Dark energy and neutrinos in perturbation theory

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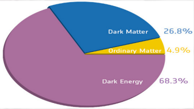
Motivations

1. Dark energy 

2. Neutrinos 

3. Cosmic emulators 

Outline

1. Higher-order perturbation theory
   - nonlinear evolution equations
   - standard perturbation theory
   - how accurate is perturbation theory?
2. Generalization to other cosmologies
   - EdS $\longrightarrow$ ΛCDM, wCDM: a lucky cancellation!
   - time-renormalization group perturbation theory
   - preliminary results: time-RG, neutrinos, and dark energy
3. Conclusions and future work

Higher-order perturbation theory

Let's work in an Einstein-de Sitter cosmology.
(EdS: $\Omega_{\rm m} = 1$, nothing else in the universe)

Continuity and Euler equations:

- $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

- $\rho \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} + \nabla \Phi = 0$

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  $\Rightarrow \frac{\partial \delta}{\partial t} + \theta = -\vec{\nabla} \cdot (\delta \vec{v})$ where $\delta = \rho/\bar{\rho} - 1$ and $\theta = \vec{\nabla} \cdot \vec{v}$

- $\rho \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} + \nabla \Phi = 0$
  $\Rightarrow \frac{\partial \theta}{\partial t} + H\theta + 4\pi G \bar{\rho} \delta = -\vec{\nabla} \cdot [(\vec{v} \cdot \vec{\nabla})\vec{v}]$

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  $\Rightarrow \frac{\partial \delta(\vec{k})}{\partial \ln a} + \frac{\theta(\vec{k})}{aH} = -\frac{1}{aH} \int \frac{d^3p\, d^3q}{(2\pi)^3} \delta_{\rm D}(\vec{k} - \vec{p} - \vec{q}) \frac{\vec{k}\cdot\vec{p}}{p^2} \theta(\vec{p}) \delta(\vec{q})$
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  $\Rightarrow \frac{\partial \theta(\vec{k})}{\partial \ln a} + \theta(\vec{k}) + \frac{3}{2} \Omega_{\rm m} aH \delta(\vec{k}) =$
  $- \frac{1}{aH} \int \frac{d^3p\, d^3q}{(2\pi)^3} \delta_{\rm D}(\vec{k} - \vec{p} - \vec{q}) \frac{k^2 (\vec{p}\cdot\vec{q})}{2p^2q^2} \theta(\vec{p}) \theta(\vec{q})$

Standard perturbation theory (SPT)

Linear theory: Throwing out nonlinear terms, we find $\ddot{\delta} + 2H\dot{\delta} = 4\pi G\delta$, which has a growing mode solution $\delta_{\rm L} \propto a$.

Standard Perturbation Theory: Solve the nonlinear equations by means of series expansions in the linear perturbation $\delta_{\rm L}$ today,

$$\delta(\vec{k}, a) = \sum_{n=1}^{\infty} a^n \delta_n(\vec{k}), \qquad \theta(\vec{k}, a) = -aH \sum_{n=1}^{\infty} a^n \theta_n(\vec{k})$$

where the δ_n and θ_n are (complicated) mode-coupling integrals over products of n $\delta_{\rm L}$s. ($\delta_1 = \delta_{\rm L}$)

Generalization beyond EdS: In a universe including other homogeneous components (dark energy), the linear overdensity $\delta_{\rm L} \propto D(a)$ (growth factor). Approximate the expansions as

$$\delta(\vec{k}, a) = \sum_{n=1}^{\infty} D(a)^n \delta_n(\vec{k}), \qquad \theta(\vec{k}, a) = -aHf \sum_{n=1}^{\infty} D(a)^n \theta_n(\vec{k})$$

where $f = d\ln D/d\ln a$ is 1 in EdS.

Standard perturbation theory (SPT)

Power spectrum: Using our expansion for $\delta(\vec{k}, a)$,

$$(2\pi)^3 \delta_{\rm D}(\vec{k} + \vec{k}') P(k, a) = \left\langle \delta(\vec{k}) \delta(\vec{k}') \right\rangle$$
$$= \left\langle \delta_1(\vec{k}) \delta_1(\vec{k}') \right\rangle + D(a)^2 \left\langle \delta_1(\vec{k}) \delta_3(\vec{k}') \right\rangle + D(a)^2 \left\langle \delta_2(\vec{k}) \delta_2(\vec{k}') \right\rangle + \ldots$$

$$\Rightarrow P(k) = P_{\rm L}(k) + P^{(1,3)}(k) + P^{(2,2)}(k) + \ldots$$

where the "1-loop" corrections are

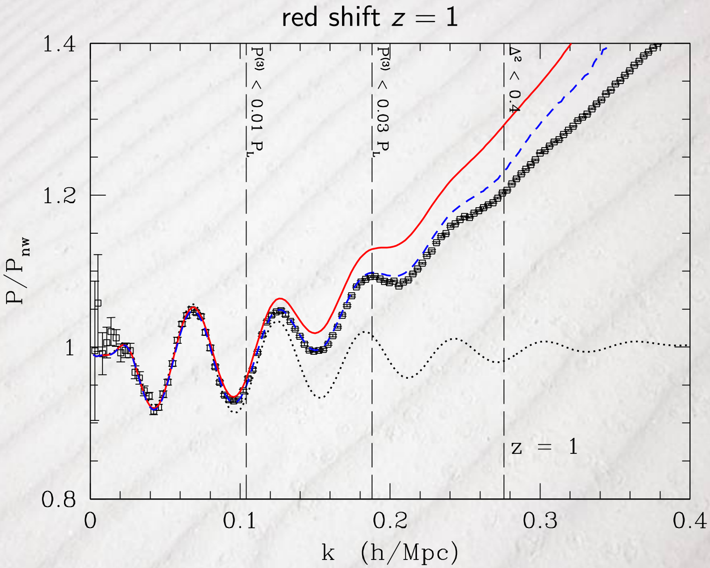
$$P^{(1,3)} = \frac{k^3 P_{\rm L}(k)}{1008\pi^2} \int_0^\infty dr P_{\rm L}(kr) \left[\frac{12}{r^2} - 158 + 100 r^2 - 42 r^4 + \frac{3(r^2-1)^3(7r^2+2)}{r^2} \ln \left| \frac{1+r}{1-r} \right| \right]$$
$$P^{(2,2)} = \frac{k^3}{392\pi^2} \int_0^\infty dr P_{\rm L}(kr) \int_{-1}^1 dx P_{\rm L}(k\sqrt{1 + r^2 - 2rx}) \frac{(3r + 7x - 10rx^2)^2}{(1 + r^2 - 2rx)^2}$$

Makino, Sasaki, Suto, PRD ***46****:585(1992).*

Similar expressions can be written down for the next order ("2-loop") terms, $P^{(1,5)}$, $P^{(2,4)}$, and $P^{(3,3)}$. They include 5-dimensional mode-coupling integrals to be evaluated numerically.

Carlson, White, Padmanabhan, PRD ***80****:043531(2009)[arXiv:0905.0479].*

How accurate is perturbation theory?



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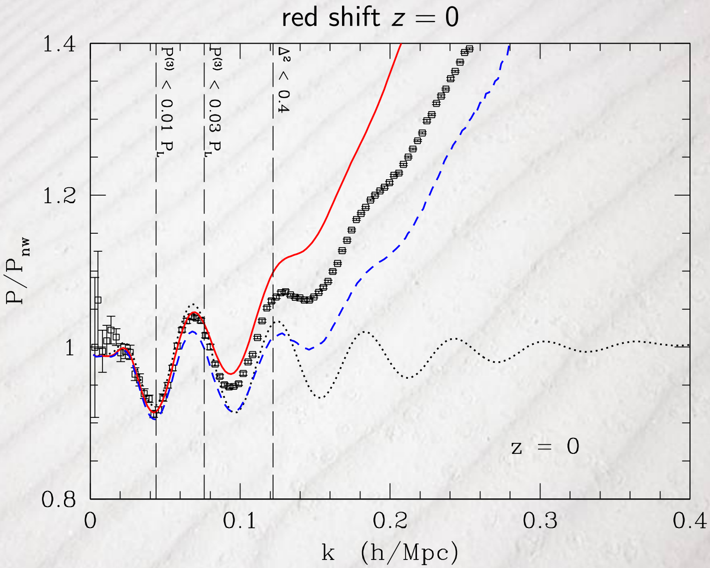

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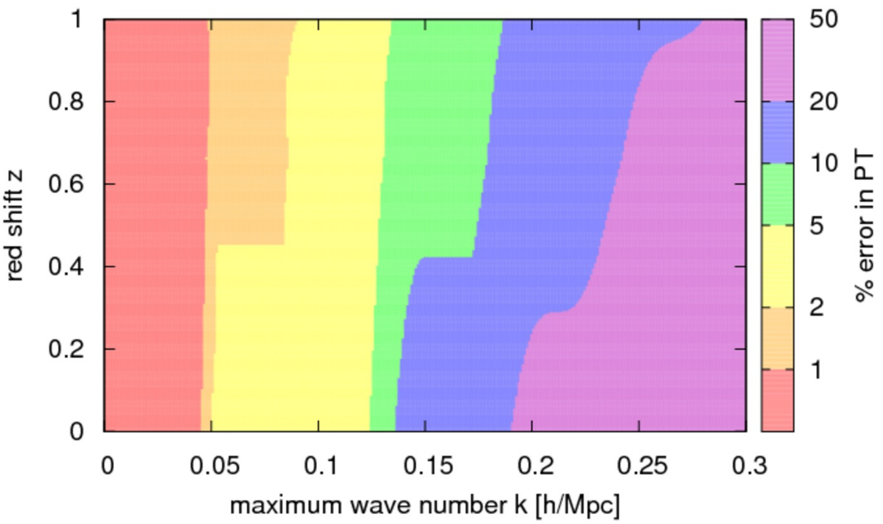
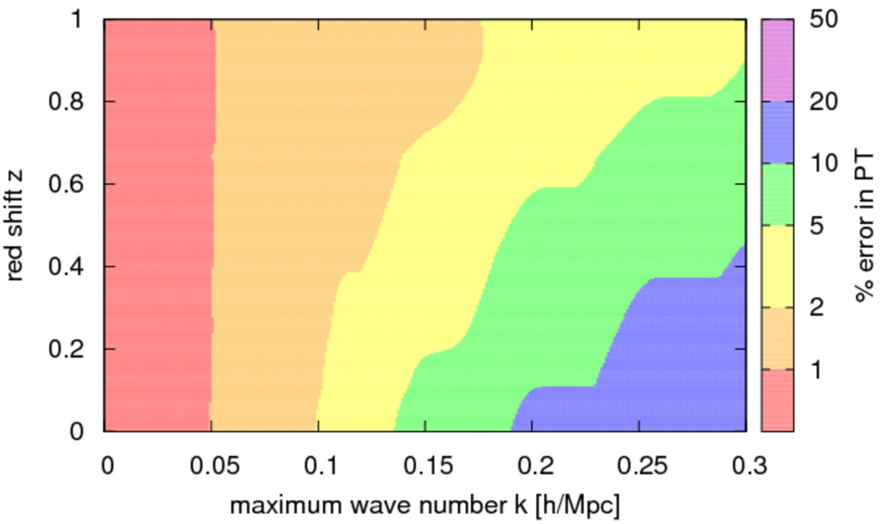
Linear theory



Heitmann, Pope, AU, Biswas (in prep.)

How accurate is perturbation theory?

Standard Perturbation Theory (1-loop)



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We developed SPT in Einstein-de Sitter, then generalized it to ΛCDM and wCDM through the simple replacement $a \to D(a)$. Why does this work so well?

First, a shorthand notation for the perturbations. $\eta = \ln(a/a_{\rm in})$ for some initial scale factor $a_{\rm in}$, and define ψ_b, Ω_{bc}, and γ_{bcd} as

$\psi_0(\vec{k},\eta) = e^{-\eta}\,\delta(\vec{k},\eta)$,
$\psi_1(\vec{k},\eta) = -e^{-\eta}\,\theta/(aH)$,
$\Omega_{00} = -\Omega_{01} = 1$
$\Omega_{10}(\vec{k},\eta) = -\frac{3\Omega_{\rm m}H_0^2}{2a^3H^2} = -\frac{3}{2}\Omega_{\rm m}(a)$
$\Omega_{11}(\eta) = 3 + \frac{d\ln H}{d\eta}$
$\gamma_{010}(\vec{k},\vec{p},\vec{q}) = \gamma_{001}(\vec{k},\vec{q},\vec{p}) = \delta_{\rm D}(\vec{k}+\vec{p}+\vec{q})(\vec{p}+\vec{q})\cdot\vec{p}/(2p^2)$
$\gamma_{111}(\vec{k},\vec{p},\vec{q}) = \delta_{\rm D}(\vec{k}+\vec{p}+\vec{q})(\vec{p}+\vec{q})^2\vec{p}\cdot\vec{q}/(2p^2q^2)$

Then the evolution equations become

$$\frac{\partial}{\partial\eta}\psi_b(\vec{k}) = -\Omega_{bc}(\vec{k})\psi_c(\vec{k}) + e^{\eta}\gamma_{bcd}(\vec{k},-\vec{p},-\vec{q})\psi_c(\vec{p})\psi_d(\vec{q})$$

Generalization to other cosmologies

In EdS, Ω_{bc} takes a particularly simple form:

$$\mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2} & \frac{3}{2} \end{pmatrix}.$$

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In a cosmology with a homogeneous dark energy, if we change our time variable from $\ln(a/a_{\rm in})$ to $\ln(D/D_{\rm in})$ and $\psi_1 \to \psi_1/f$,

$$\mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -\frac{3\Omega_{\rm m}(a)}{2f^2} & \frac{3\Omega_{\rm m}(a)}{2f^2} \end{pmatrix} \text{ with } f = \frac{d\ln D}{d\ln a}$$

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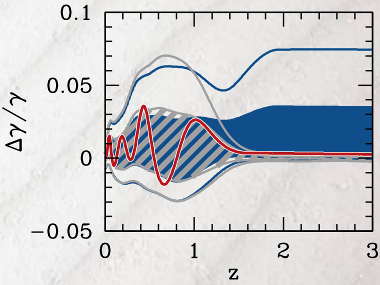
For a large range of cosmologies, $f(a) \approx \Omega_{\rm m}(a)^{0.55}$ to excellent precision. Then $\Omega_{\rm m}(a)/f(a)^2 \approx \Omega_{\rm m}(a)^{-0.1} \approx 1$ to $\approx 10\%$ since decoupling.

When does $f = \Omega_m(a)^{0.55}$ fail?

Massive neutrinos behave as a warm dark matter component. In the late universe they cluster like matter for k below a free-streaming scale $k_{fs}(a)$, and don't cluster for k much greater than k_{fs}. Growth becomes scale-dependent.

Exotic dark energy models such as early dark energy can change $f(a)$ to $\Omega_m(a)^{\gamma(a)}$ with $\gamma \neq 0.55$: *Mortonson, Hu, Huterer, PRD* **81***:063007(2010)[arXiv:0912.3816].*



We need a perturbation theory that doesn't rely on this close cancellation!

Time Renormalization Group (TRG) perturbation theory

Rather than treating D as a time variable, integrate the evolution equations for the power spectrum directly:

$$\frac{\partial}{\partial\eta}\langle\psi_a\psi_b\rangle = -\Omega_{ac}\langle\psi_c\psi_b\rangle - \Omega_{bc}\langle\psi_a\psi_c\rangle + e^\eta\gamma_{acd}\langle\psi_c\psi_d\psi_b\rangle + e^\eta\gamma_{bcd}\langle\psi_a\psi_c\psi_d\rangle$$

$$\frac{\partial}{\partial\eta}\langle\psi_a\psi_b\psi_d\rangle = -\Omega_{ad}\langle\psi_d\psi_b\psi_c\rangle - \Omega_{bd}\langle\psi_a\psi_d\psi_c\rangle - \Omega_{cd}\langle\psi_a\psi_b\psi_d\rangle + e^\eta\gamma_{ade}\langle\psi_d\psi_e\psi_b\psi_c\rangle + e^\eta\gamma_{bde}\langle\psi_a\psi_d\psi_e\psi_c\rangle + e^\eta\gamma_{cde}\langle\psi_a\psi_b\psi_d\psi_e\rangle$$

...

Linear theory: Neglect the bispectrum $\sim \langle\psi_a\psi_b\psi_c\rangle$. Then the infinite tower of evolution equations truncates, and we can integrae to find the power spectrum.

Time-RG is the next level of approximation. Keep the bispectrum but neglect the trispectrum, the connected part of $\langle\psi_a\psi_b\psi_c\psi_d\rangle$. This includes the 1-loop terms and some of the 2-loop terms.

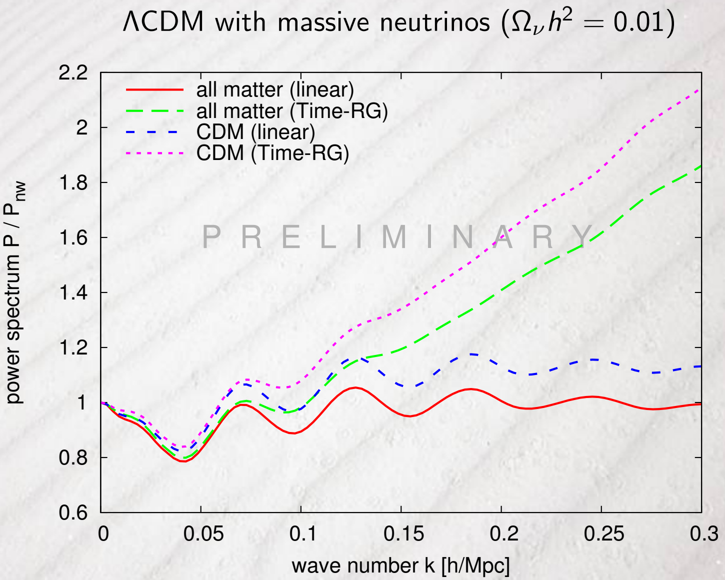
Pietroni, JCAP **810***:36(2008)[arXiv:0806.0971].*

Dark energy can be included trivially. Just use the actual linear evolution matrix $\mathbf{\Omega}$ rather than its EdS approximation.

Neutrinos modify the calculation several ways:

1. Homogeneous evolution $H(a)$ is modified;
2. Growth becomes scale-dependent, $D(a) \longrightarrow D(\vec{k}, a)$;
3. ψ_0 becomes $e^{-\eta}\,\delta_{\rm cb}$, the CDM+baryon density contrast, and ψ_1 becomes the CDM+baryon velocity divergence;
4. Ω_{10} changes from $-\frac{3}{2}\Omega_{\rm m}(a)$ to $-\frac{3}{2}\Omega_{\rm m}(a)\left[(1-f_\nu)+f_\nu\frac{\delta_\nu}{\delta_{\rm cb}}\right]$.

Time-RG and massive neutrinos



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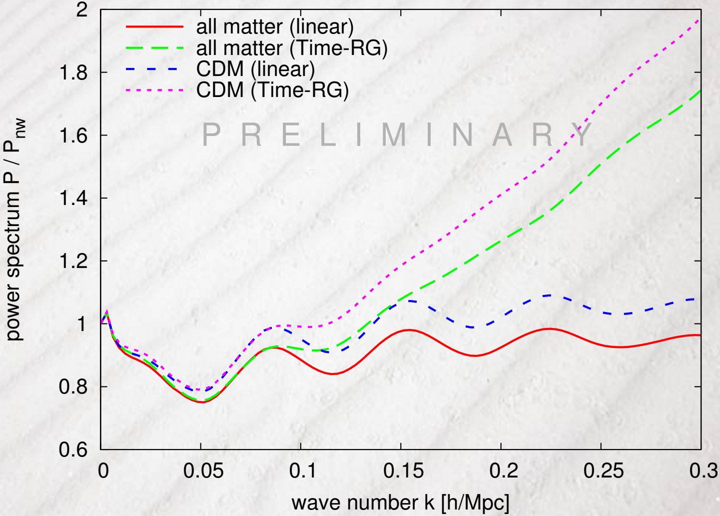
Time-RG, massive neutrinos, and early dark energy

wCDM ($w(a) = w_0 + w_a(1-a)$, $w_0 = -0.7$, $w_a = 0.67$)
with massive neutrinos ($\Omega_\nu h^2 = 0.01$)



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Conclusions and future work

- Higher-order perturbation theory is useful for extending power spectrum predictions into the quasilinear regime, especially at early times $z > 0.5$.
- N-body simulations and emulators can be used to verify the accuracy of perturbation theory techniques.
- Time-Renormalization Group perturbation theory allows rapidly evolving dark energy and massive neutrinos to be incorporated into higher-order perturbation theory.
- Future work:
  - compare my time-RG calculations to N-body simulations
  - build an emulator with w_0, w_a, and $\Omega_\nu h^2$
  - other models of scale-dependent growth? RSD in time-RG?