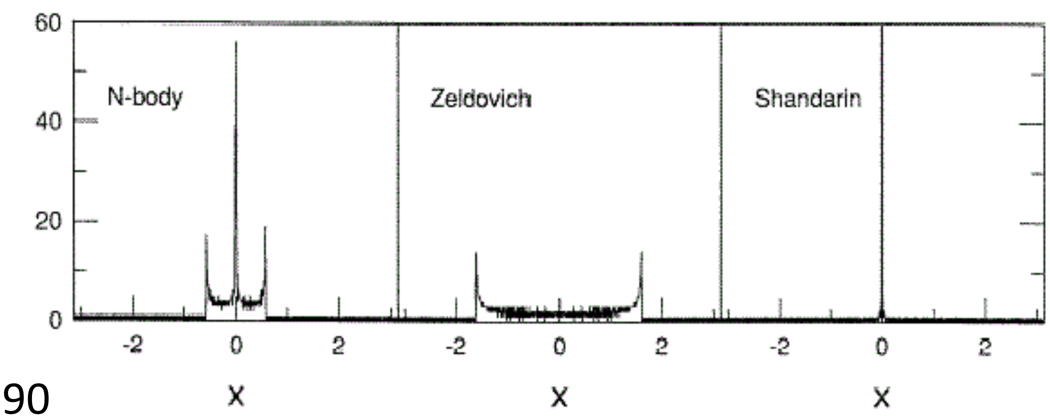
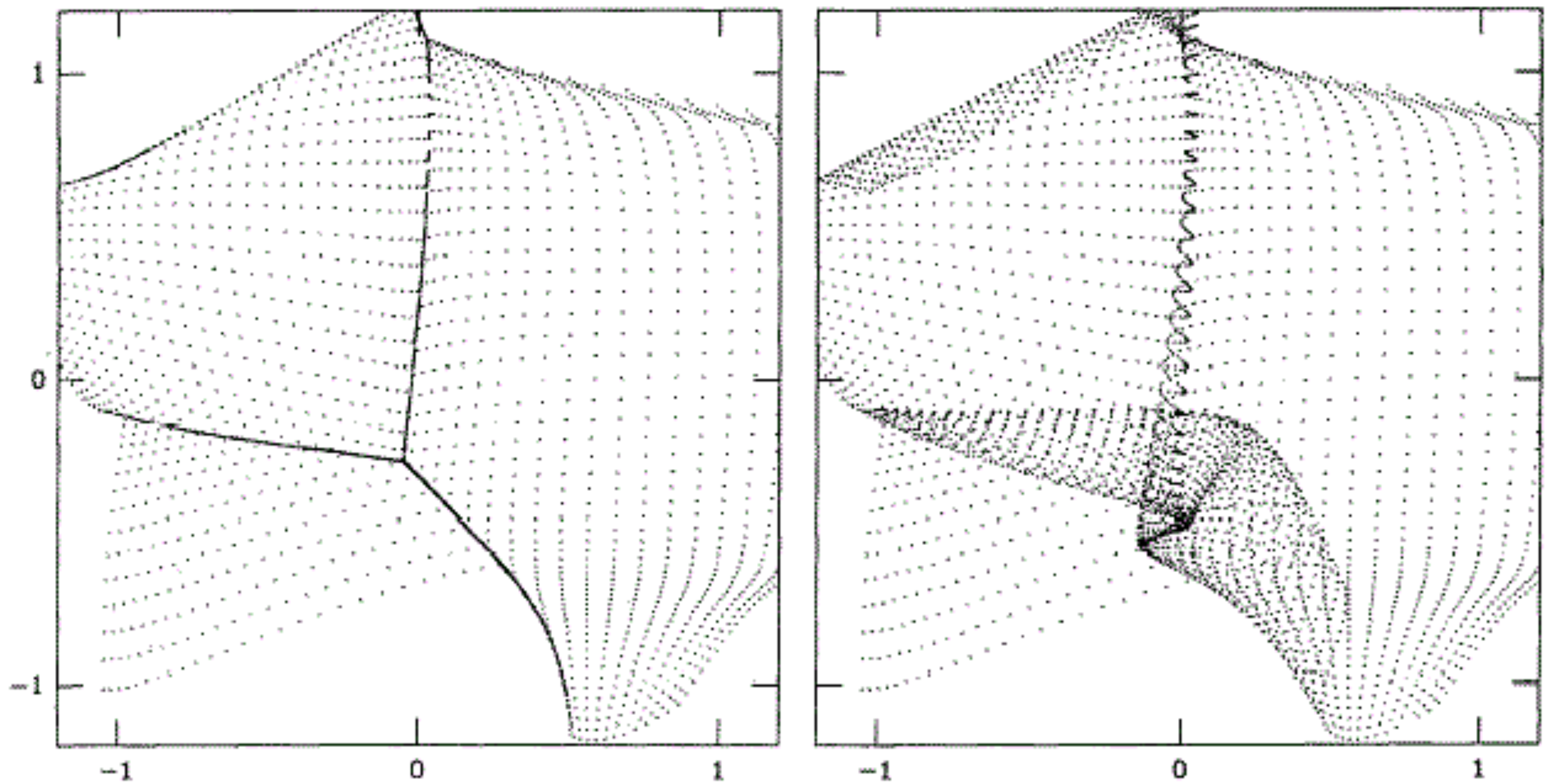


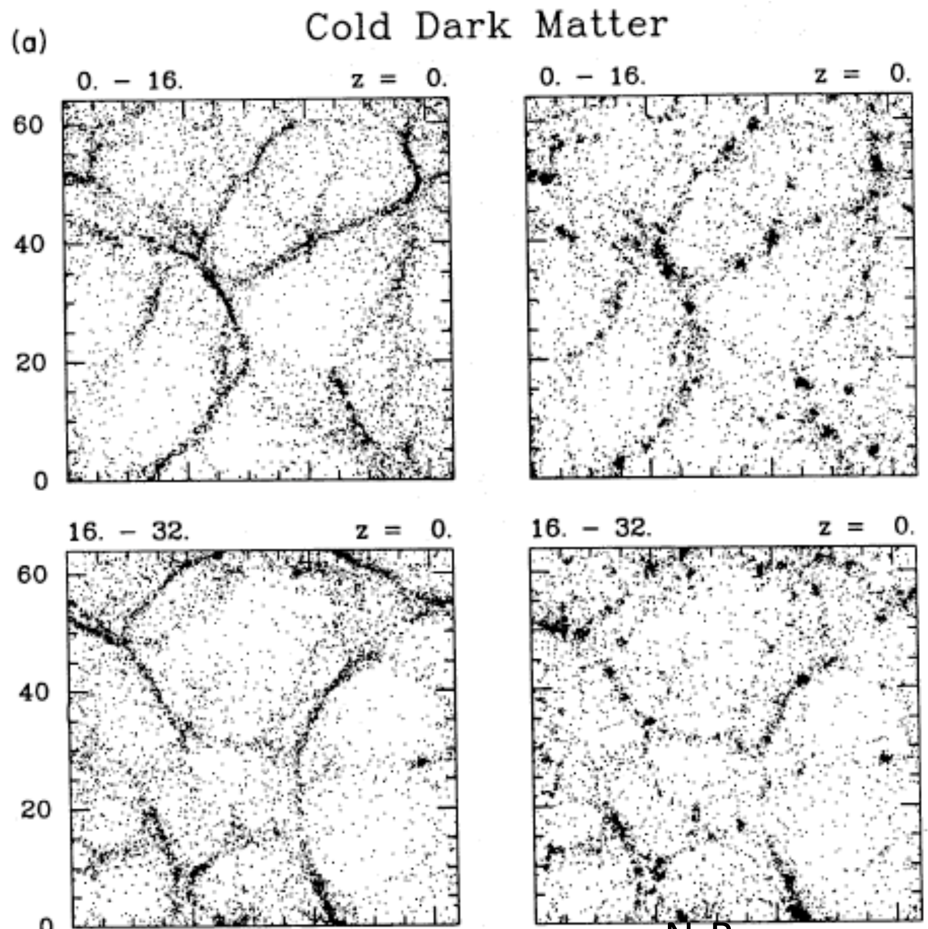
FIG. 1b



Nusser, Dekel 1990

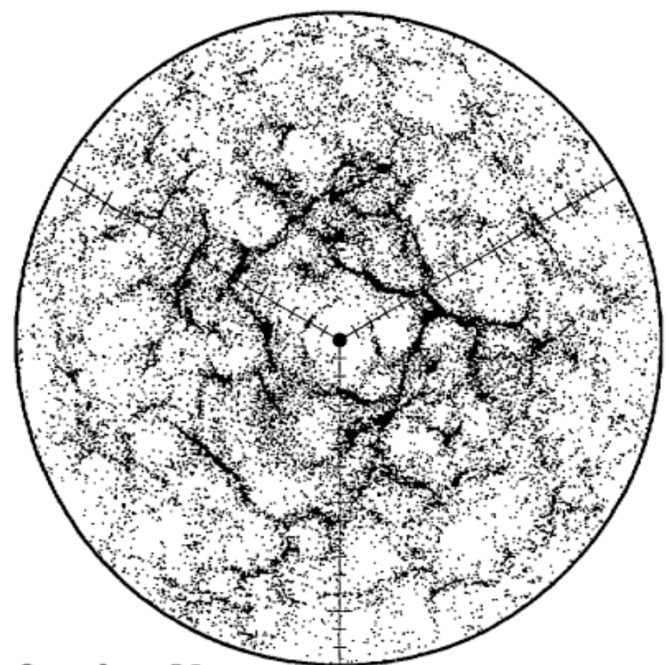

1
0
−1
−1
0
1
−1
0
1

Adhesion approximation in 3D



AA

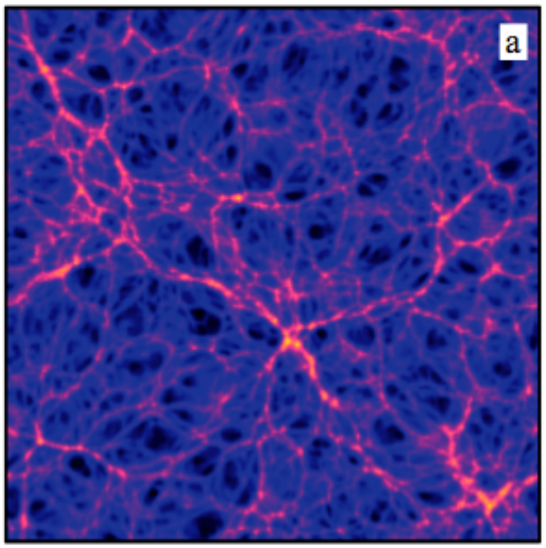
N-B



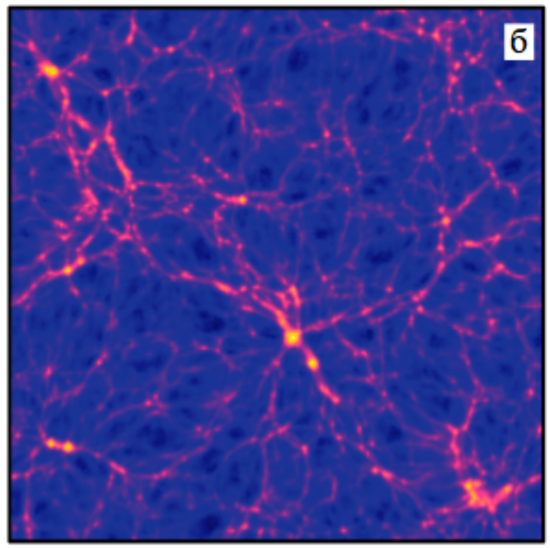
0 < δ < 20
m < 16.5
32803 galaxies

Weinberg, Gunn 1990

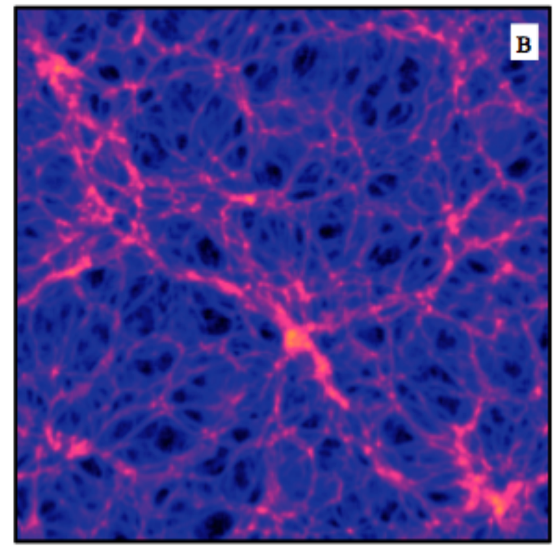
Adhesion Approximation



N-body Simulation



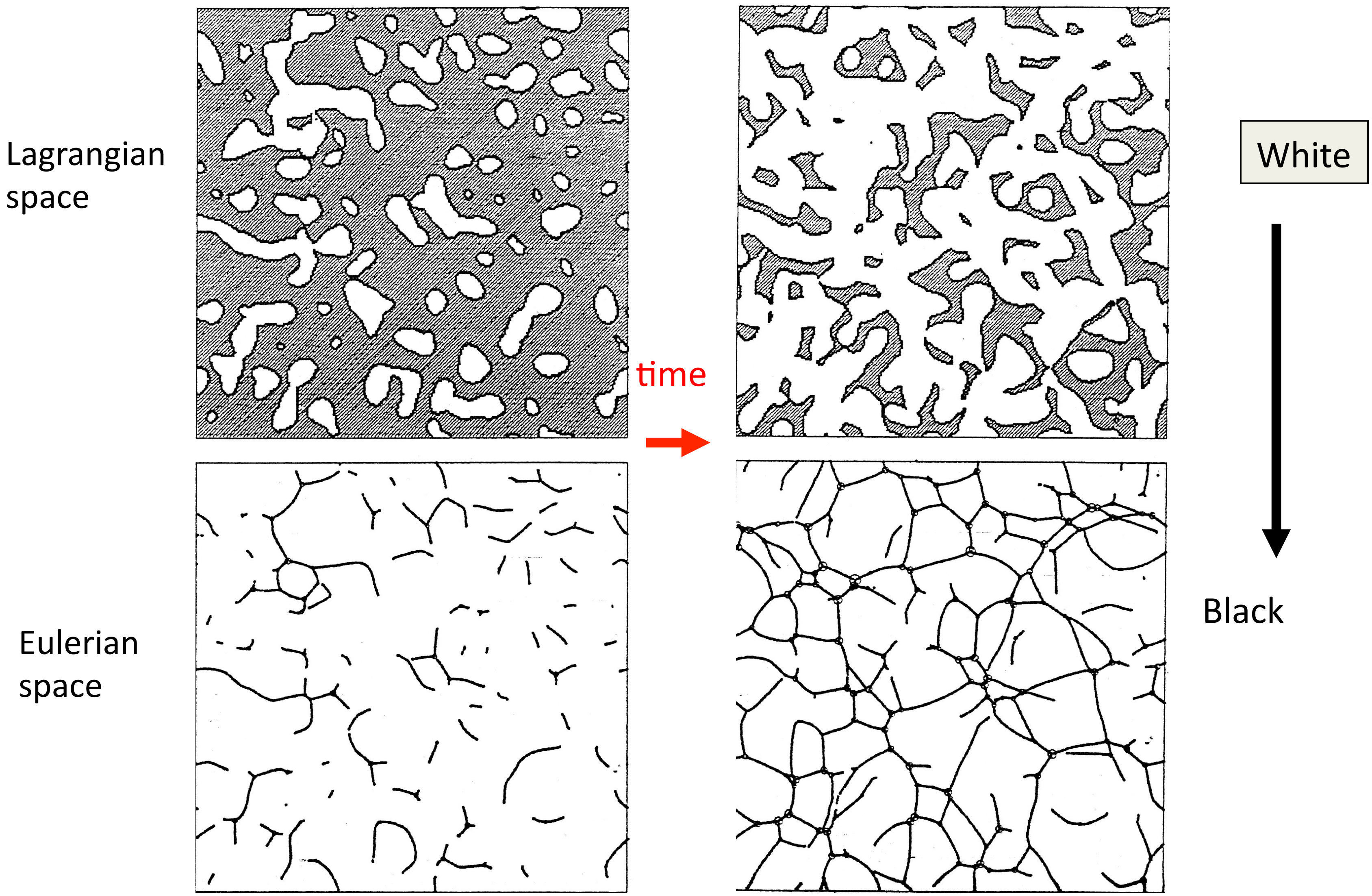
Zeldovich Approximation



Hidding 2011

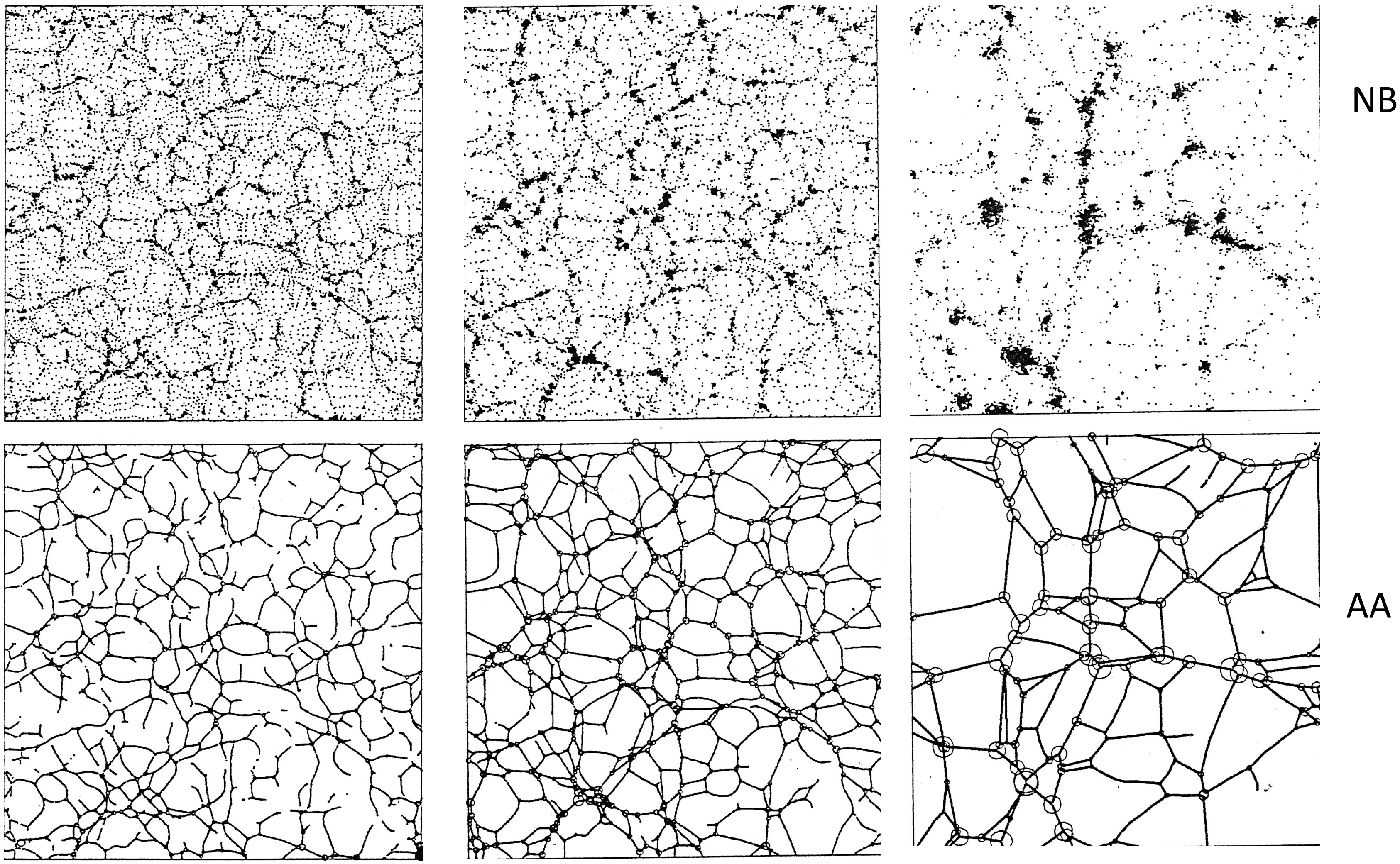
Adhesion model

Pogosyan PhD 1990

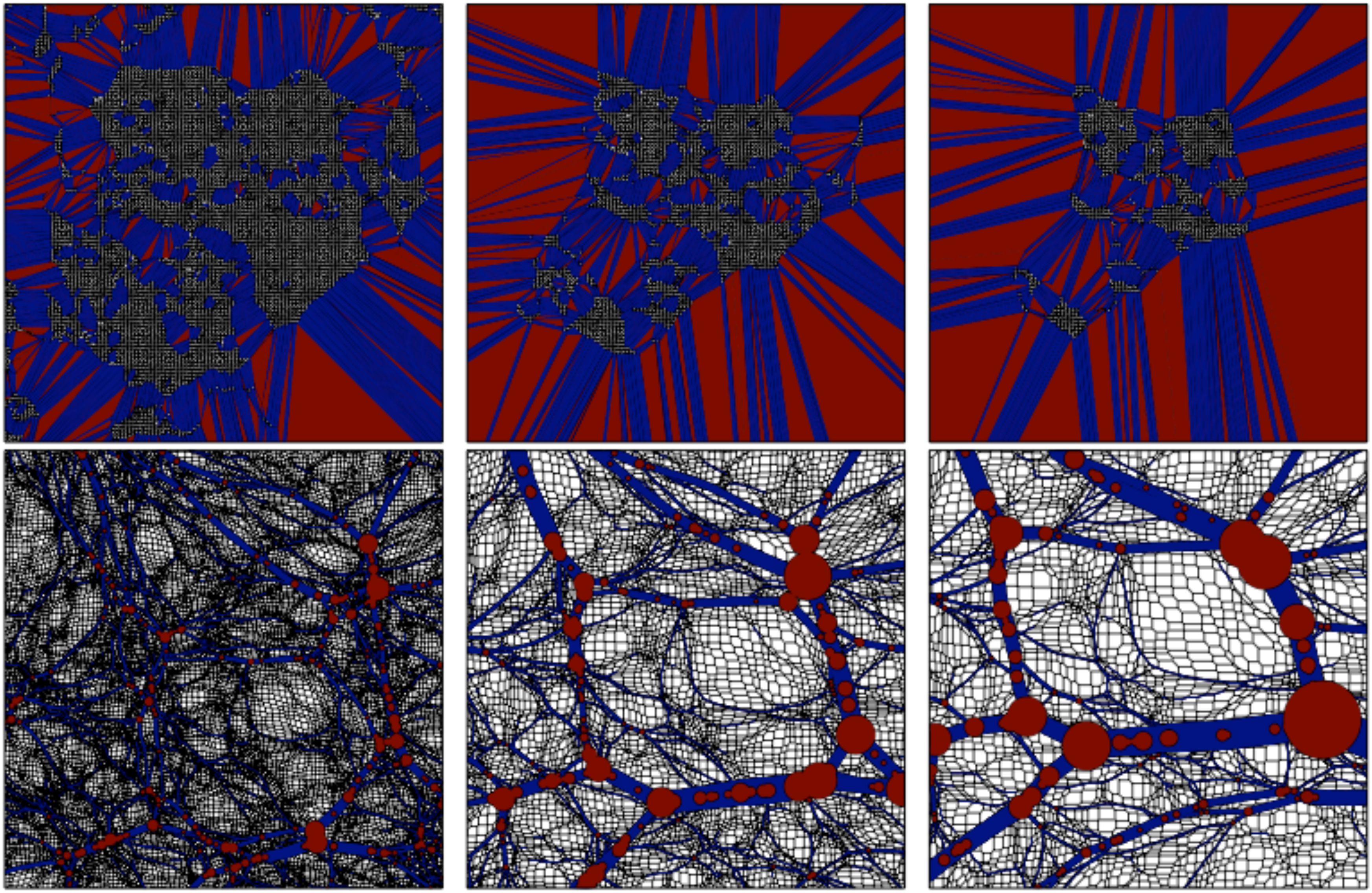


Evolution in adhesion model

Pogosyan PhD



Evolution in Lagrangian Space



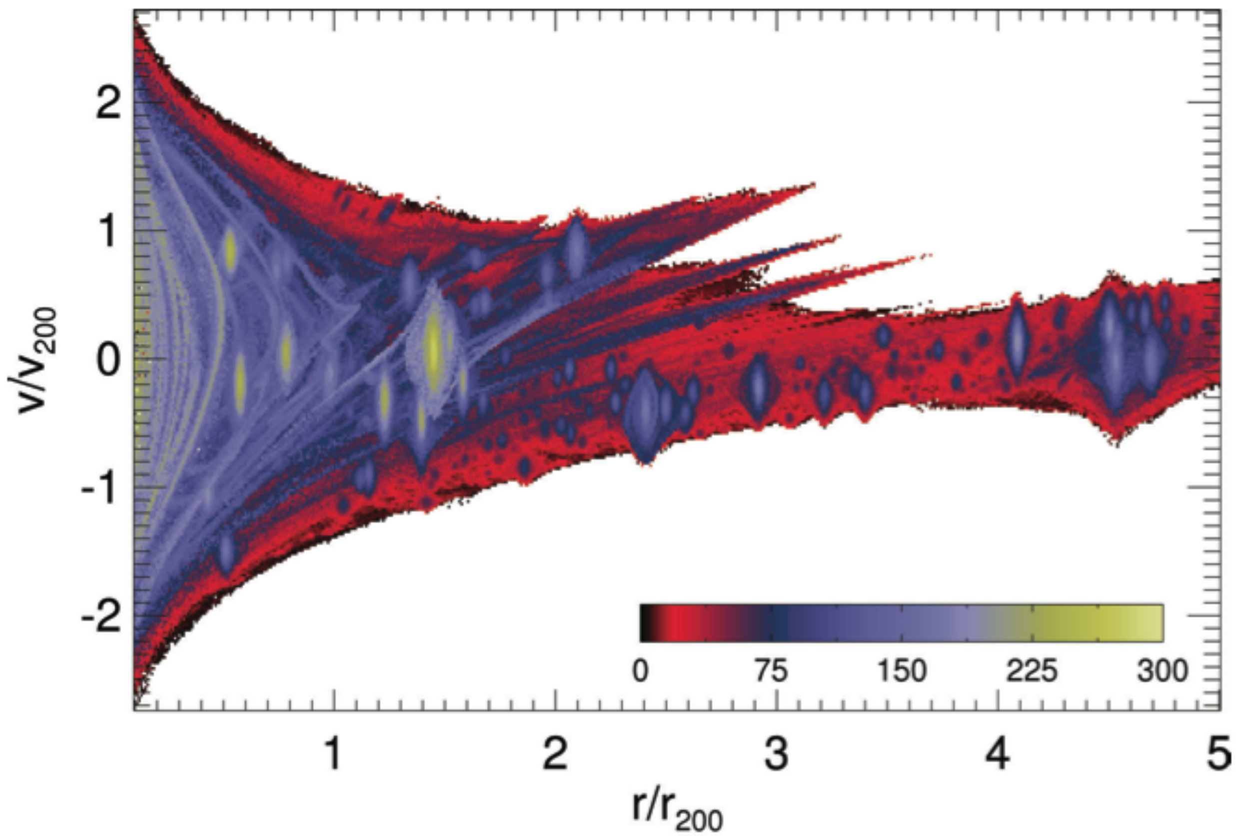
Evolution in Eulerian Space

Hidding, van de Weygaert, Vegter, Jones 2012

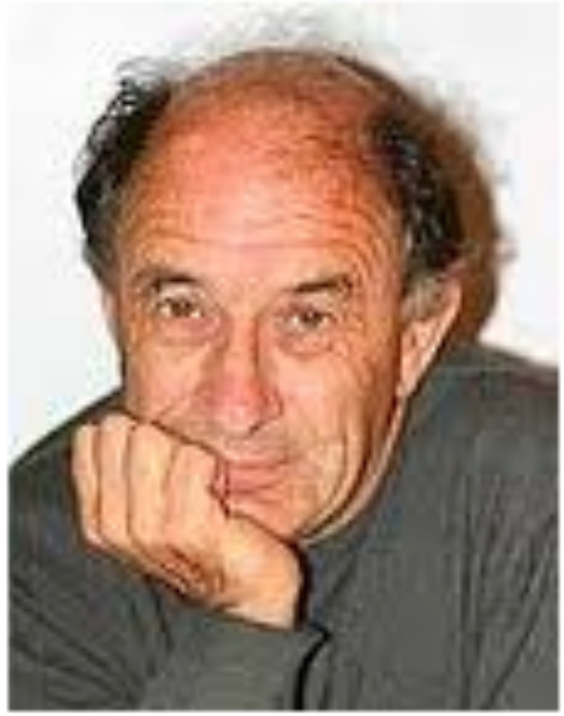
Vogelsberger & White 2011

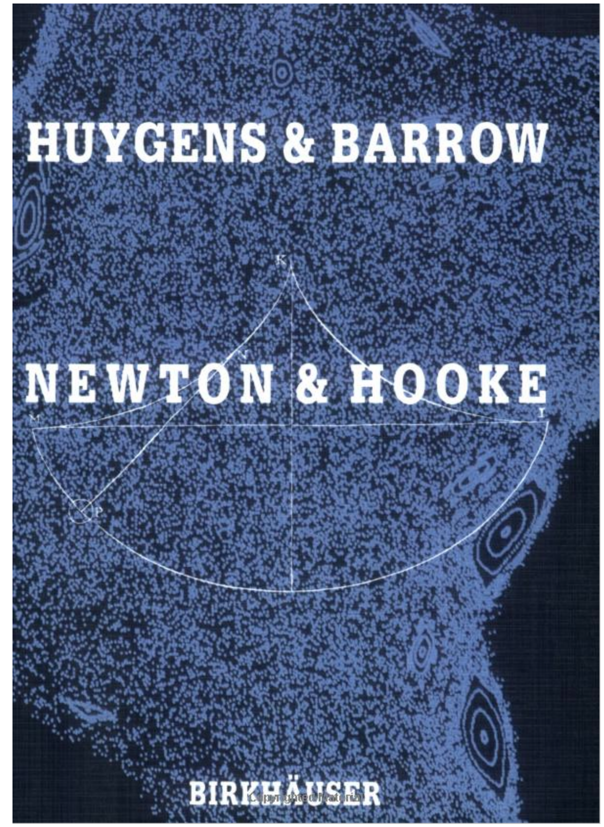
‘Streams and caustics:

the fine-grained structure of cold dark matter haloes’



At 8 kpc from the halo centre, a typical point intersects about 10^14 streams with a very broad range of individual densities;
the ~10^6 most- massive streams contribute about half of the local dark matter density.





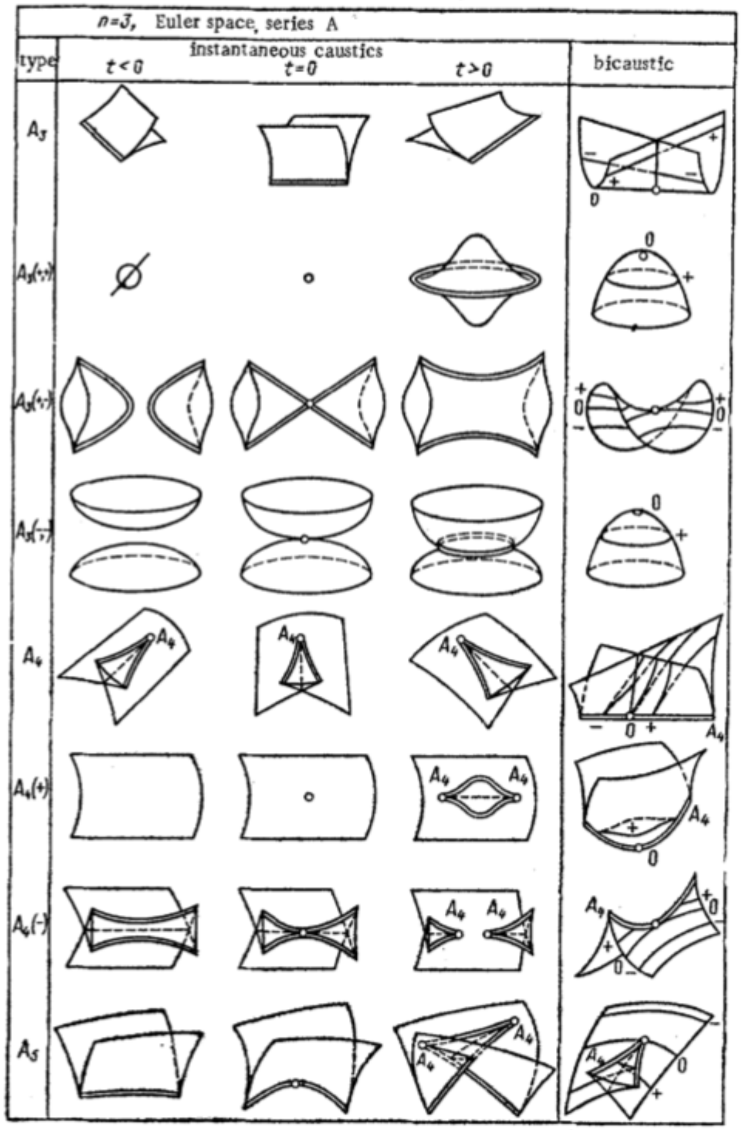
Vladimir Igorevich
Arnold
1937 - 2010

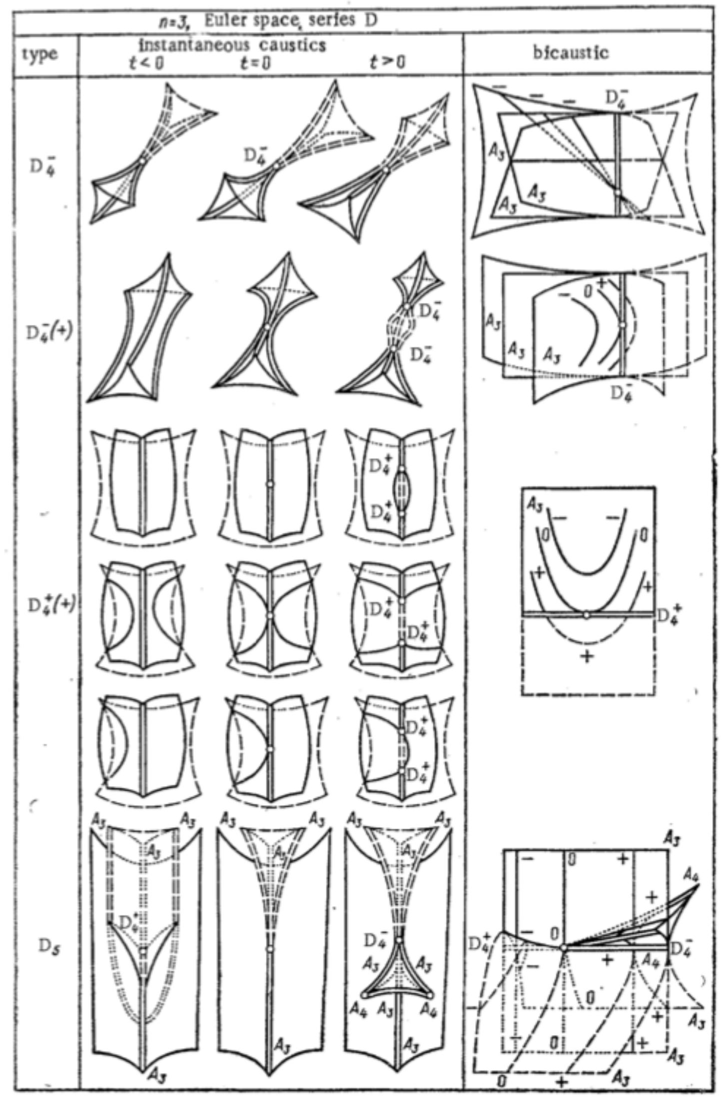
"Mathematics is a part of physics.

Physics is an experimental science, a part of natural science.

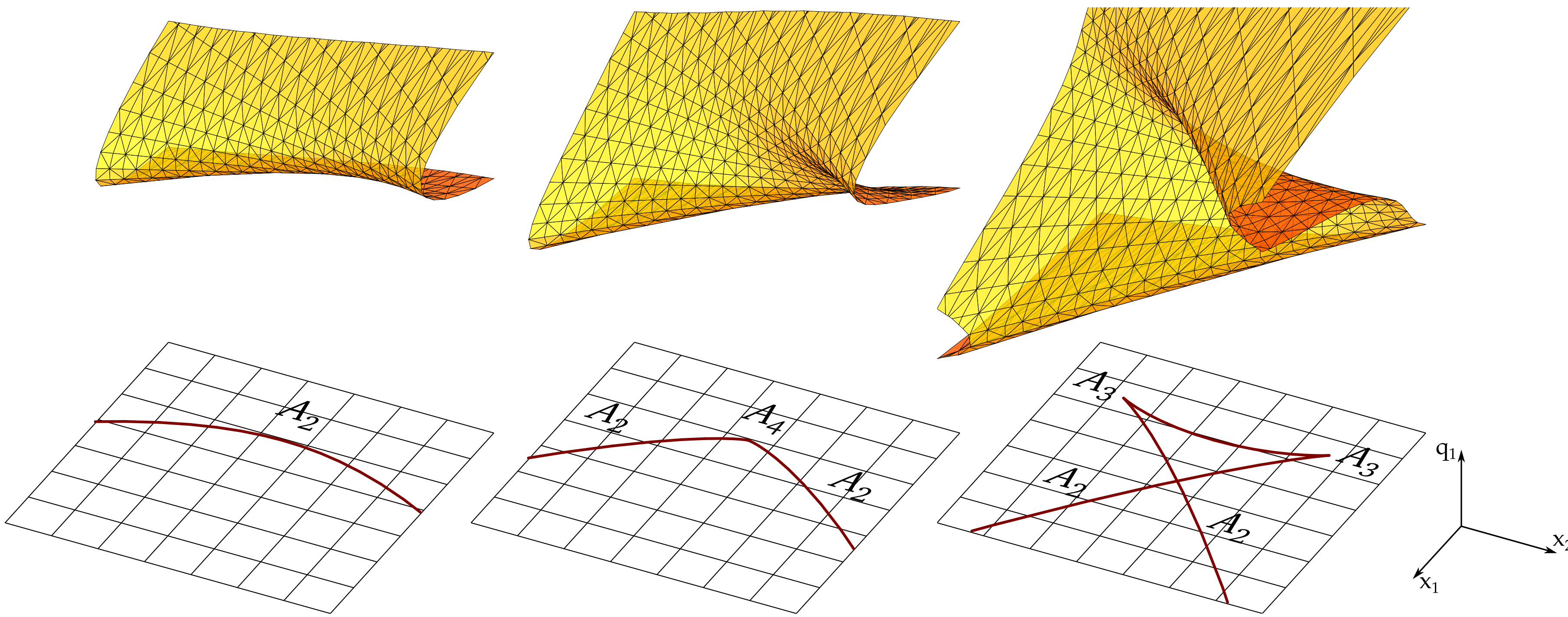
Mathematics is the part of physics where experiments are cheap."

ADE classification of caustics in 3D (Arnold 19





Normal forms are the simplest polynomial describing the surfaces, projection of which result in a caustic with right topology.



Geophys. Astrophys. Fluid Dynamics, 1982, Vol. 20, pp. 111–130
0309-1929/82/2002-0111 $06.50/0

Printed in Great Britain

The Large Scale Structure of the Universe I. General Properties. One- and Two-Dimensional Models

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Moscow State University, U.S.S.R.

and

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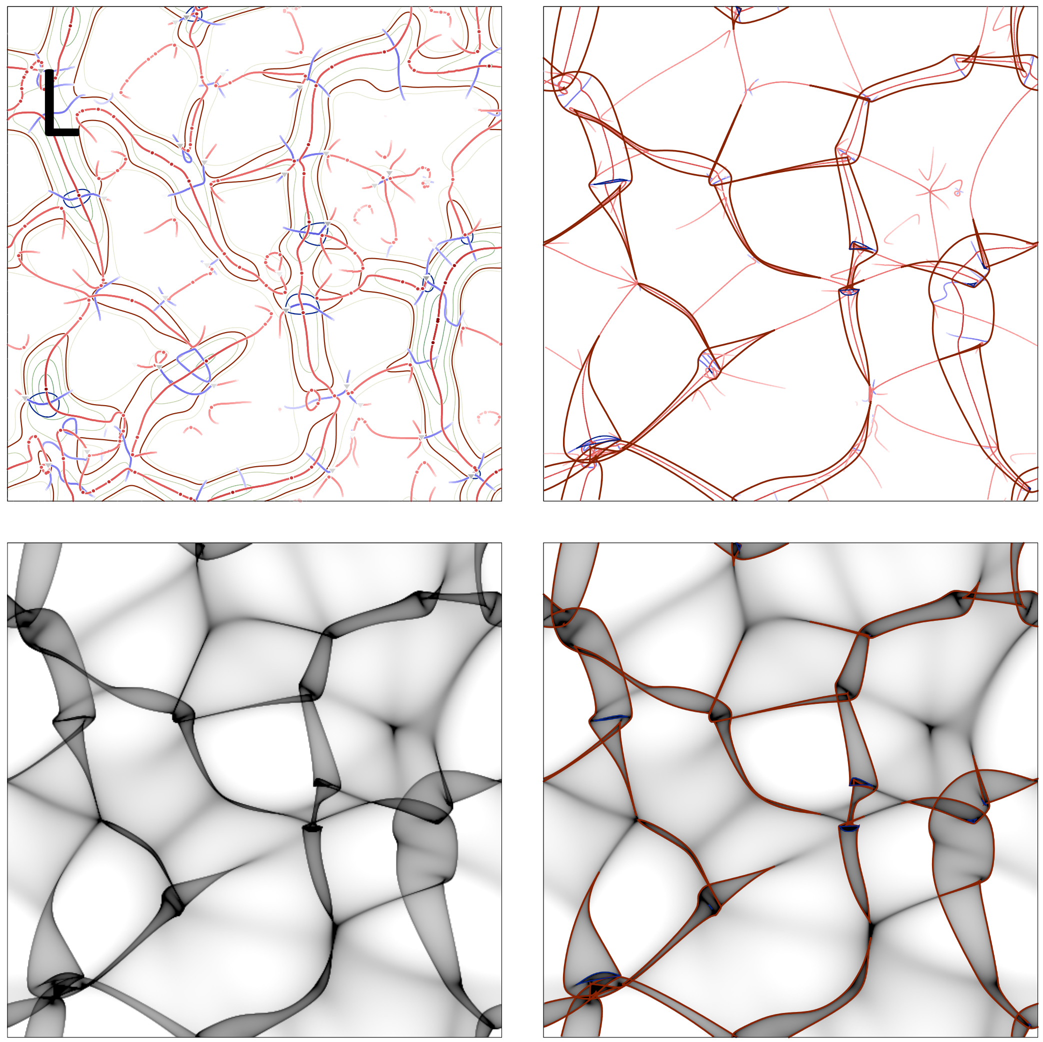
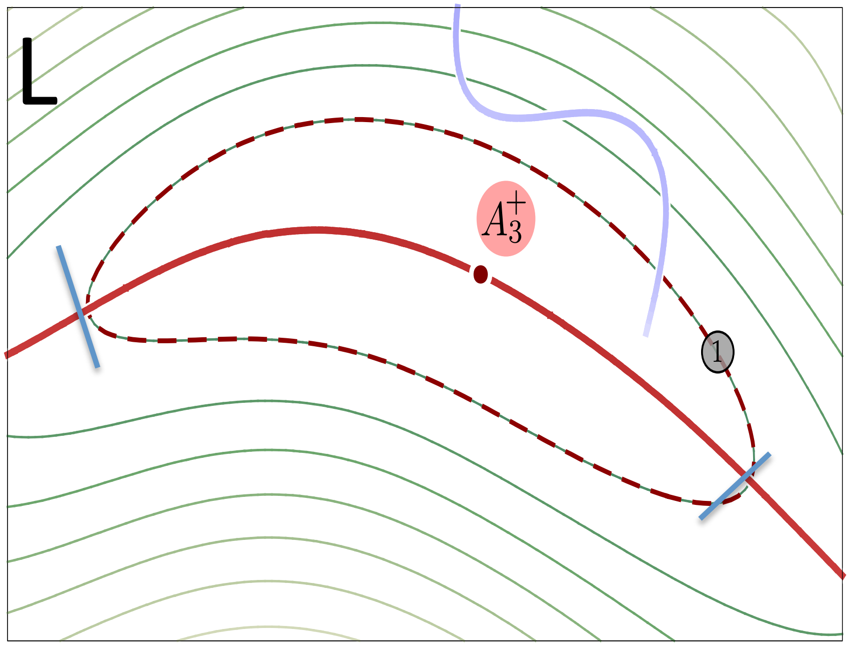
Institute of Applied Mathematics, Moscow, U.S.S.R.

(*Received August* 11, 1981)

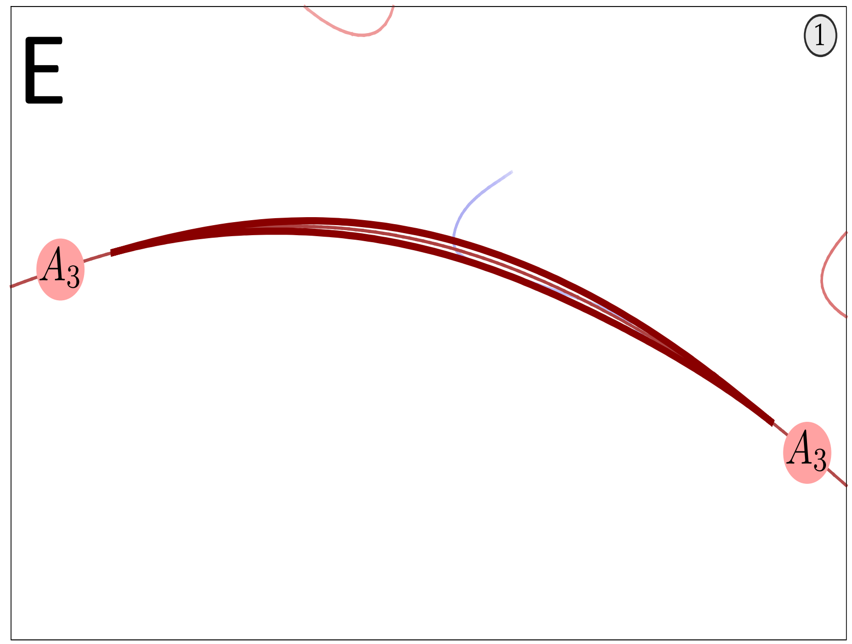
Zeldovich Approximation in 2D



Zeldovich's pancake (2D)

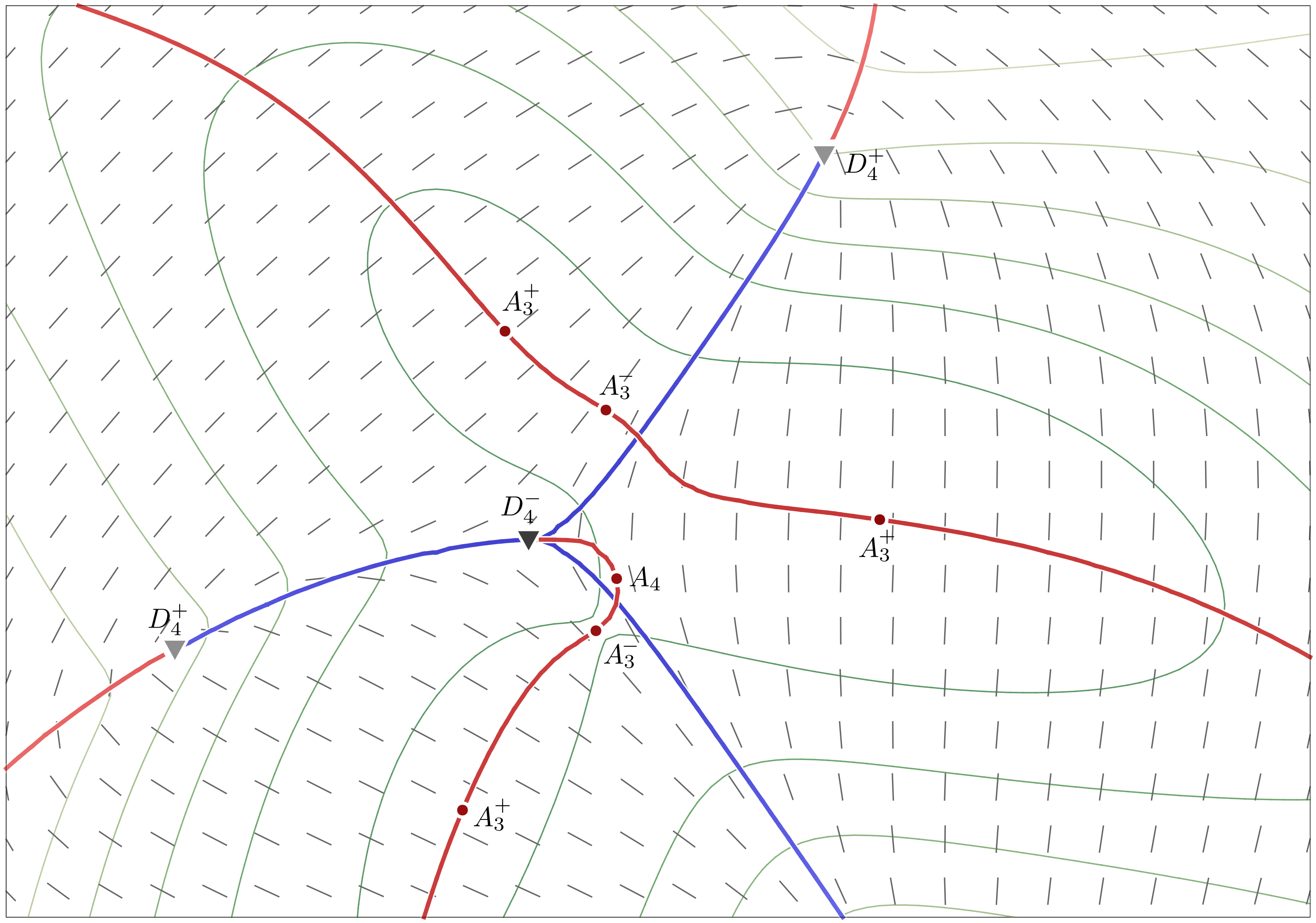


\alpha – contours in Lagrangian space

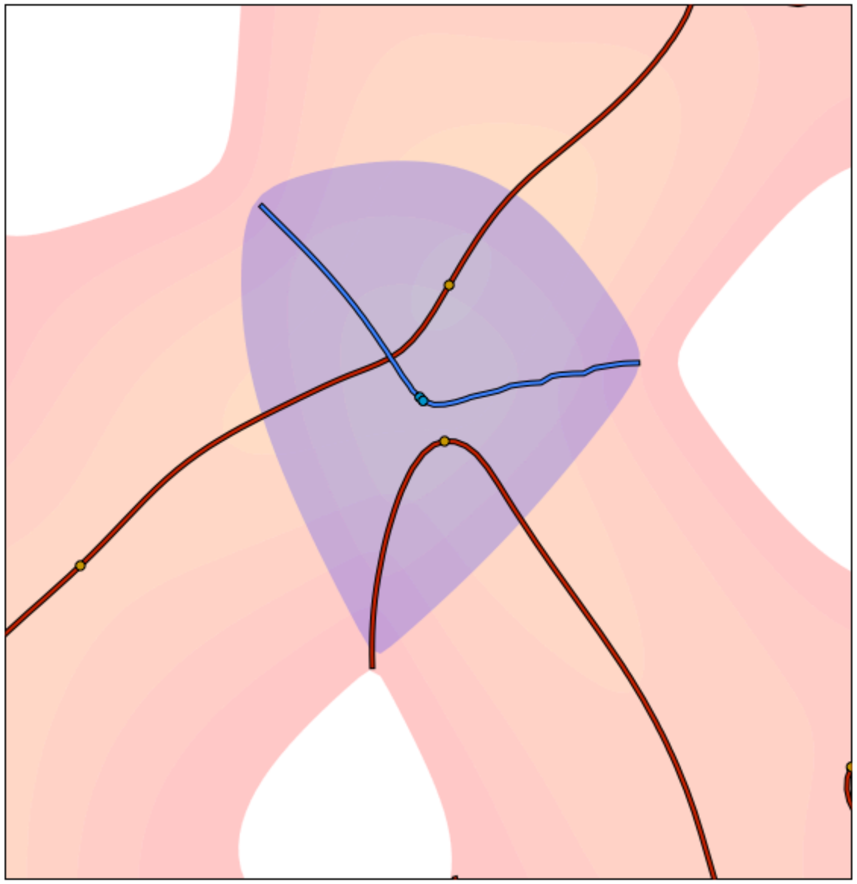


The \alpha – contour in Eulerian space

1 - D(t)\alpha(q) = 0

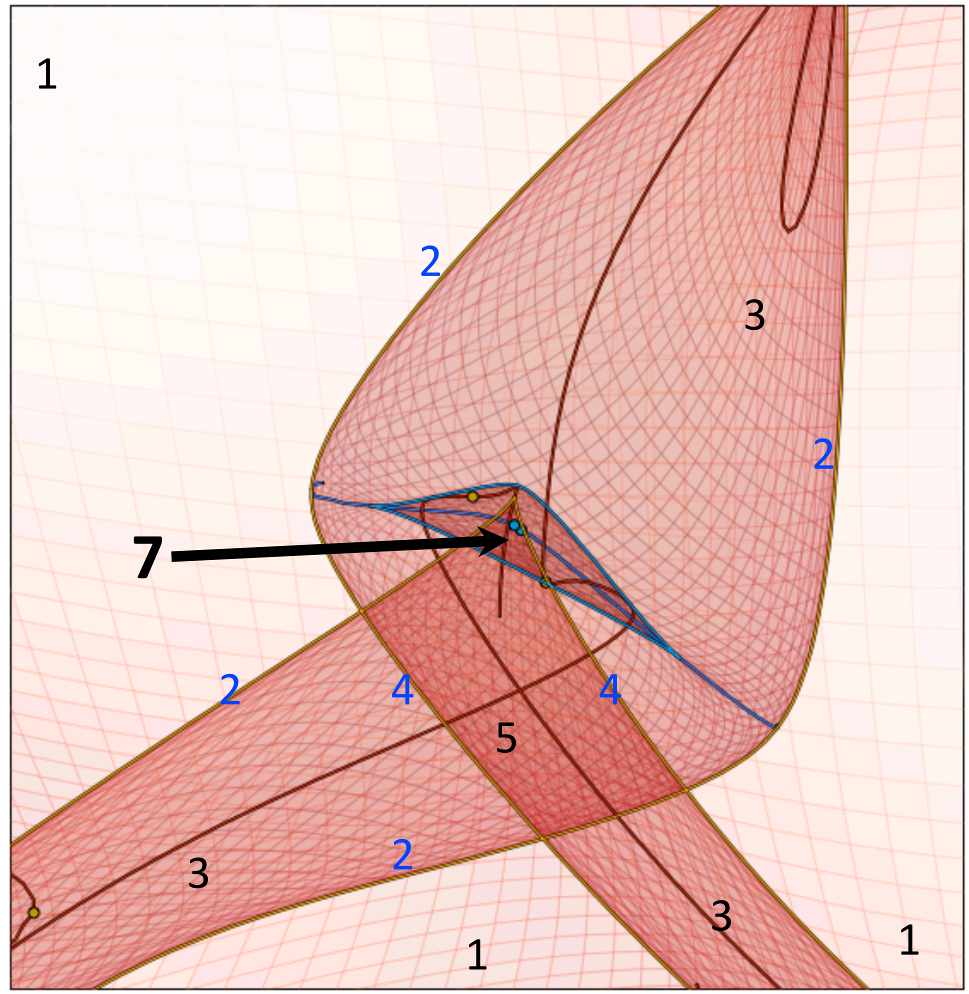


A_3^+
A_3^-
D_4^+
D_4^-
A_3^+
A_4
D_4^+
A_3^-
A_3^+



Lagrangian space

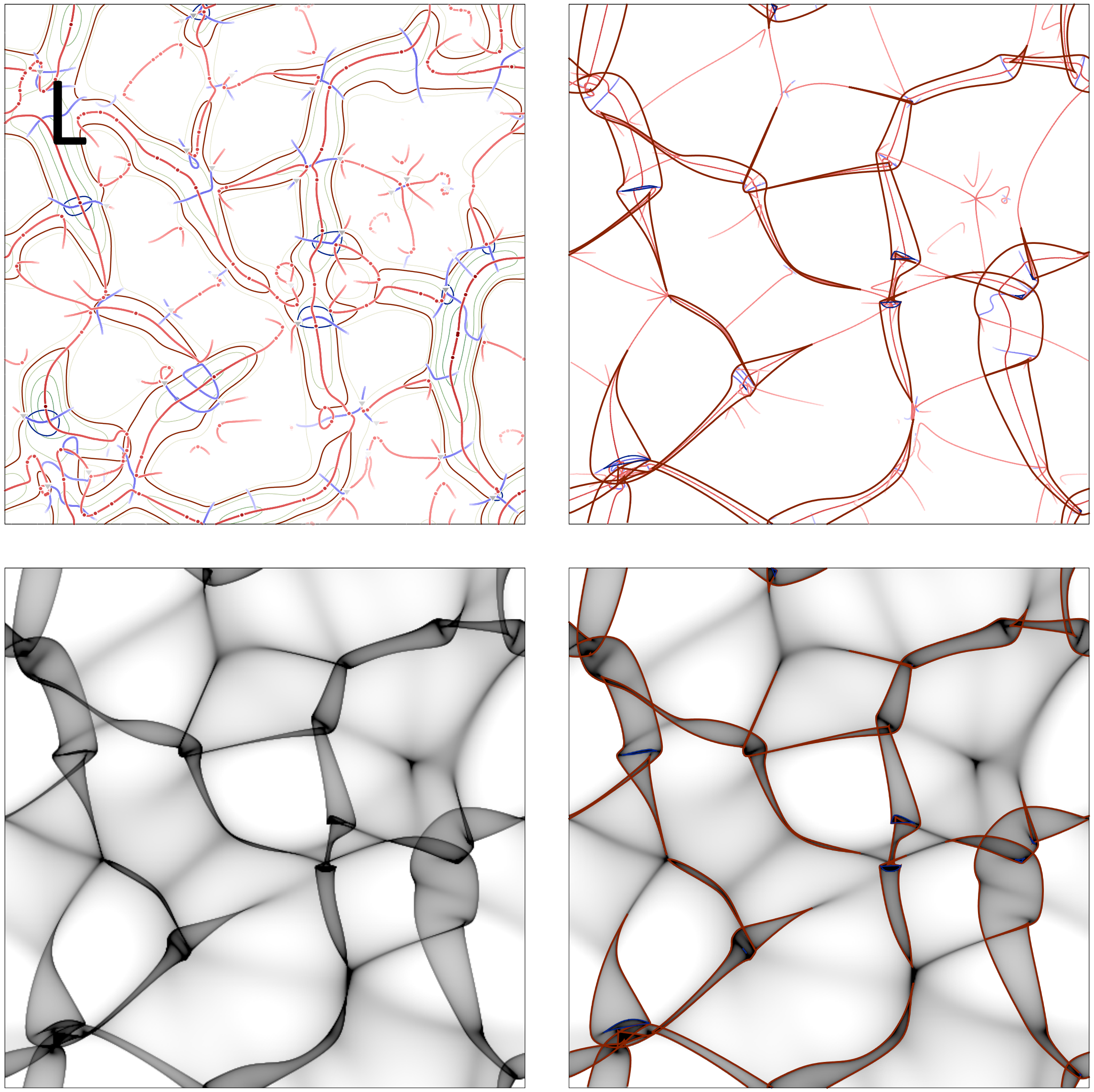
Number of streams in
Eulerian space



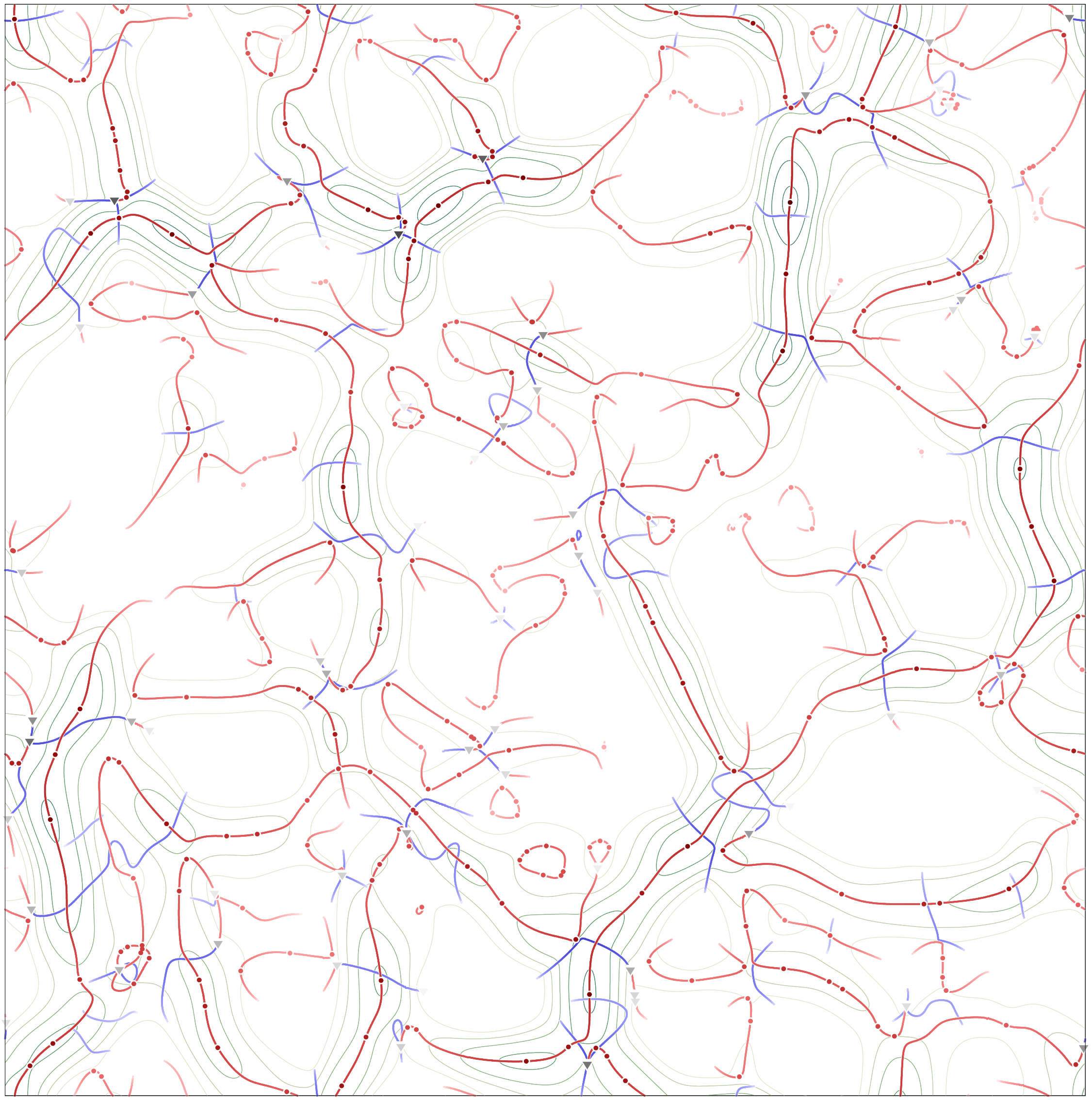
Hidding,
Shandarin,
van de Weygaert

work in progress

Zeldovich Approximation in 2D



A_3 lines make the progenitor of the skeleton of the Cosmic Web in 2D



Generic singularities in 1,2,3D

$A_2 : \rho \propto \delta r^{-1/2}$ Surfaces (2D) at a generic instant of time (1,2,3 D)

$A_3 : \rho \propto \delta r^{-2/3}$ Lines (1D) at a generic instant of time (1,2,3 D)

$A_4 : \rho \propto \delta r^{-3/4}$ Points (0D) at a generic instant of time (2,3 D)
Candidate to be a filament

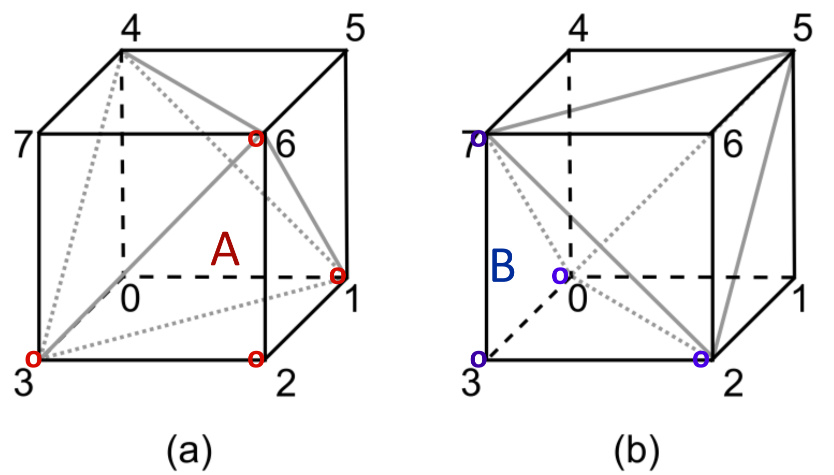
$A_5 : \rho \propto \delta r^{-4/5}$ Points (0D) at particular instants of time (3D)

$D_4 : \rho \propto \delta r^{-1}$ Points (0D) at a generic instant of time (2,3 D)
Candidate to be a filament

$D_5 : \rho \propto \delta r^{-1} \ln \delta r$ Points (0D) at particular instants of time (3D)

Arnol'd, Shandarin, Zel'dovich 1982

Computing Caustics in 3D



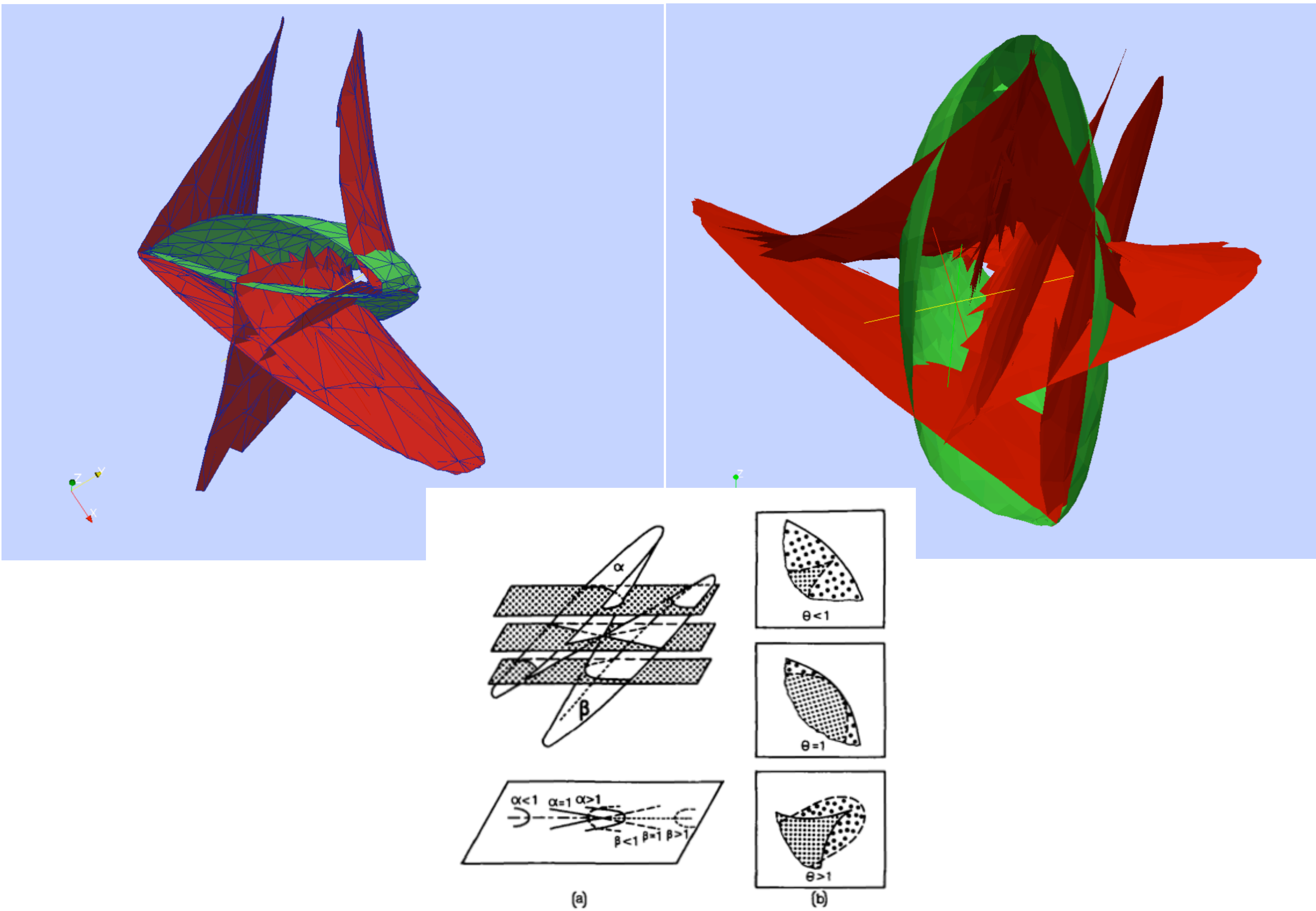
Neighboring tetrahedra A=[a1,a2,a3,a6] and B=[b0,b2,b3,b7]

share a common face [a1,a2,a6] = [b0,b3,b7].

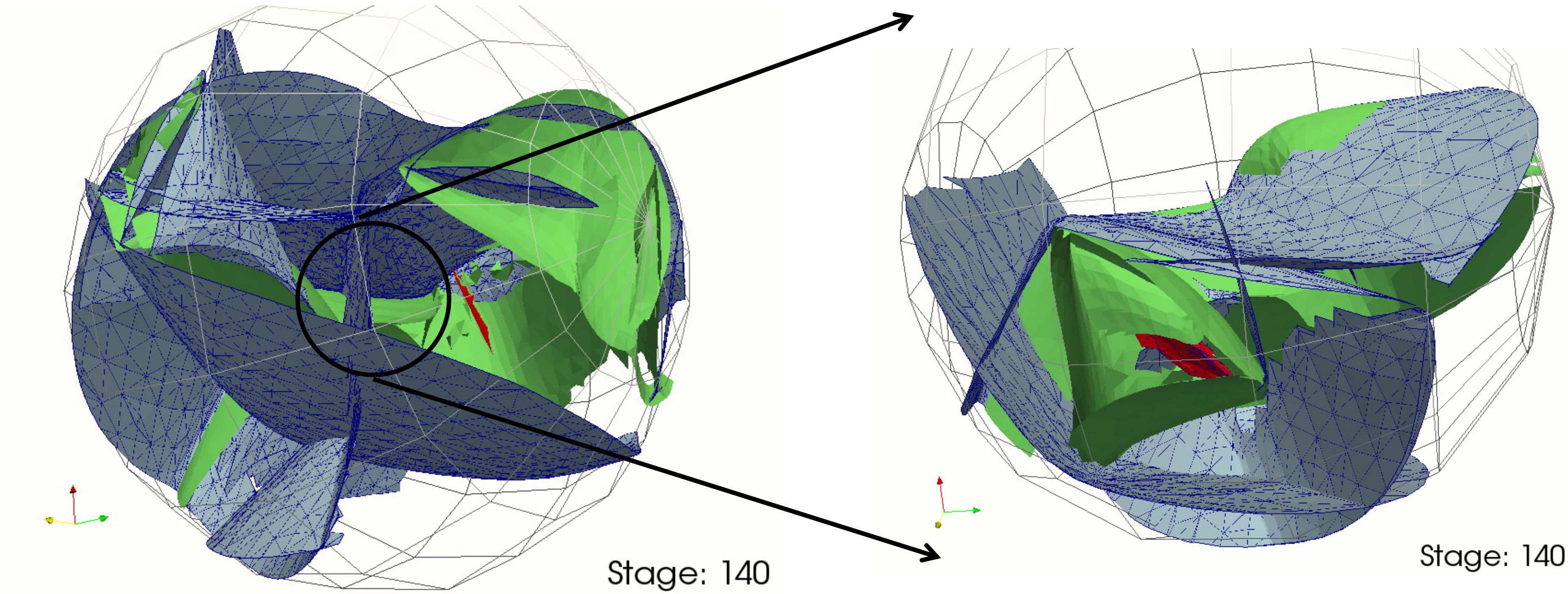
When the parity of A is opposite of the parity of B

the common face is an element of a caustic surface

D_4 singularity (Arnold et al 1982)



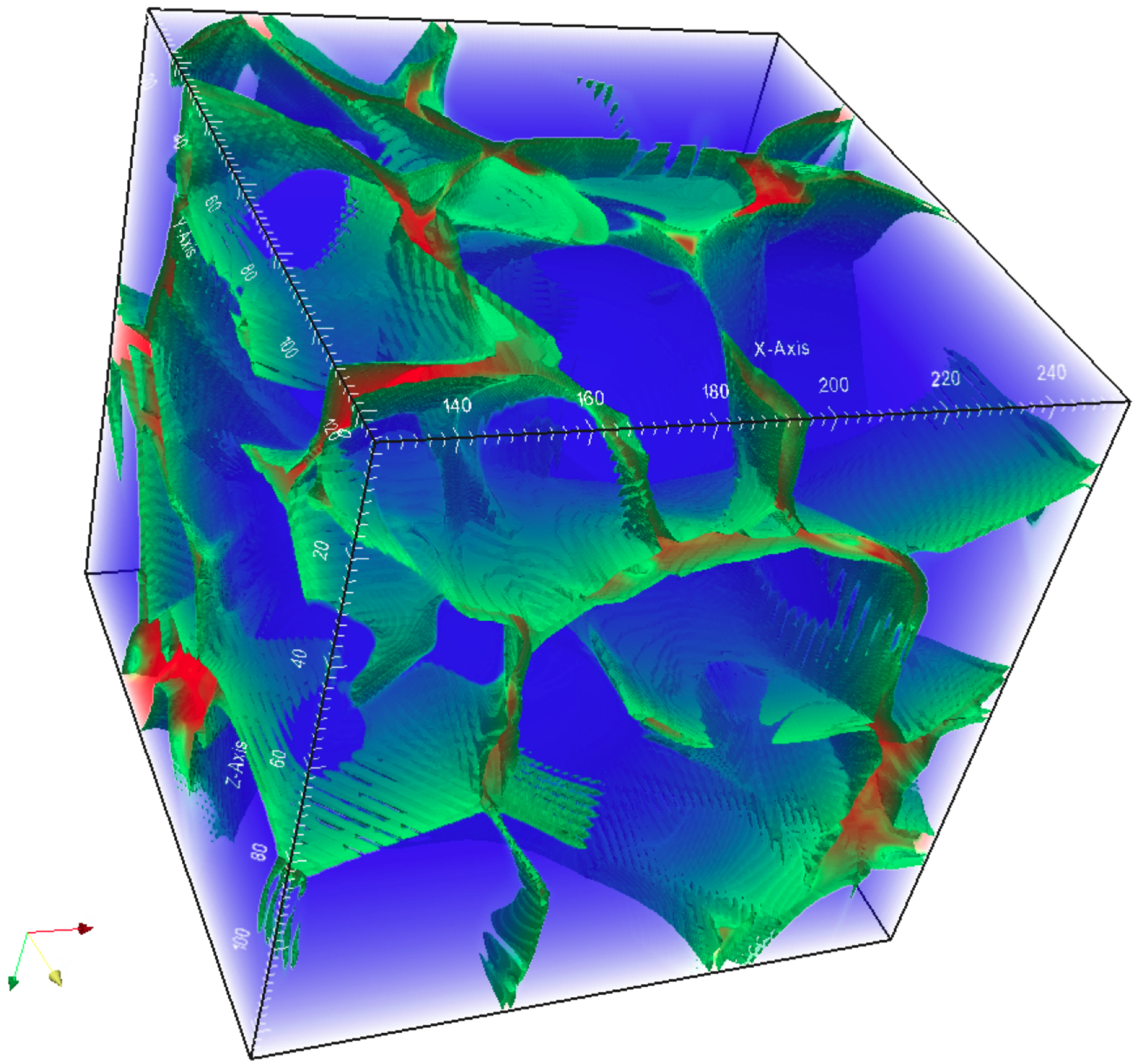
Phase-space Caustics



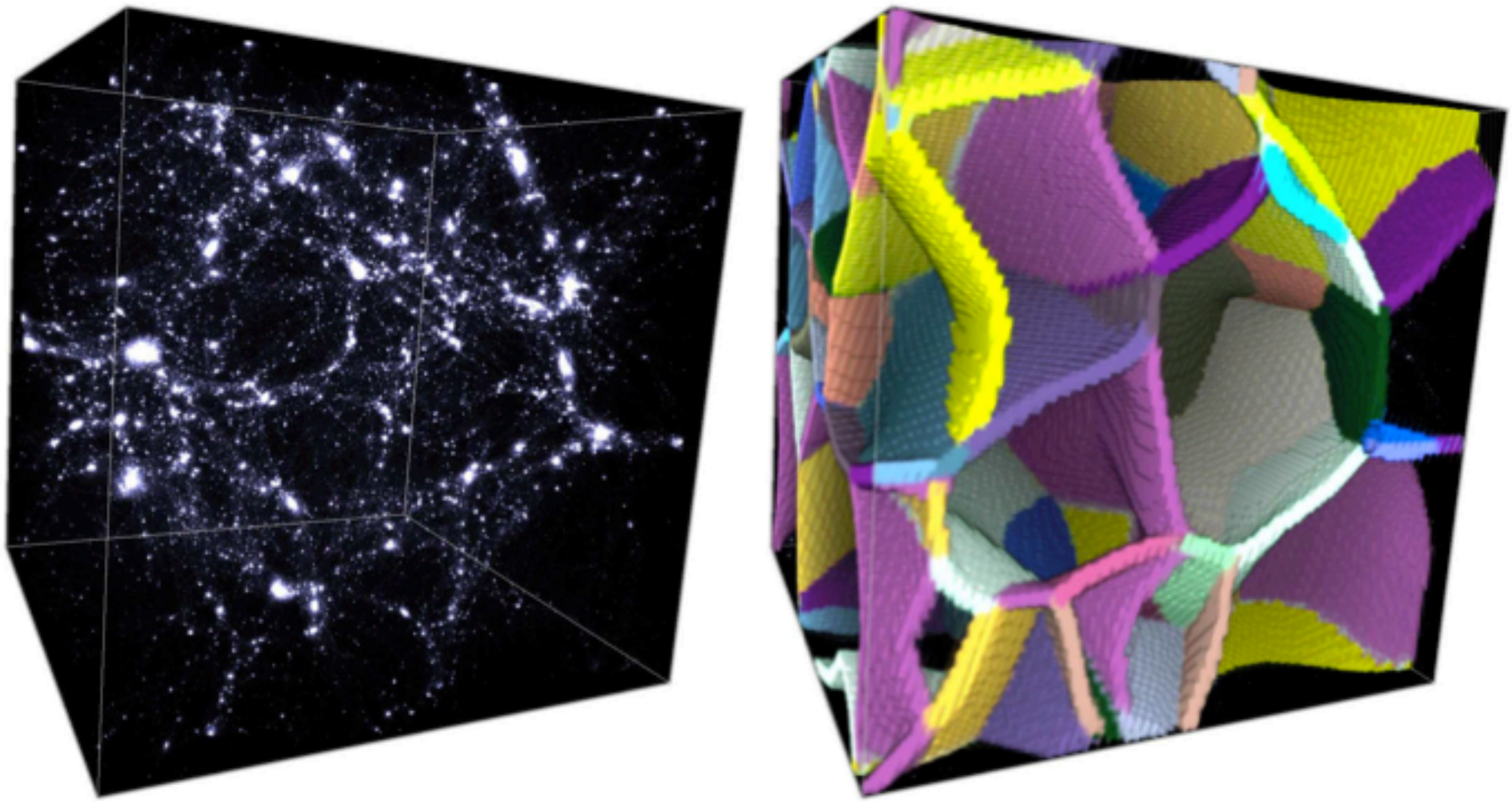
Three different families of caustic surfaces
in matter distribution (blue, green, red)

Shandarin 2012

Pancakes in Zeldovich Approximation in 3D



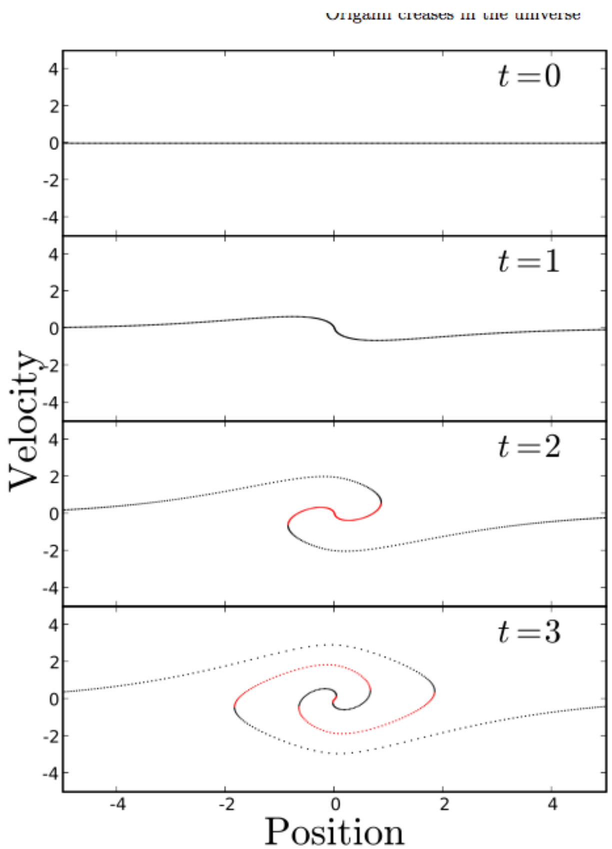
“Walls” in LCDM N-body simulation

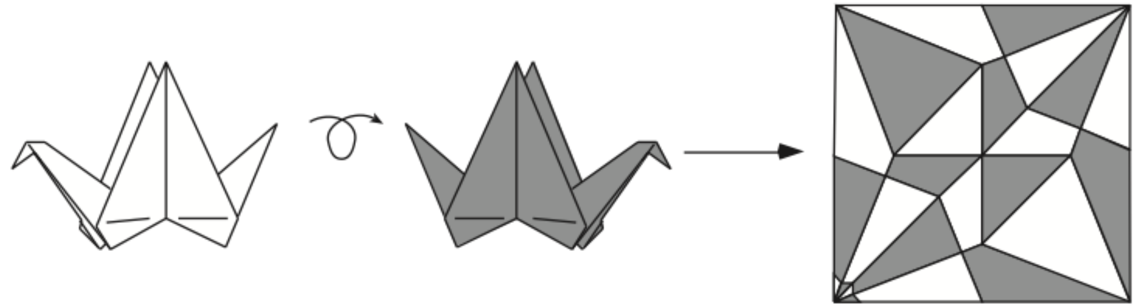


Sausbie 2010, MNRAS: “The persistent cosmic web and its filamentary structure I: Theory and implementation”.

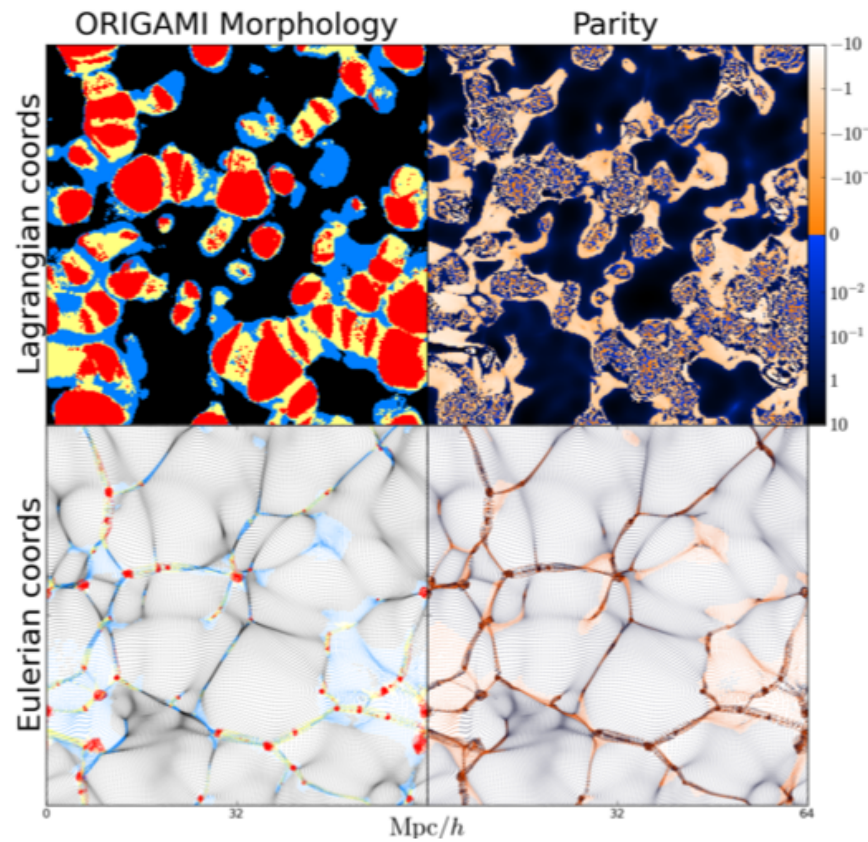
Based on Discrete Morse Theory and Topological Persistence.

Parity = sign of nD volume of nD simplex



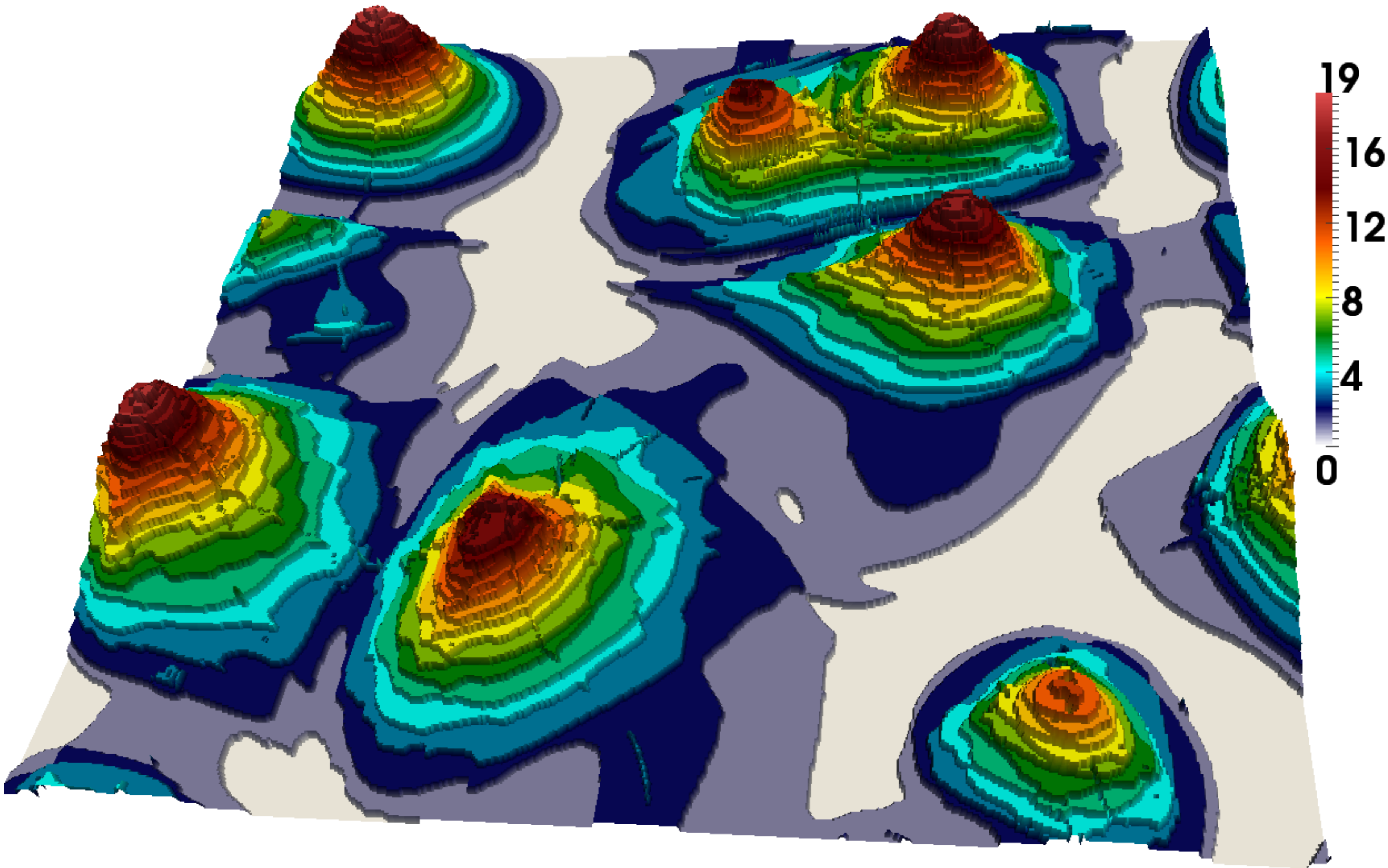


Origami

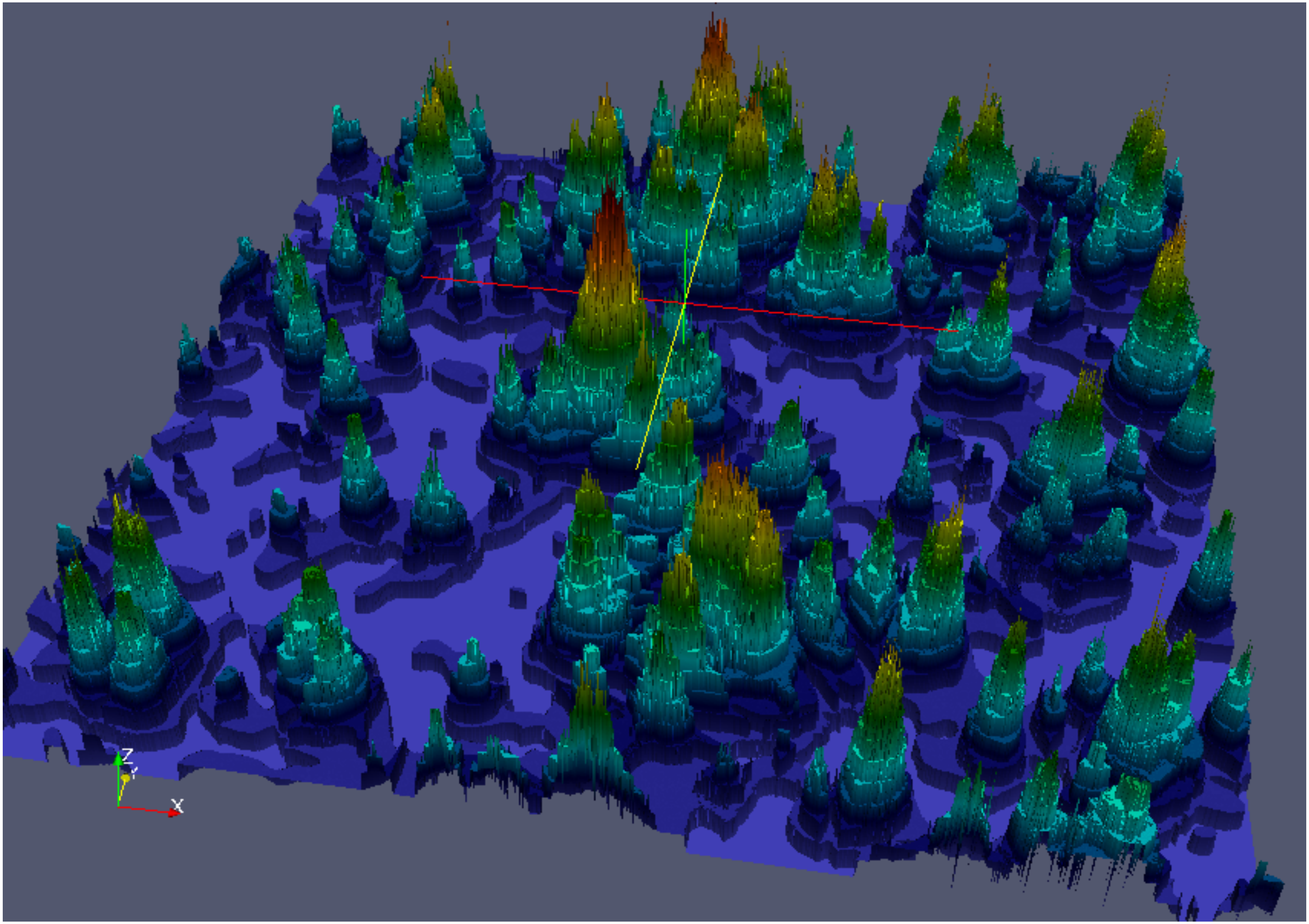


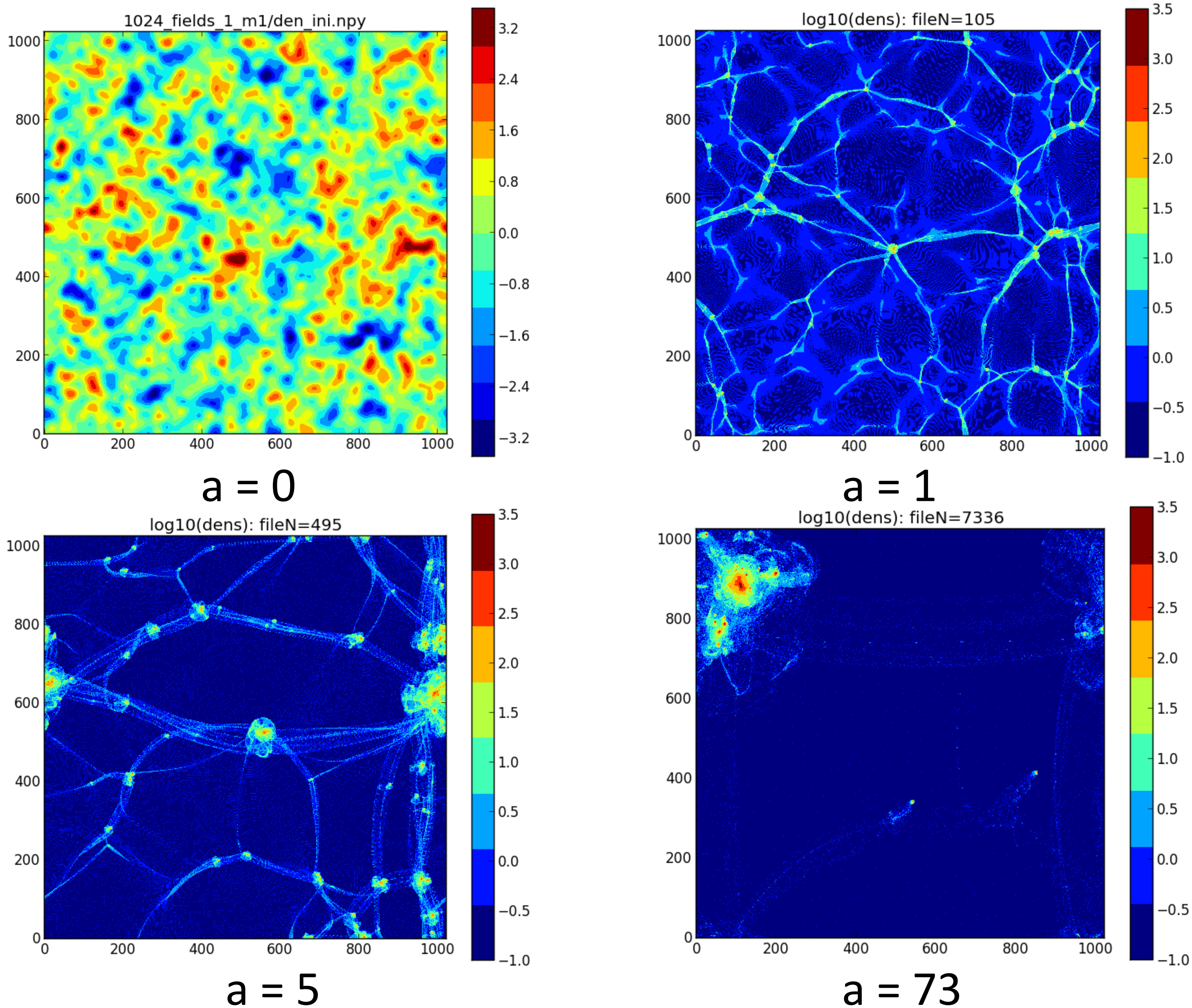
Neyrinck 12

Number of flip-flop in Lagrangian space in 512^2 simulation (smooth initial conditions)

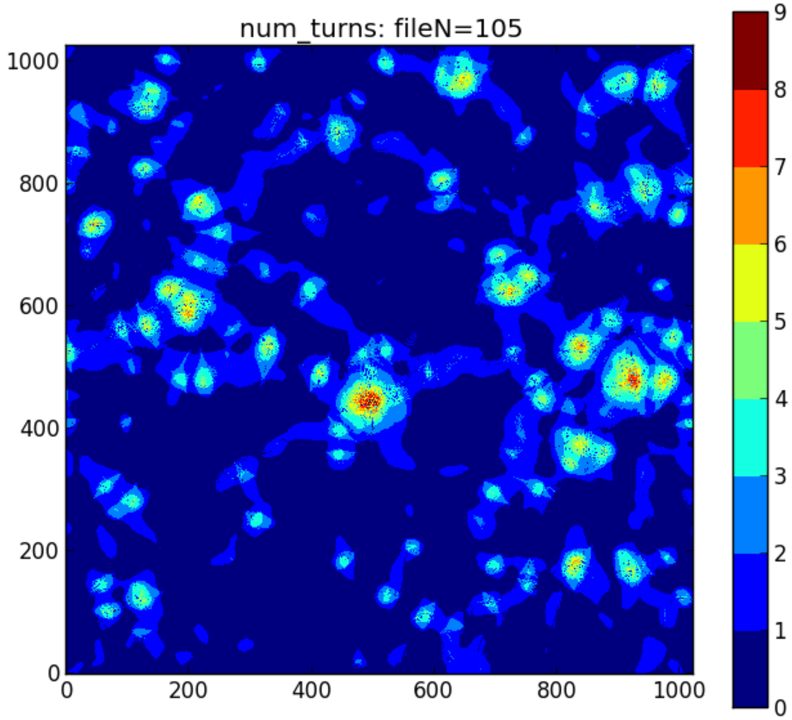


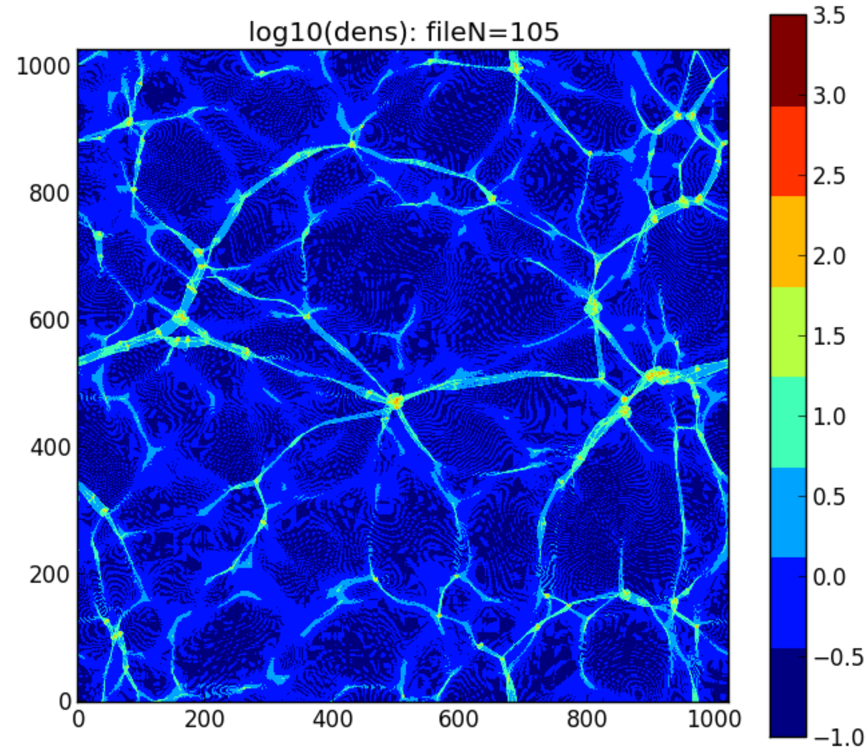
Number of flip-flop in Lagrangian space in 1024^2 simulation

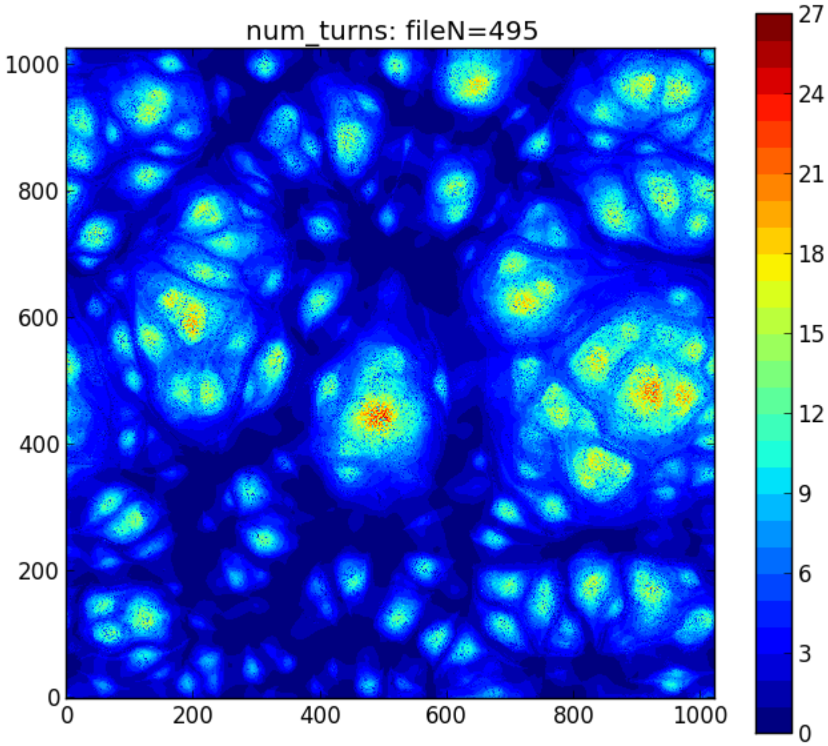



1024_fields_1_m1/den_ini.npy
a = 0
log10(dens): fileN=105
a = 1
log10(dens): fileN=495
a = 5
log10(dens): fileN=7336
a = 73

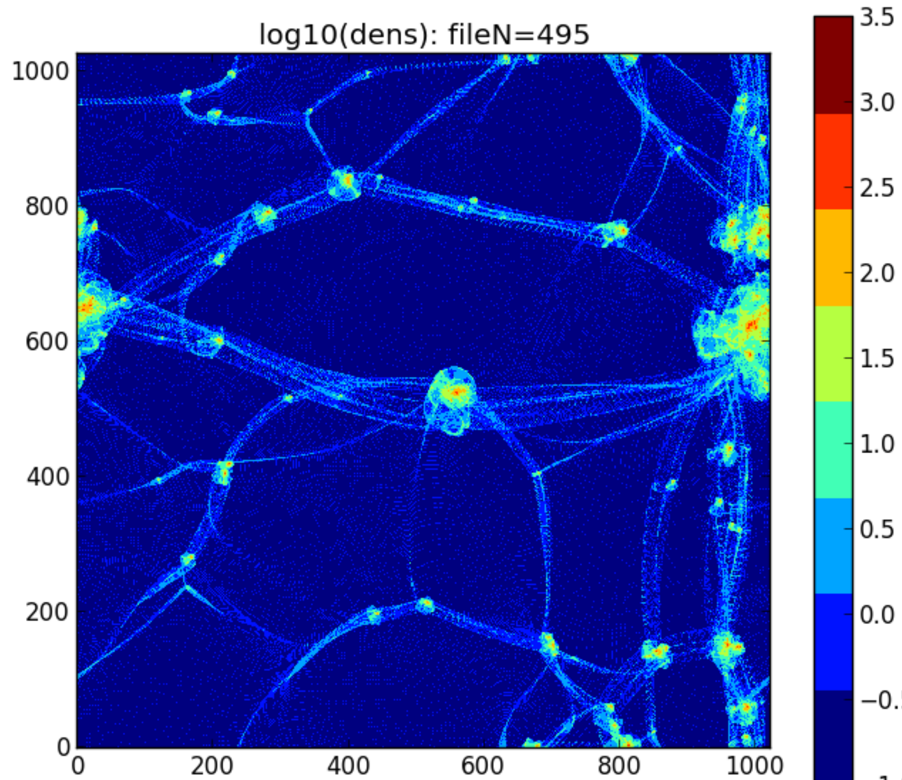
a = 1

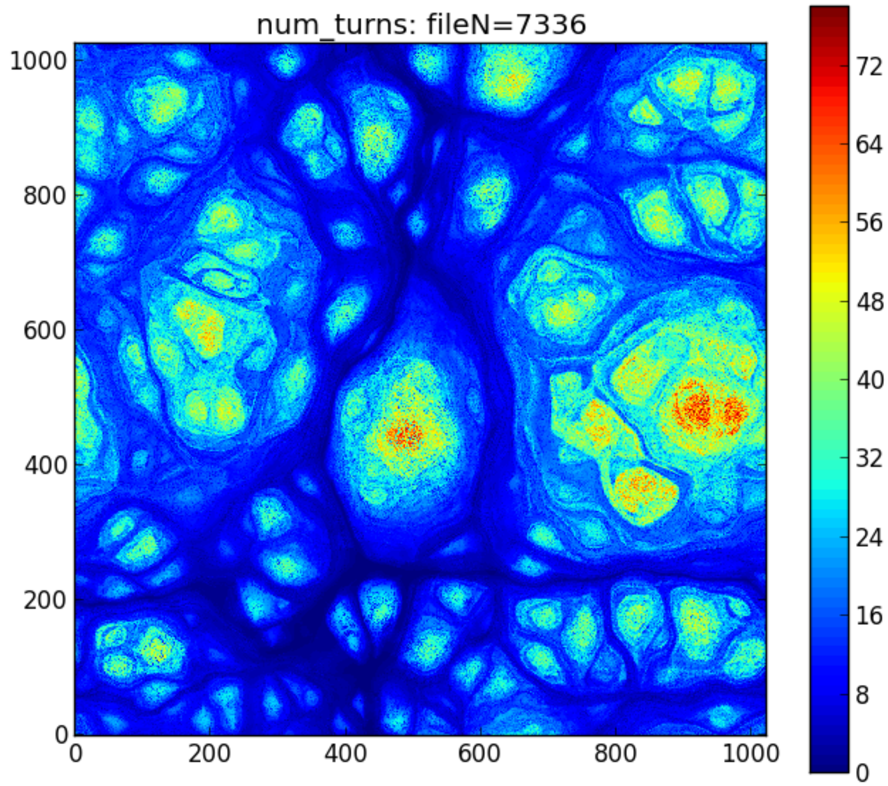




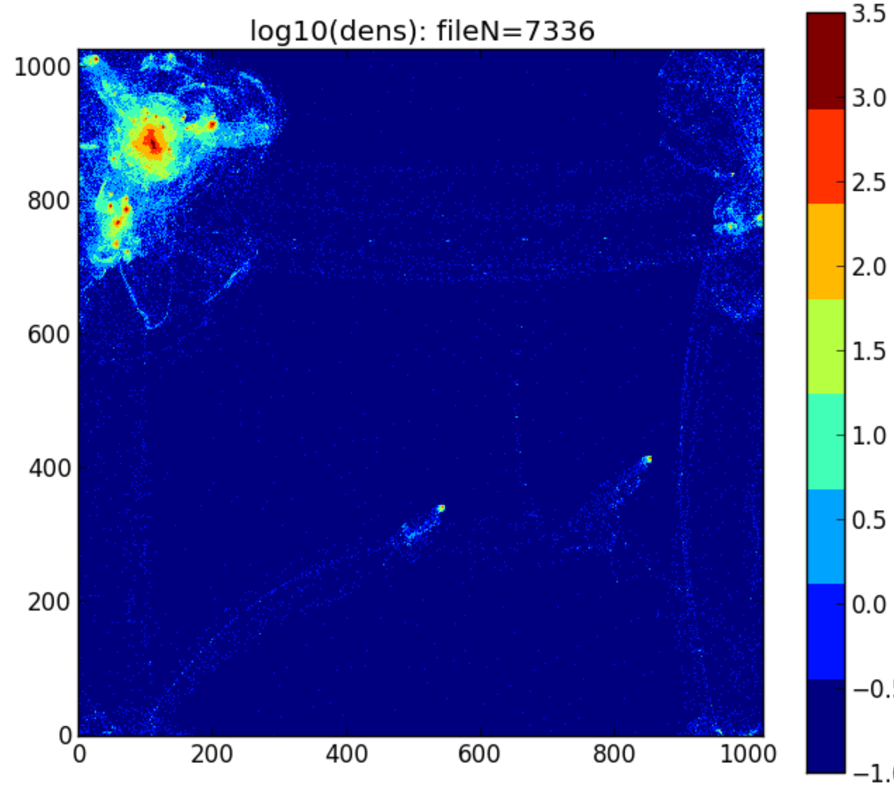

num_turns: fileN=495
0
200
400
600
800
1000
27
24
21
18
15
12
9
6
3

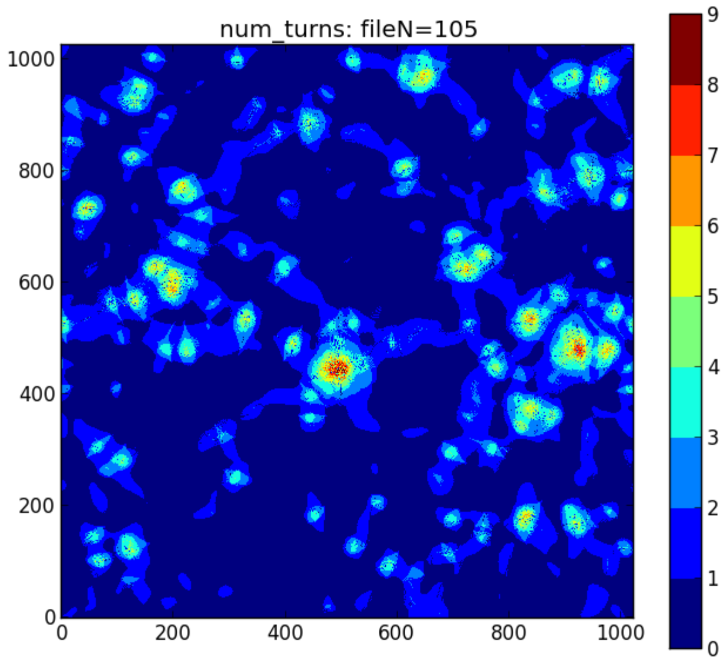
a = 5


log10(dens): fileN=495
0
200
400
600
800
1000
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0


num_turns: fileN=7336
1000
800
600
400
200
0
0
200
400
600
800
1000
72
64
56
48
40
32
24
16
8
0

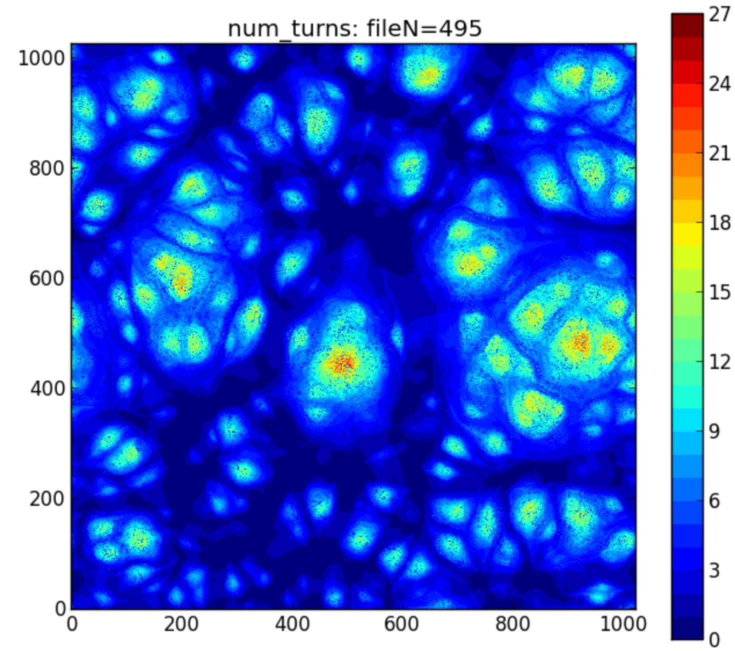
a = 73


log10(dens): fileN=7336
1000
800
600
400
200
0
0
200
400
600
800
1000
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0

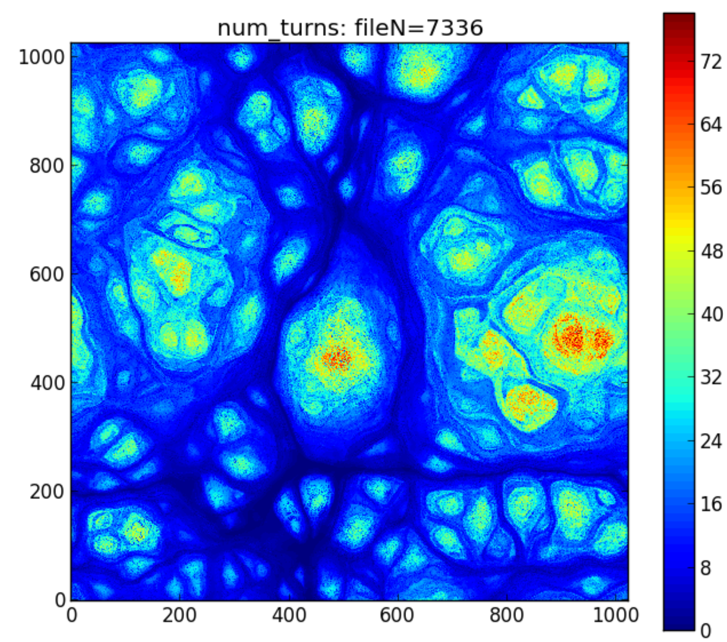

num_turns: fileN=105
0
200
400
600
800
1000
0
1
2
3
4
5
6
7
8
9

a = 1

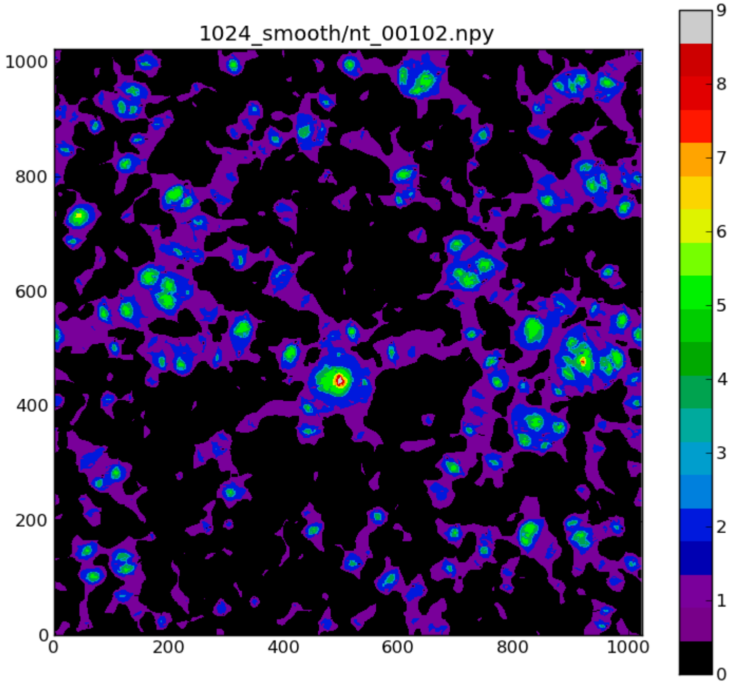
a = 5

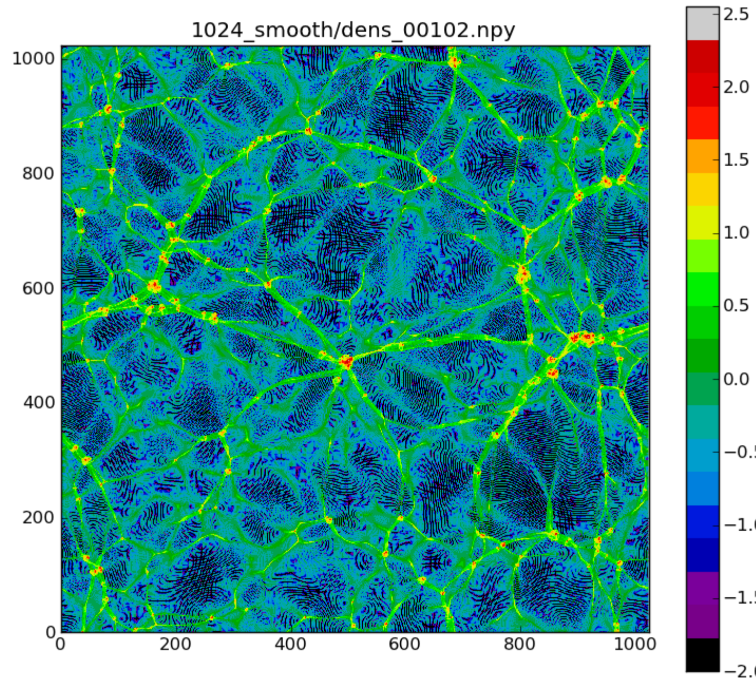

num_turns: fileN=495
0
200
400
600
800
1000
0
3
6
9
12
15
18
21
24
27

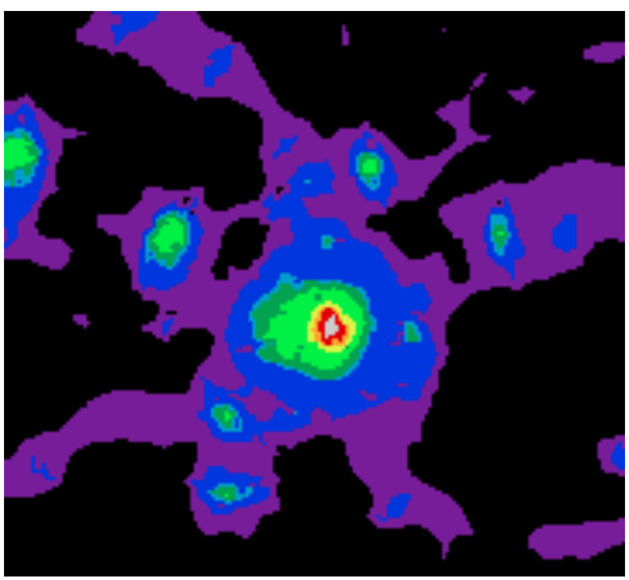
a = 73

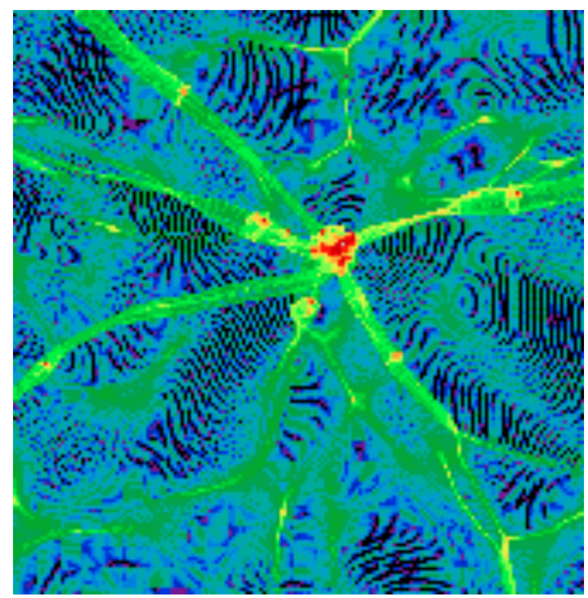

num_turns: fileN=7336
0
200
400
600
800
1000
0
8
16
24
32
40
48
56
64
72

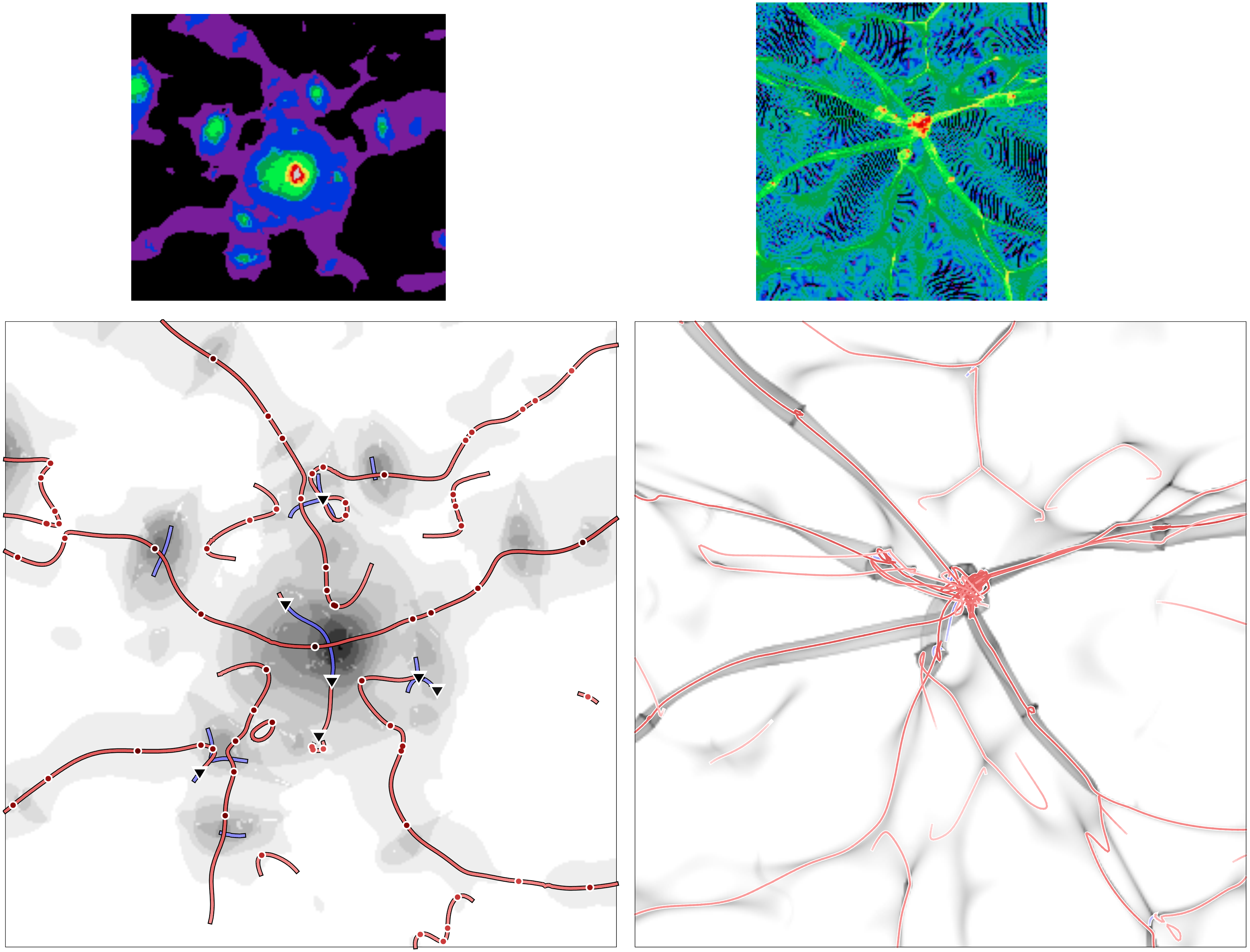
a = 1


1024_smooth/nt_00102.npy
0
200
400
600
800
1000
0
1
2
3
4
5
6
7
8
9

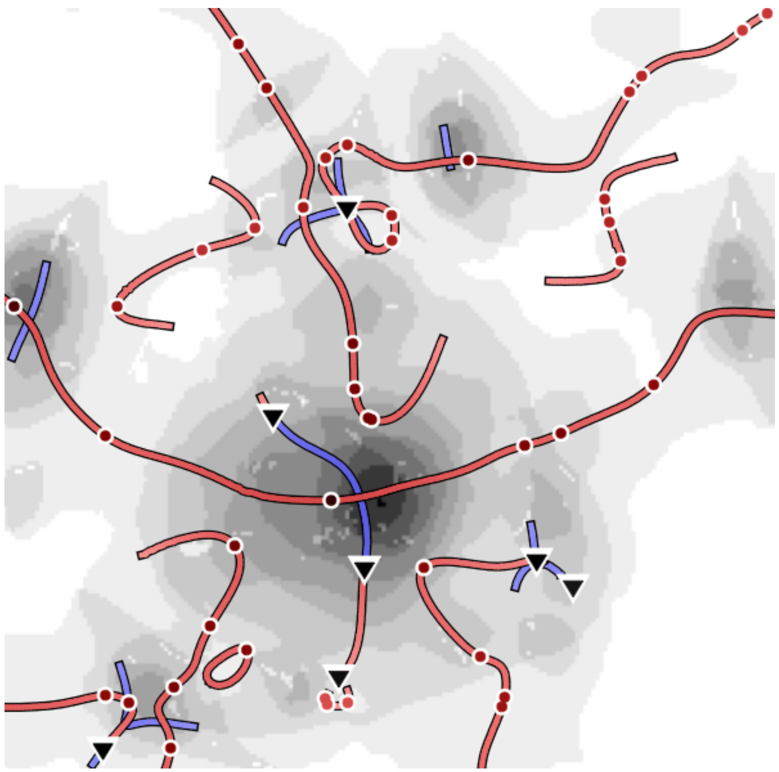

1024_smooth/dens_00102.npy
0
200
400
600
800
1000
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5

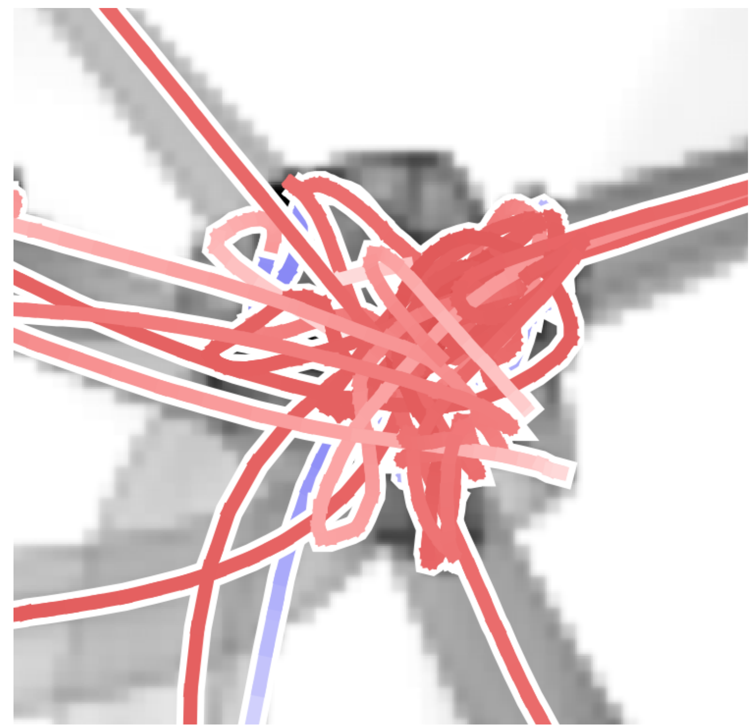






A3 lines are tied in complex knots in halos





Summary

New fields: number of streams, parity, number of flip-flops reveal new properties of the cosmic web. These fields are easy to obtain from current cosmological N-body simulations.

Number of streams field as a function of Eulerian coordinates allows to set physical limit on the the total volume of the voids:
for LCDM model: volume fraction is 93% and the mass fraction is 24%
for the halos with mass $> \sim (1.-3.)\ 10^{11}\ M_{sun}$.

The number of flip-flops as a function of Lagrangian coordinates may be a good quantitative indicator of substructure in DM halos.

The skeleton of the Cosmic Web in L can be obtained using the Zeldovich Approximation: A3 lines in 2D or A3 surfaces in 3D. The skeleton in E should be obtained using the N-body simulation map of the Lagrangian skeleton to Eulerian space.

The Zeldovich Approximation provides all the necessary concepts and language needed for analysis of cosmological N-body simulation, including: phase space, Lagrangian submanifold, number-of-stream field, number of flip-flop field, and caustic surfaces.