Evolution of Cosmic Web Kinematics
-- from potential to vortical flow

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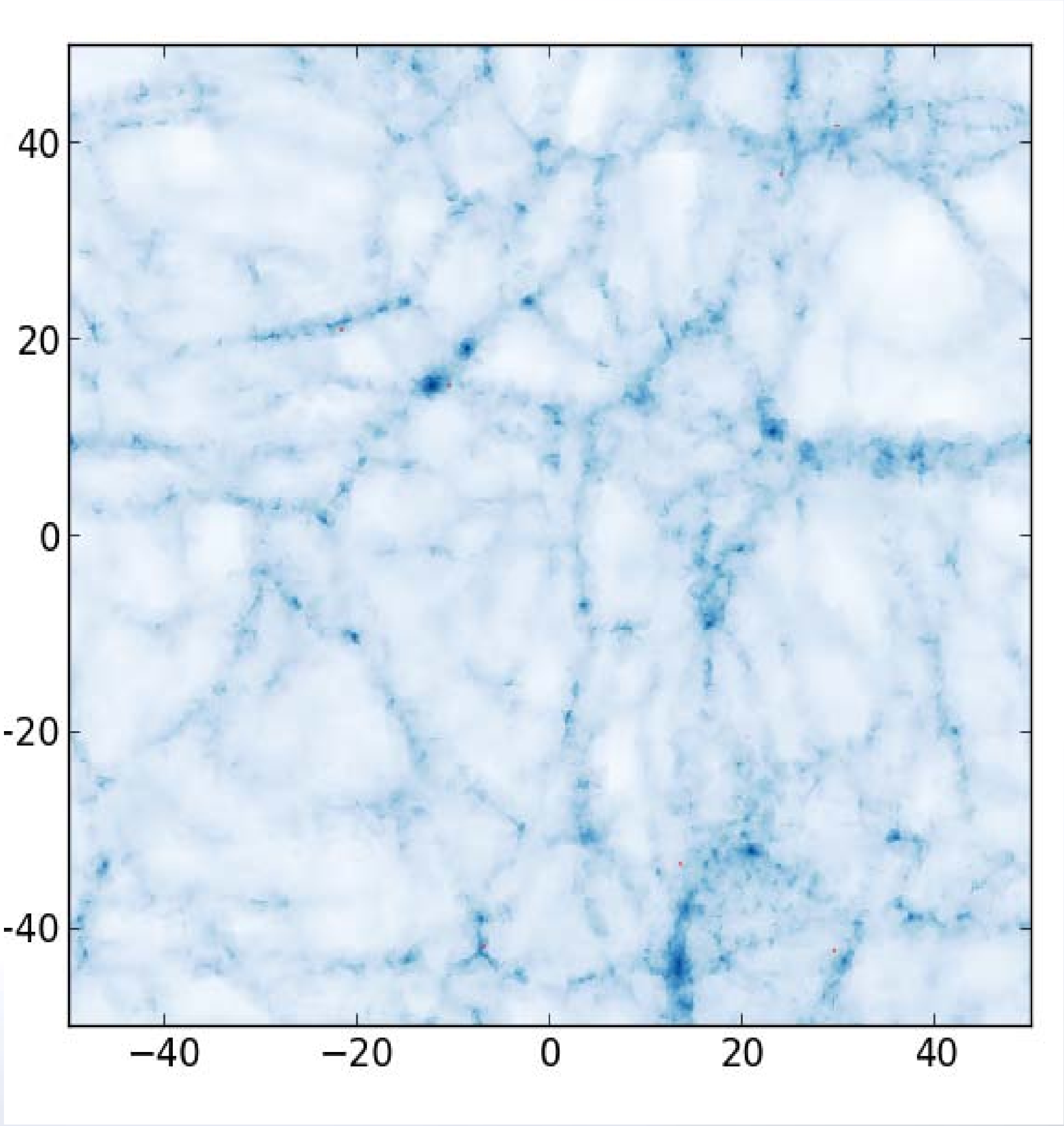
Alex Szalay(JHU), Miguel A. Arogon-Calvo(JHU), Mark Neyrinck(JHU)

Large-scale Structure

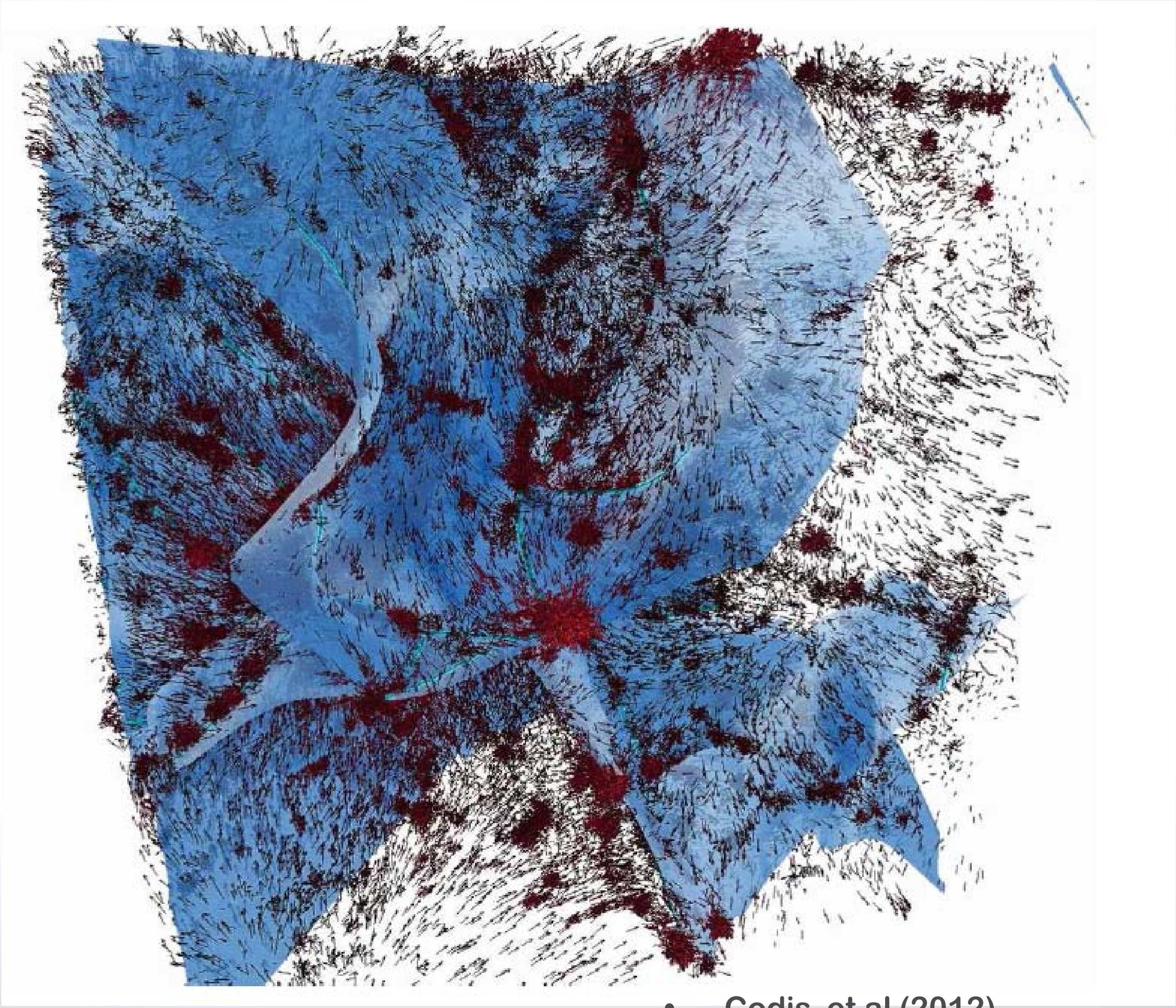
- **Cosmic web structure**
  - **Density**
  - **Halo, filament, wall**

- **Coherent evolution of velocity and density**
  - **Halo, filament, wall …**



Large-scale Structure

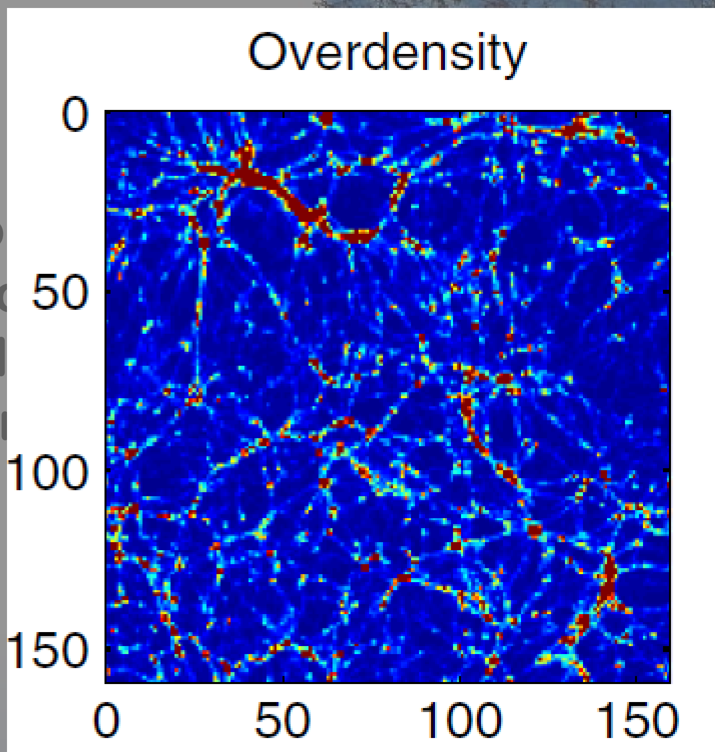
- **Coherent evolution of velocity and density**

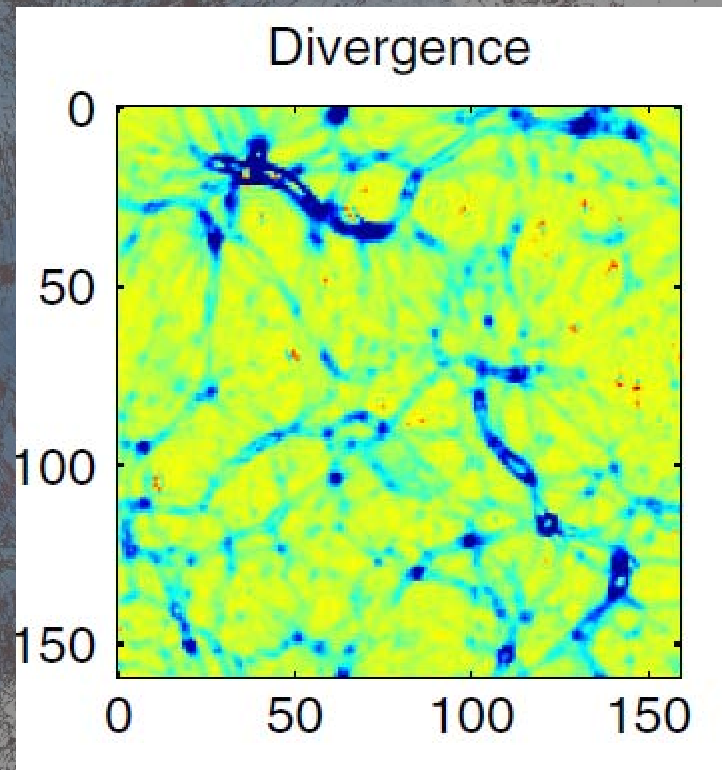


- **Codis et al (2012)**

Large-scale Structure

- Co ev vel de

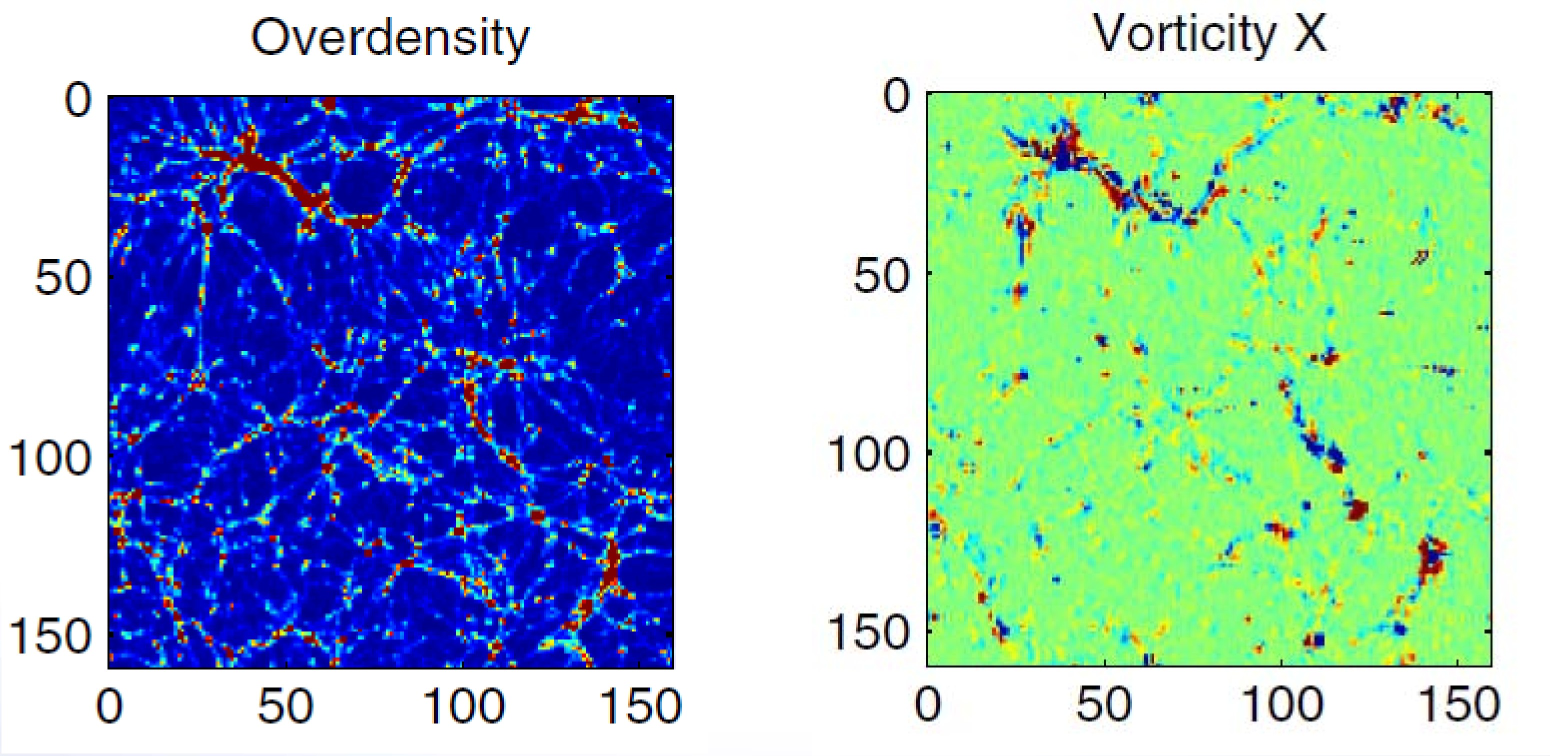




- Pueblas et al (2009)

Vortical flow

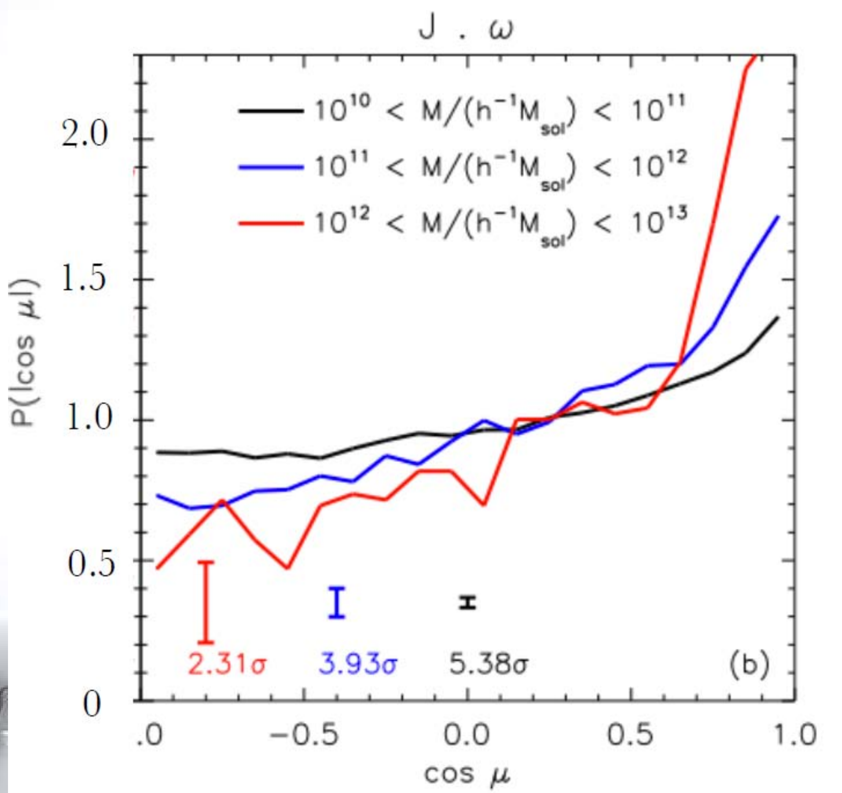
- cosmic web & vorticity
  - After Shell-crossing

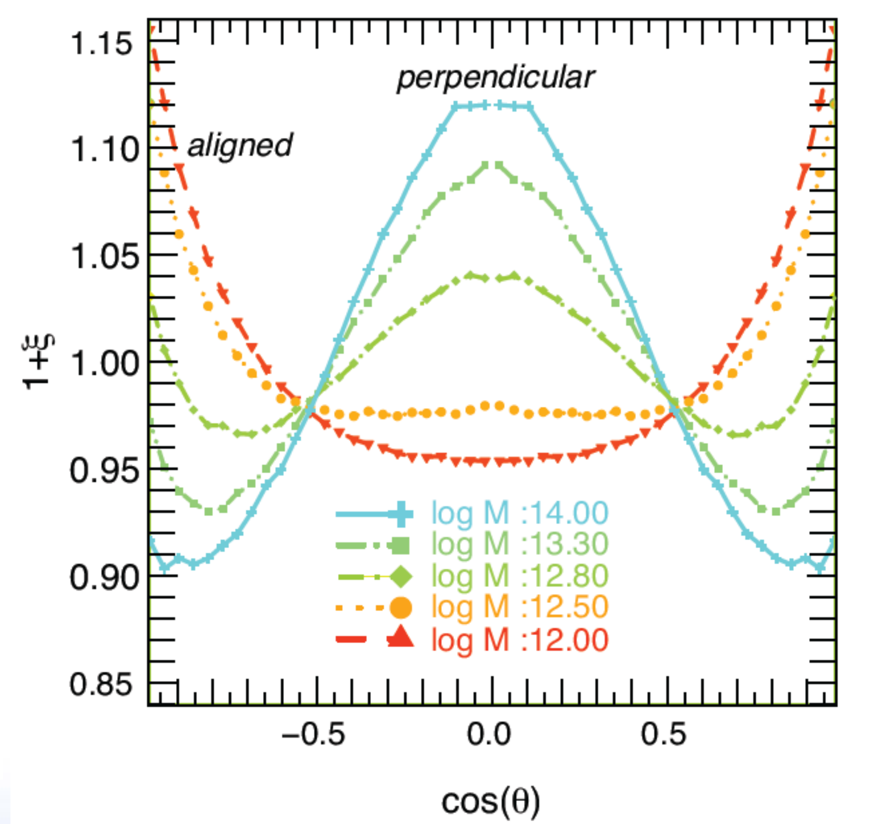


- Pueblas et al (2009)

Vortical flow – *vorticity orientation*

- cosmic web → vorticity → halo spin





- Libeskind et al. (2013)
- Codis et al (2012)

Invariants of velocity gradient

- ***How to study?***
  - *Evolution of peculiar velocity?*

- **Decomposition:**
  - divergence, vorticity:

$$\theta = \nabla \cdot \mathbf{u} \qquad \mathbf{w} = \nabla \times \mathbf{u}$$

  - Evolution equation

$$\frac{\partial \mathbf{u}(\mathbf{x},\tau)}{\partial \tau} + \mathscr{H}(\tau)\mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = -\nabla\Phi(\mathbf{x},\tau) - \frac{1}{\rho}\nabla_j(\rho\sigma_{ij}) \ ,$$

  - Linear regime

$$\frac{\partial \theta(\mathbf{x},\tau)}{\partial \tau} + \mathscr{H}(\tau)\theta(\mathbf{x},\tau) + \frac{3}{2}\Omega_m(\tau)\mathscr{H}^2(\tau)\delta(\mathbf{x},\tau) = 0 \ ,$$

$$\frac{\partial \mathbf{w}(\mathbf{x},\tau)}{\partial \tau} + \mathscr{H}(\tau)\mathbf{w}(\mathbf{x},\tau) = 0 \ .$$

Cosmic flow

- **In general:**

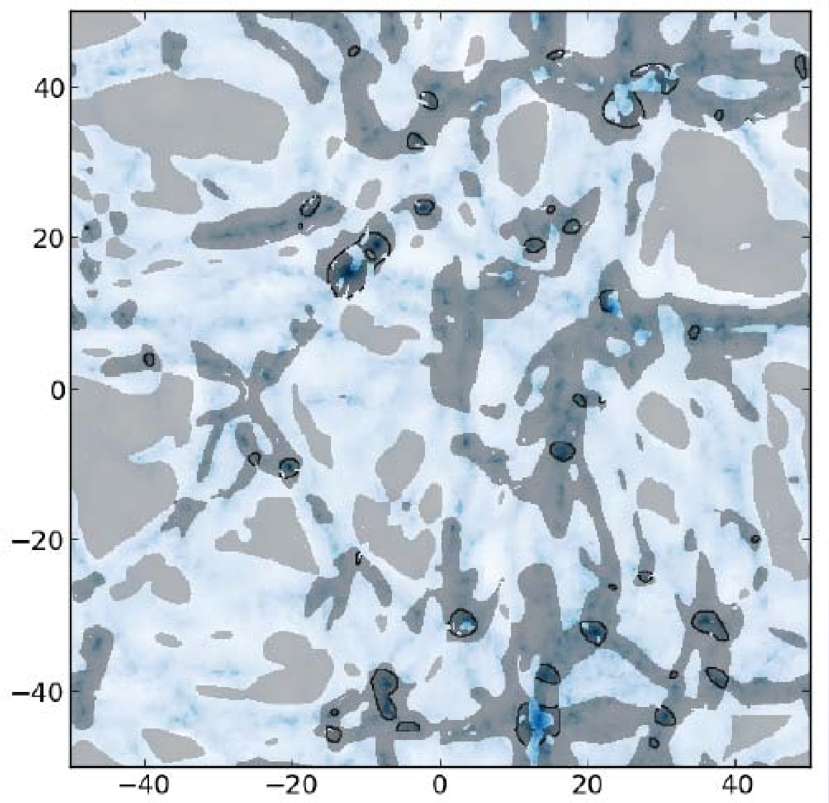
$$A_{ij}(\mathbf{x},\tau) = \frac{\partial u_i}{\partial x_j}(\mathbf{x},\tau).$$

- **Kinematical classification**

$$\frac{dx_i}{d\tau}(\mathbf{x}) = u_i(\mathbf{x}) = u_i(\mathbf{x}_0) + A_{ij}(\mathbf{x}_0)x_j + \cdots$$

- **where** $$\mathbf{A} = \mathbf{R}^{-1}\begin{pmatrix}\lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3\end{pmatrix}\mathbf{R},$$

  - **halo:** $\lambda_i < 0,$
  - **void:** $\lambda_i > 0$
  - **filament /wall:** indefinite



Cosmic flow

- **Velocity decomposition**
  – divergence, vorticity: $\theta = \nabla \cdot \mathbf{u}$, $\mathbf{w} = \nabla \times \mathbf{u}$
- in g
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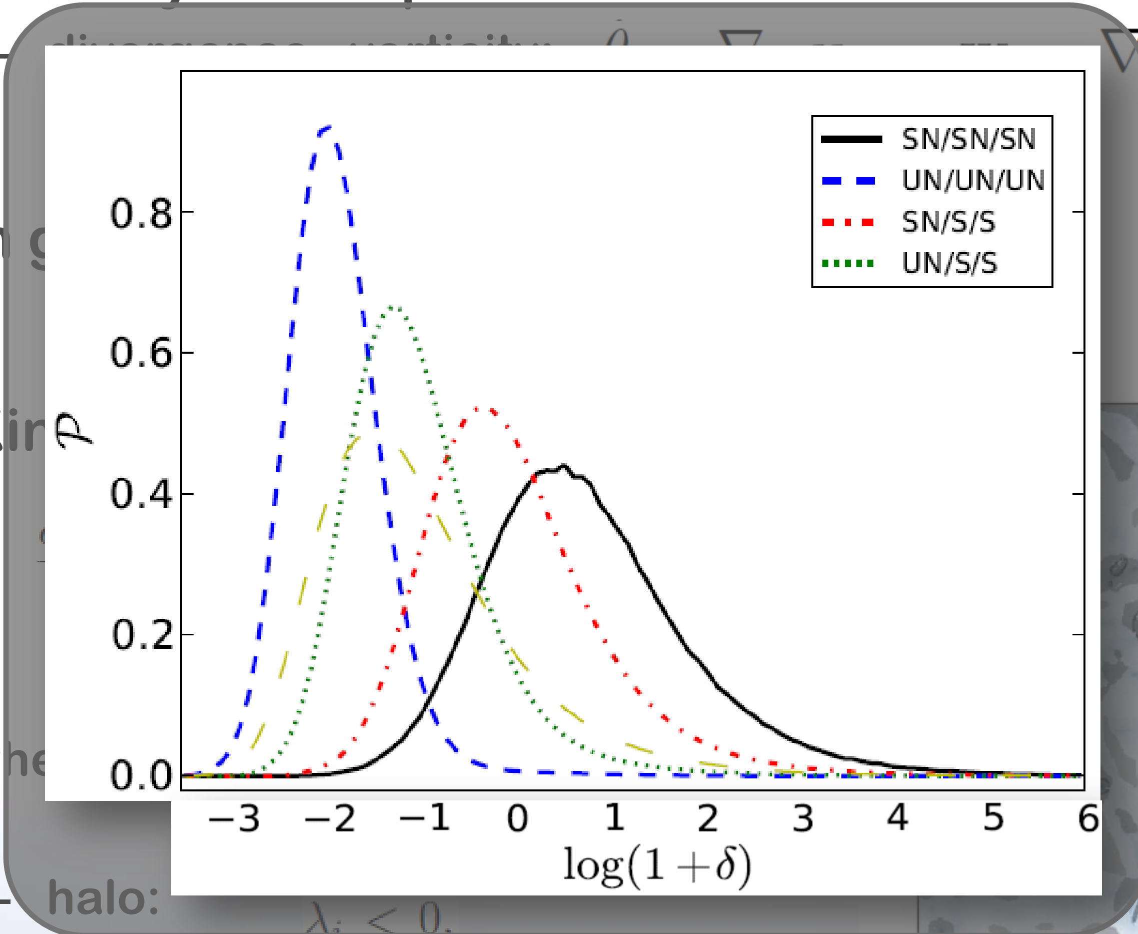
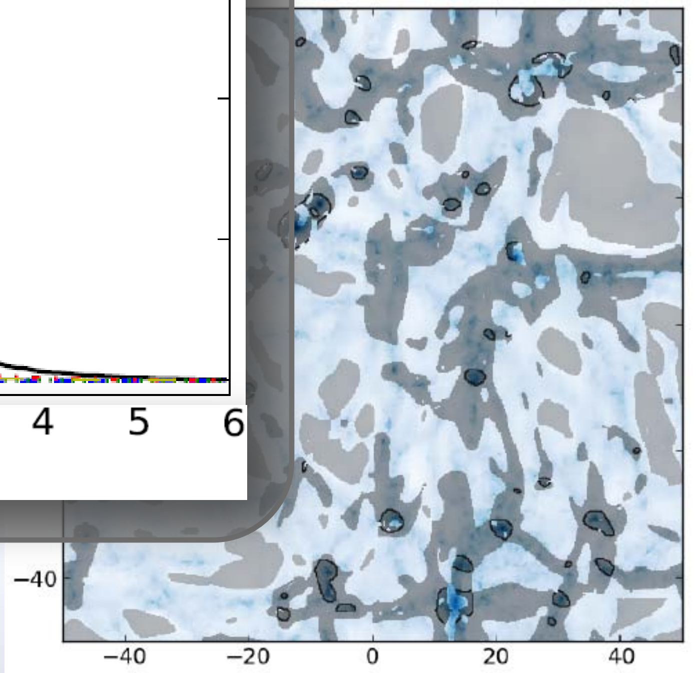
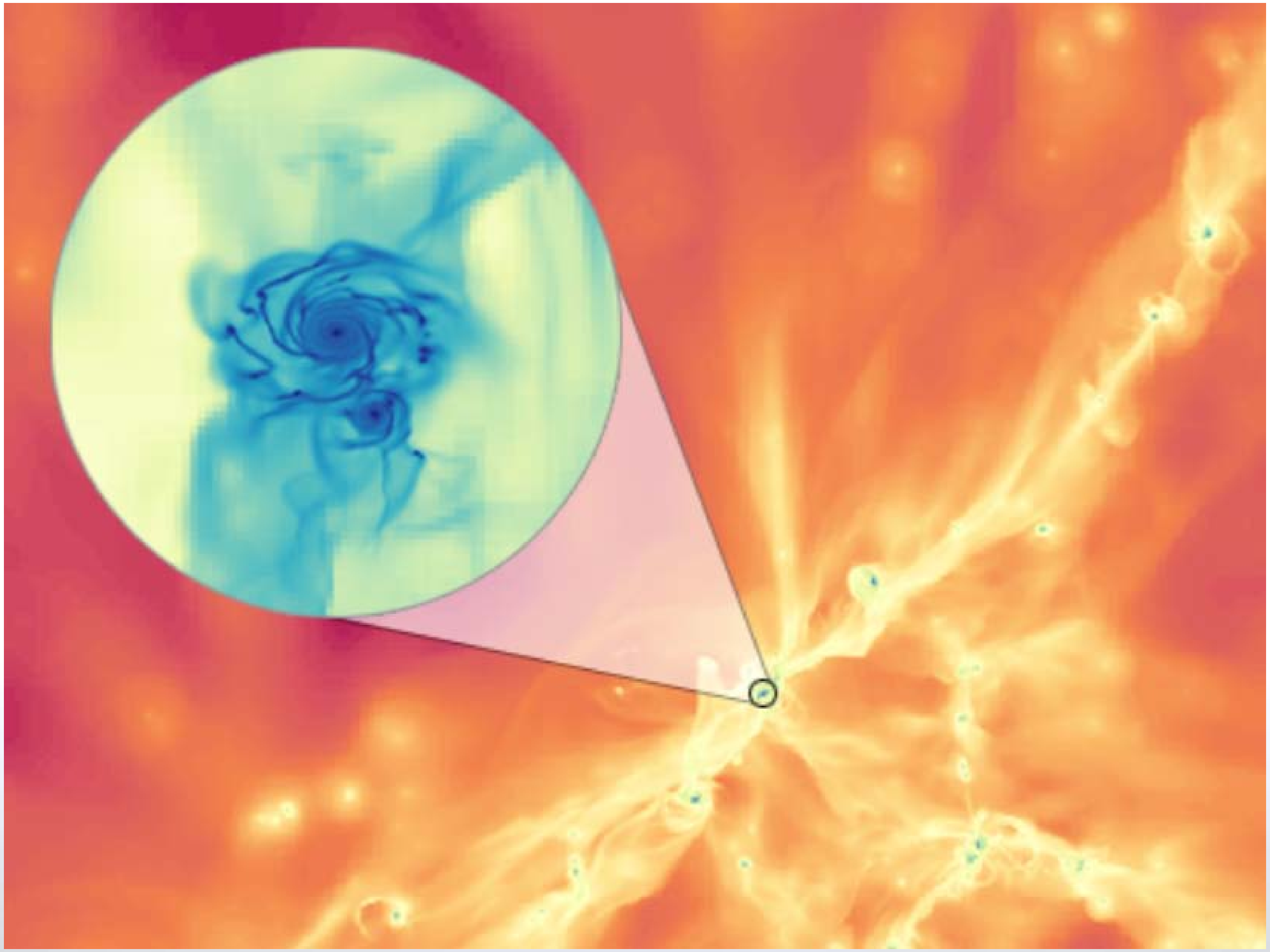
- whe
  – halo: $\lambda_i < 0$,
  – void: $\lambda_i > 0$
  – filament /wall: indefinite





Invariants of velocity gradient

- Vortical flow?
  - Complex eigenvalues



Invariants of velocity gradient

- Vortical flow?
  - Complex eigenvalues

- Alternatively, more general

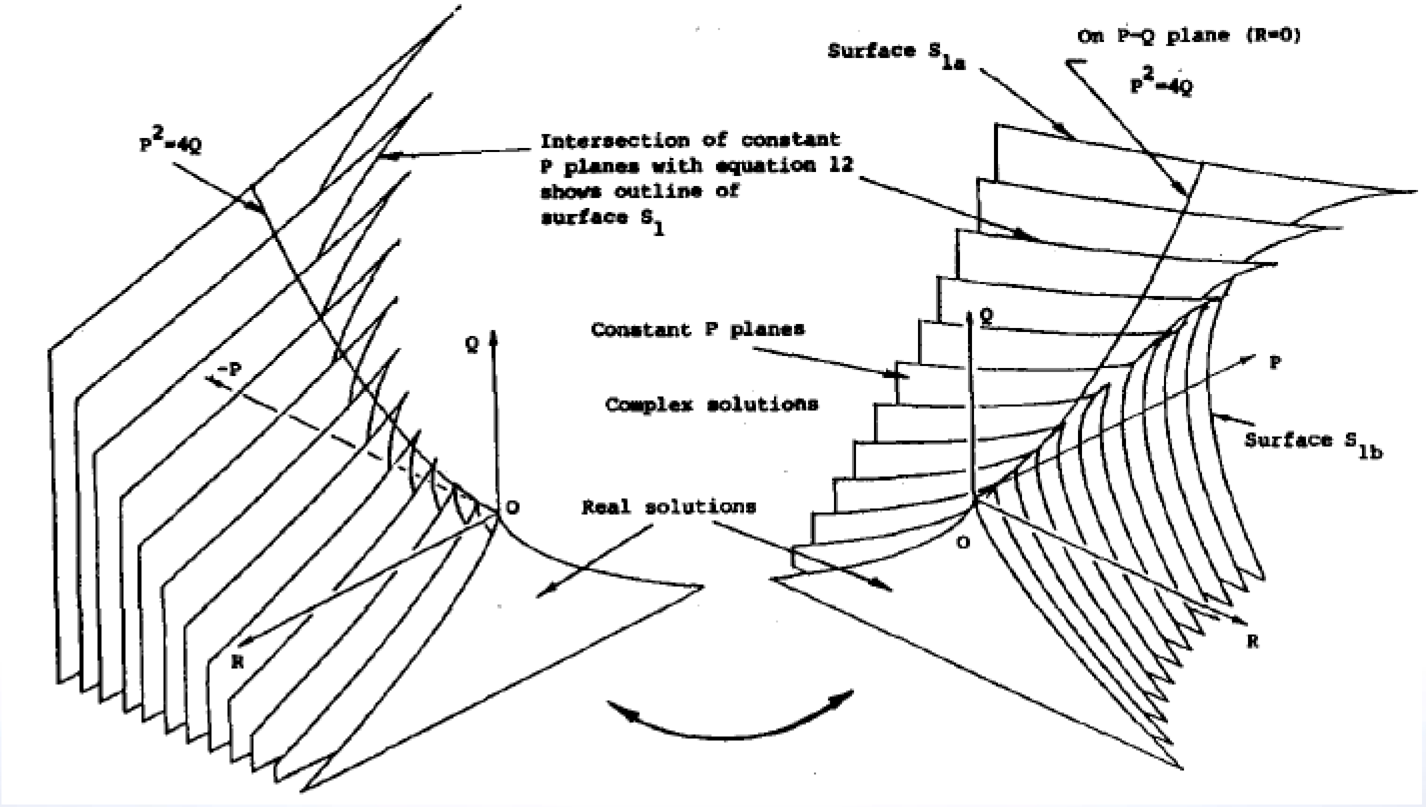
$$\det[\mathbf{A} - \lambda \mathbf{I}] = \lambda^3 + s_1\lambda^2 + s_2\lambda + s_3 = 0,$$

- Define <u>**invariants of velocity gradient**</u>
  - (Turbulence 20+ years)

$$\begin{aligned}
s_1 &= -\mathrm{tr}[\mathbf{A}] = -\theta_{\mathrm{ii}}, \\
s_2 &= \frac{1}{2}\left(s_1^2 - \mathrm{tr}[\mathbf{A}^2]\right) = \frac{1}{2}(s_1^2 - \theta_{ij}\theta_{ji} - \omega_{ij}\omega_{ji}) \\
s_3 &= -\det[\mathbf{A}] = \frac{1}{3}\left(-s_1^3 + 3s_1s_2 - \mathrm{tr}[\mathbf{A}^3]\right) \\
&= \frac{1}{3}\left(-s_1^3 + 3s_1s_2 - \theta_{ij}\theta_{jk}\theta_{ki} - 3\omega_{ij}\omega_{jk}\theta_{kl}\right),
\end{aligned}$$

Invariants of velocity gradient

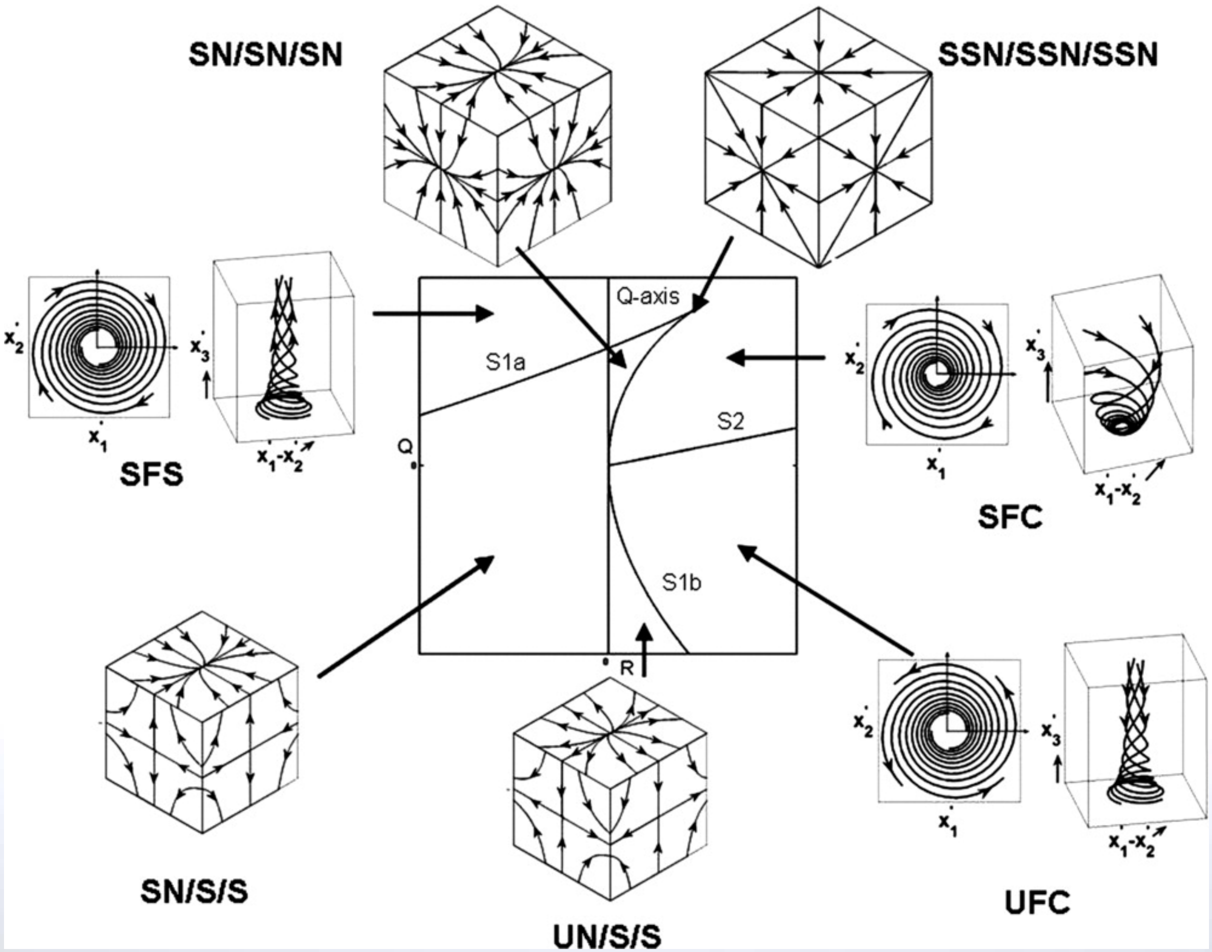
- Invariants-space



- Turbulence, Chong et. al, 1990

Kinematics of Cosmic Web

- **Mapping:** **eigenvalue space → Invariant space**



Kinematics of Cosmic Web

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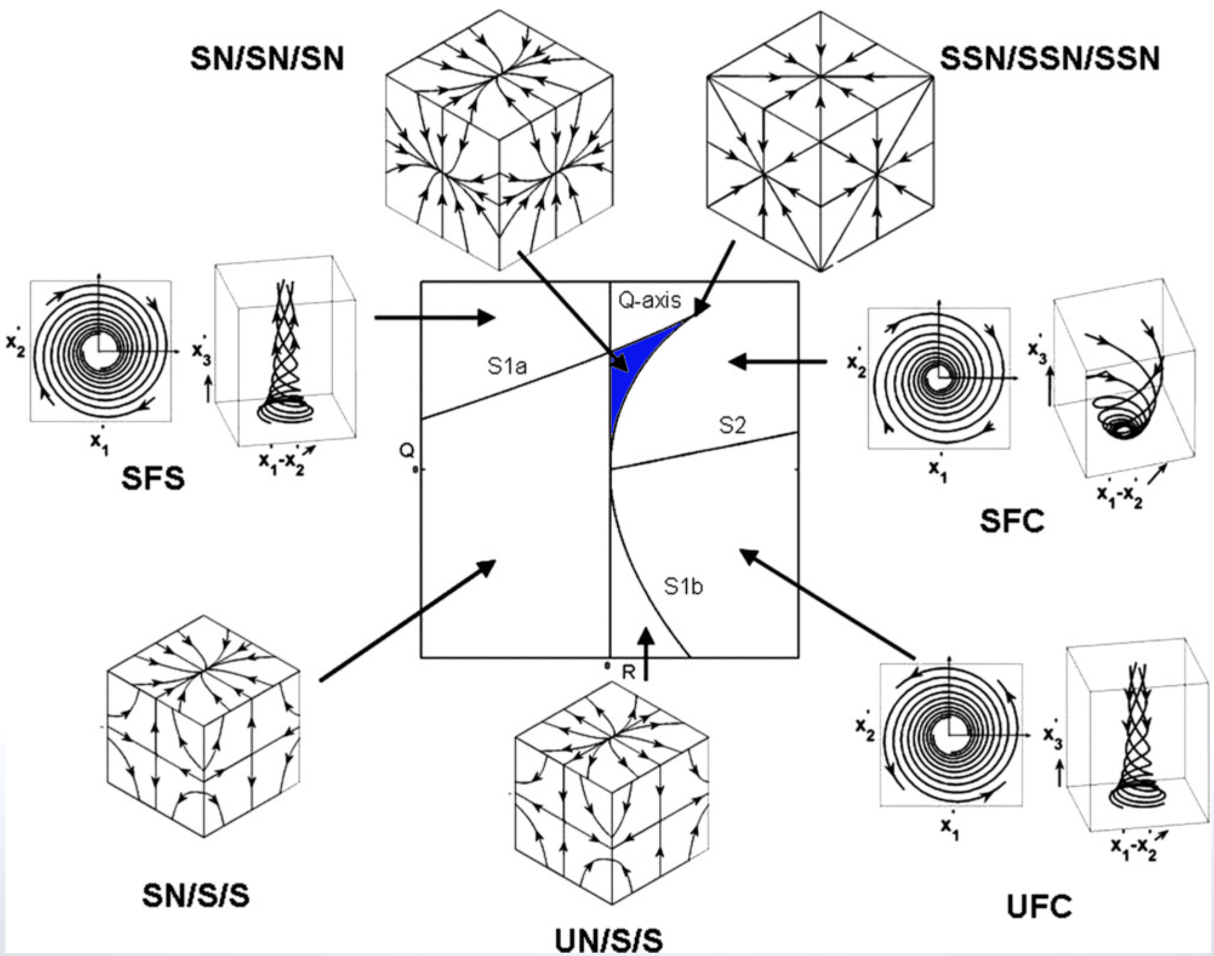
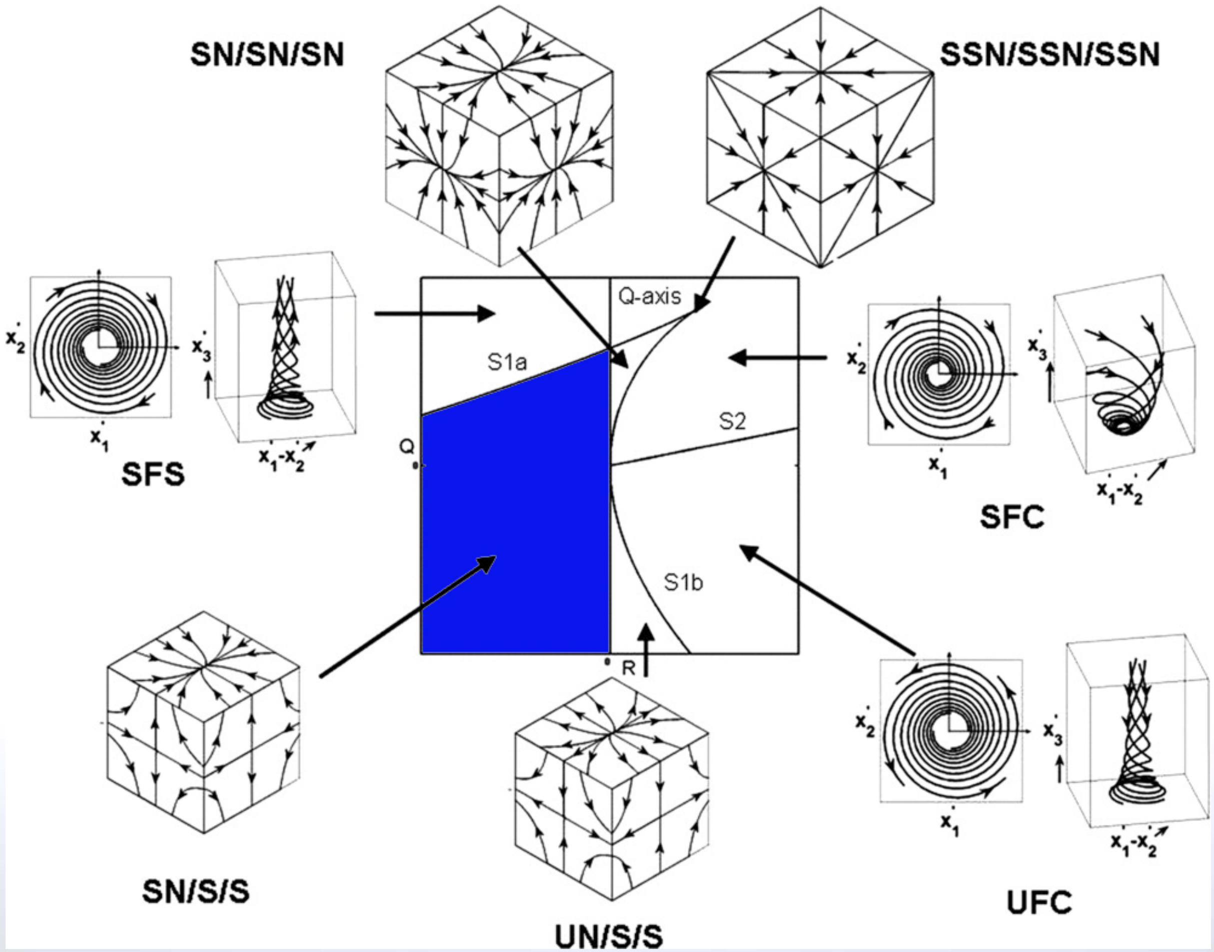


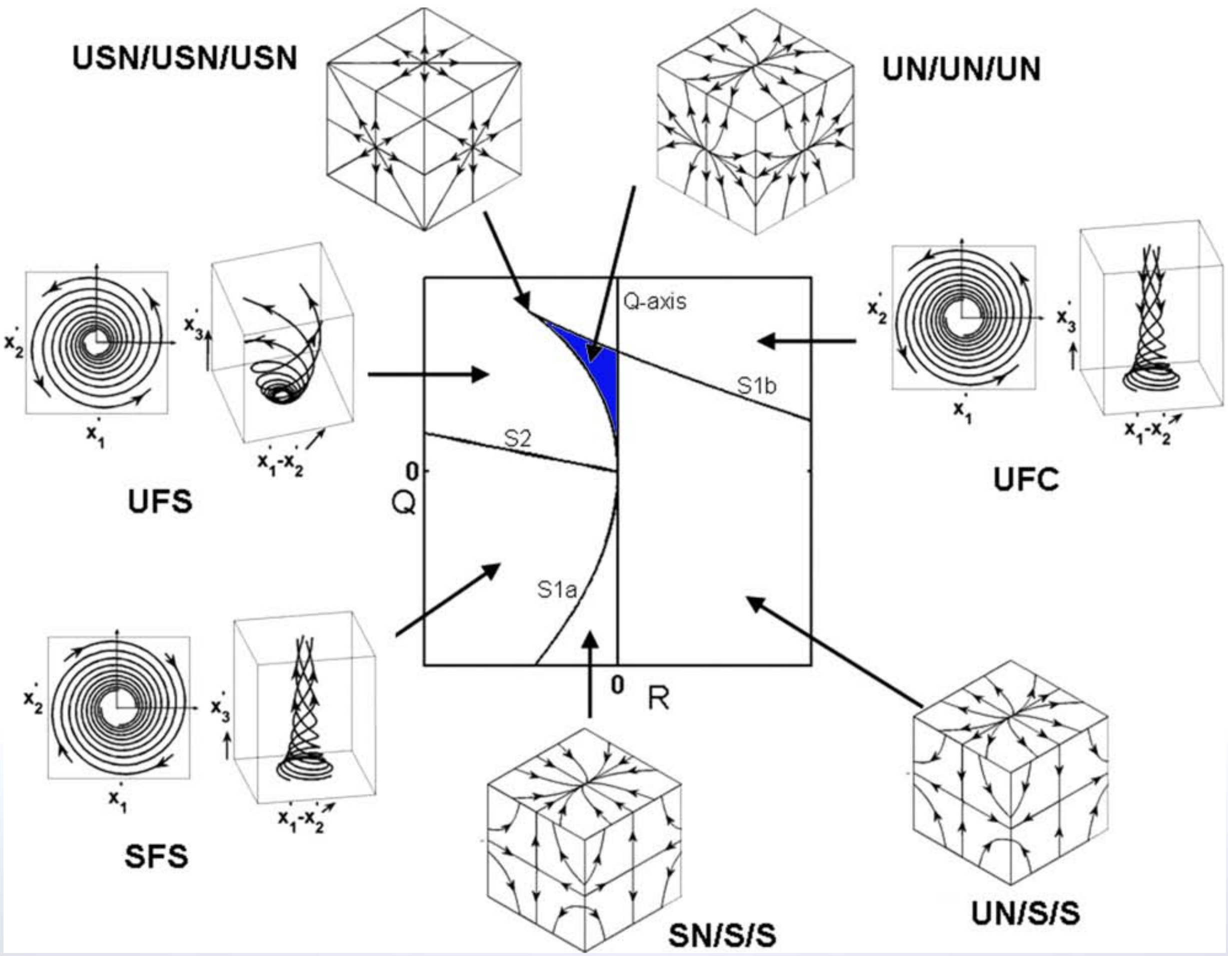
Kinematics of Cosmic Web

- **Mapping:** **eigenvalue space → Invariant space**



Kinematics of Cosmic Web

- **Mapping:** eigenvalue space → Invariant space



Evolution of Cosmic Web
Dynamics
--- potential flow

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Dynamics of Cosmic-Web

- **Dynamical evolution:**
  - Euler-Poisson system

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = -\nabla\Phi(\mathbf{x},\tau)$$

$$\frac{dA_{ij}}{d\tau} + \mathcal{H}(\tau)A_{ij} + A_{ik}A_{kj} = \varpi_{ij}$$, where $\varpi_{ij}(\mathbf{x},\tau) = -\nabla_i\nabla_j\Phi(\mathbf{x},\tau)$

- **Invariants evolution**

$$\frac{d}{d\tau}s_1 + \mathcal{H}(\tau)s_1 - s_1^2 + 2s_2 = -\varpi$$

$$\frac{d}{d\tau}s_2 + 2\mathcal{H}(\tau)s_2 - s_1 s_2 + 3s_3 = -s_1\varpi - \varpi_A$$

$$\frac{d}{d\tau}s_3 + 3\mathcal{H}(\tau)s_3 - s_1 s_3 = -s_2\varpi - s_1\varpi_A - \varpi_{A^2}$$

  - where

$$\varpi = \varpi_{ii} = -\nabla^2\Phi \qquad \varpi_{A^2} = \varpi_{ik}A_{kj}A_{ji}$$

Dynamics of Cosmic-Web

- **Coupled equations**
  - **Newton potential**

$$\varpi = \varpi_{ii} = -\nabla^2 \Phi$$

$$\frac{d}{d\tau}\varpi + \left[\frac{d}{d\tau}\ln\mathcal{N}(\tau) - s_1\right]\varpi - \frac{1}{\mathcal{N}}(\tau)s_1 = 0,$$

  - **Tidal field**

$$\frac{d}{d\tau}\varepsilon_{ij} + \mathcal{H}(\tau)\varepsilon_{ij} + \nabla_k \epsilon^{kl}{}_{(i}H_{j)l} + \theta\varepsilon_{ij} + \delta_{ij}\sigma^{kl}\varepsilon_{kl}$$
$$-3\sigma^k{}_{(i}\varepsilon_{j)k} - \omega^k{}_{(i}\varepsilon_{j)k} = -\varpi\sigma_{ij}.$$

where H_{ij} is Newtonian limit of magnetic part of Weyl tensor

Dynamics of Cosmic-Web

- **Zel'dovich approximation**

$$\mathbf{x}(\tau) = \mathbf{q} + \mathbf{\Psi}(\mathbf{q}, \tau) \approx \mathbf{q} + D(\tau)\mathbf{\Psi}_1(\mathbf{q}),$$

  - Velocity evolution

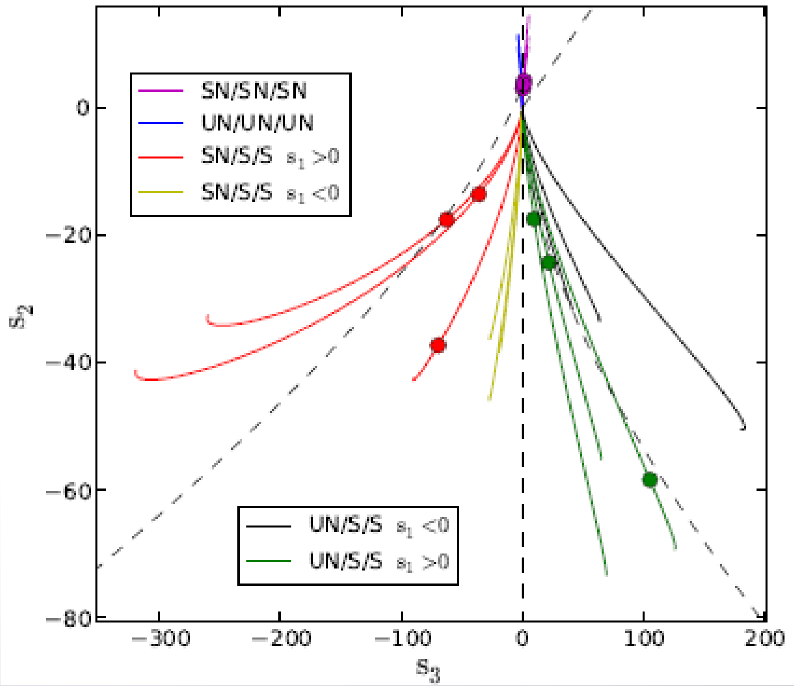
$$\tilde{\mathbf{u}}' = \frac{d\tilde{\mathbf{u}}}{dD} = \left(\frac{\partial}{\partial D} + \tilde{\mathbf{u}} \cdot \nabla\right) \tilde{\mathbf{u}} = 0,$$

  - Velocity gradient evolution

$$\frac{d\tilde{A}_{ij}}{dD} + \tilde{A}_{ik}\tilde{A}_{kj} = 0$$

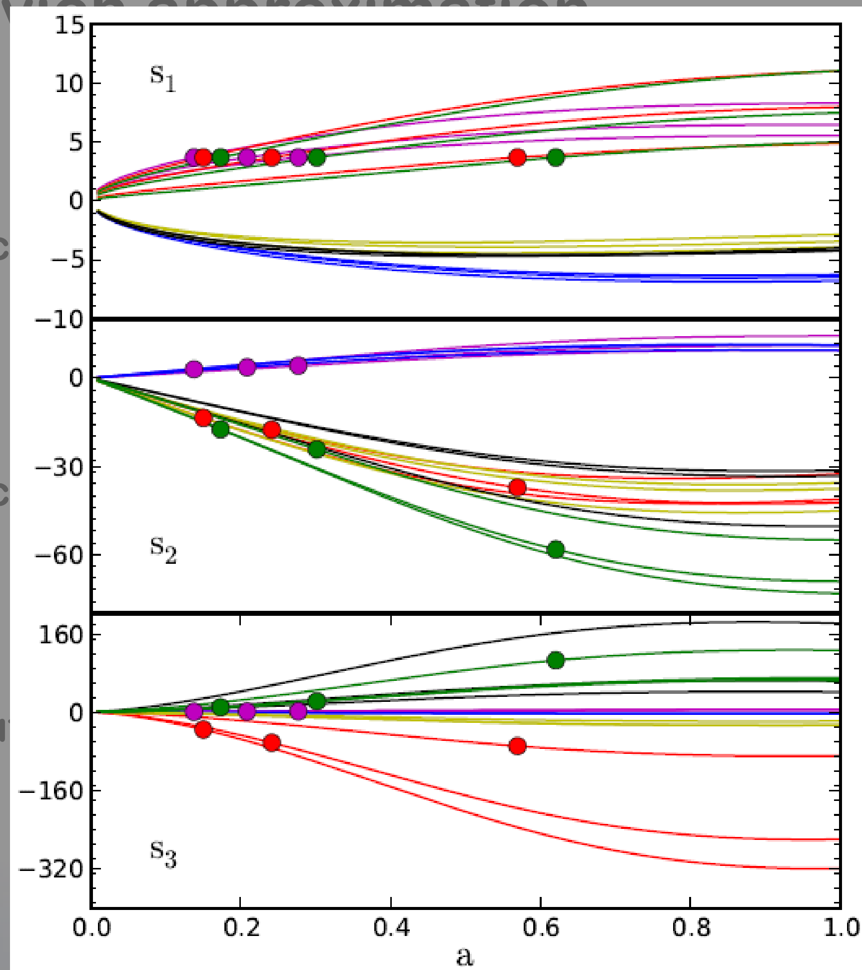
  - Evolution of invariants

$$\begin{aligned} \tilde{s}_1' - \tilde{s}_1^2 + 2\tilde{s}_2 &= 0, \\ \tilde{s}_2' - \tilde{s}_1\tilde{s}_2 + 3\tilde{s}_3 &= 0, \\ \tilde{s}_3' - \tilde{s}_1\tilde{s}_3 &= 0. \end{aligned}$$

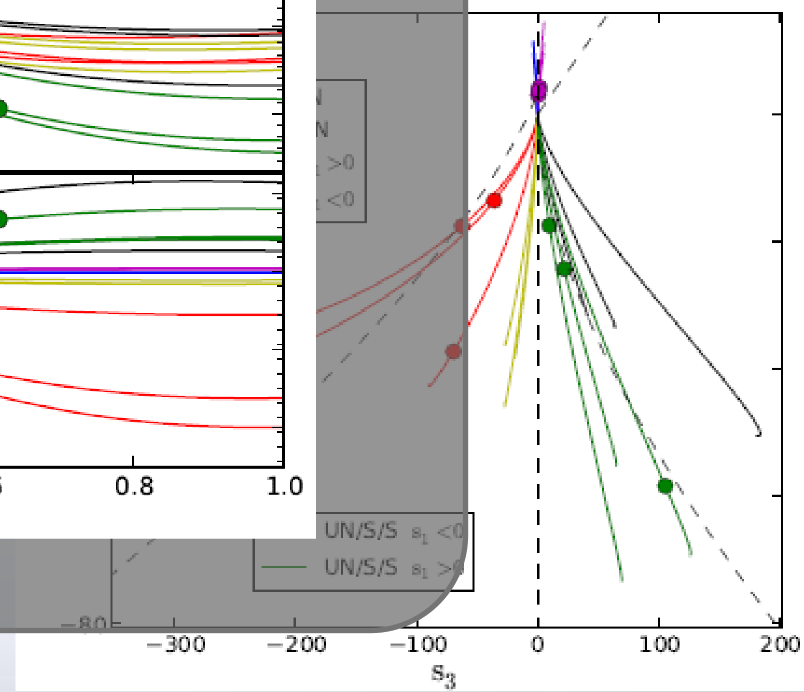


Dynamics of Cosmic-Web

- Zel'dovich approximation
  - Veloc
  - Veloc
  - Evolu



$$s_3' - s_1 s_3 = 0.$$



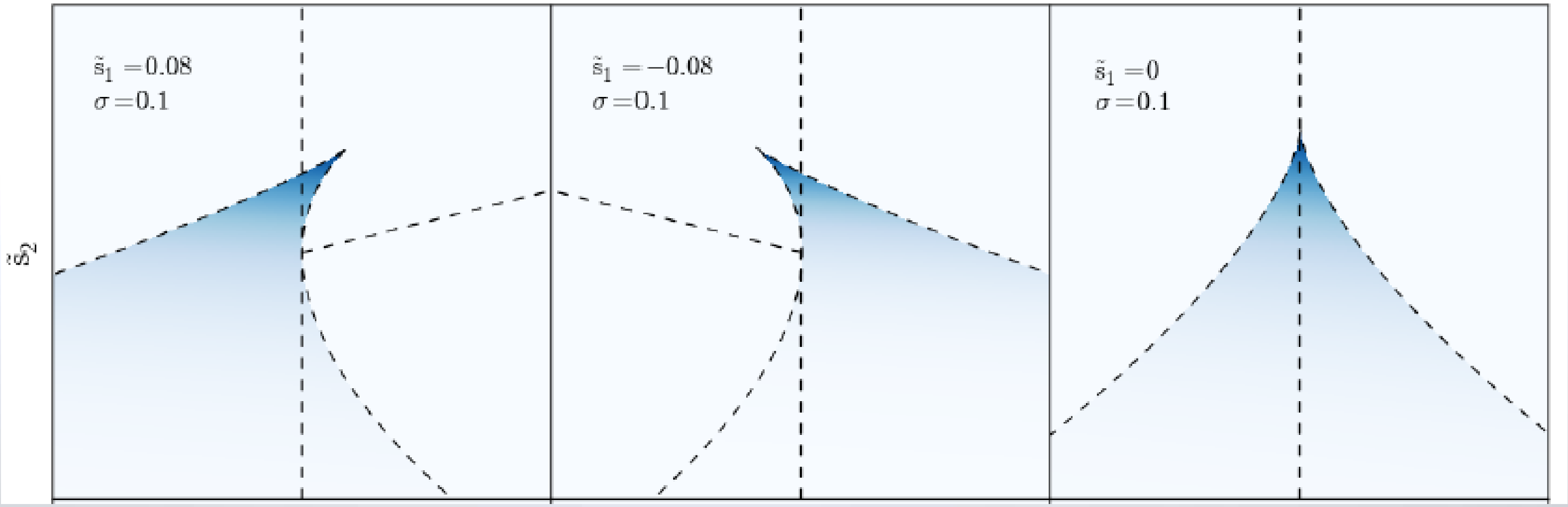
Probability Distribution

- **Initial condition**
  - **Gaussian distribution**

$$\mathcal{P}\left(\tilde{A}_{ij}^{(\xi)}\right) = \frac{1}{(2\pi)^3\sqrt{|\mathbf{C}|}} e^{-(\mathbf{X}^T\cdot\mathbf{C}^{-1}\cdot\mathbf{X})/2},$$

$$\mathcal{P}\left(\tilde{s}_i^{(\xi)}\right)\prod_i d\tilde{s}_i^{(\xi)} d\Omega_{S_3} = \mathcal{P}\left(\tilde{A}_{ij}^{(\xi)}\right)\prod_{i\leq j} d\tilde{A}_{ij}^{(\xi)},$$

$$\mathcal{P}\left(\tilde{s}_i^{(\xi)}\right) = \frac{15^3}{8\pi\sqrt{5}\,\sigma^6} e^{-\frac{3}{2\sigma^2}\left[2\left(\tilde{s}_1^{(\xi)}\right)^2 - 5\tilde{s}_2^{(\xi)}\right]}\, Real\left(\tilde{s}_i^{(\xi)}\right)$$

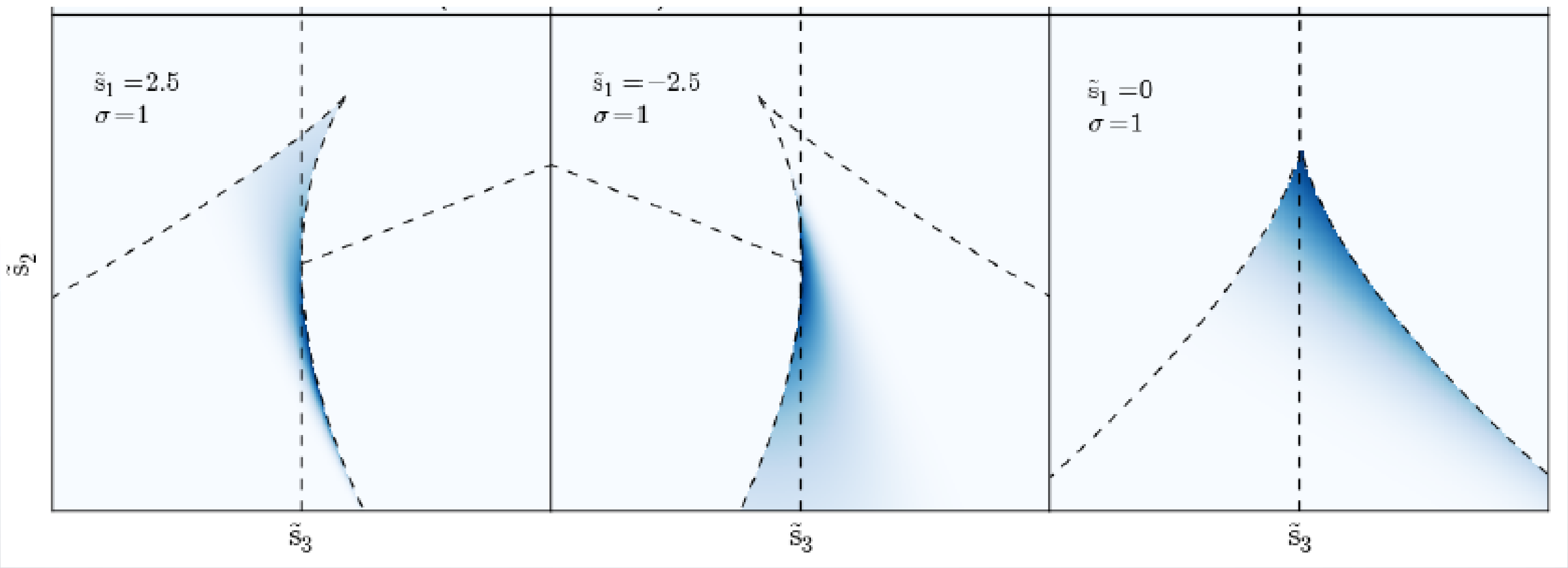


Probability Distribution

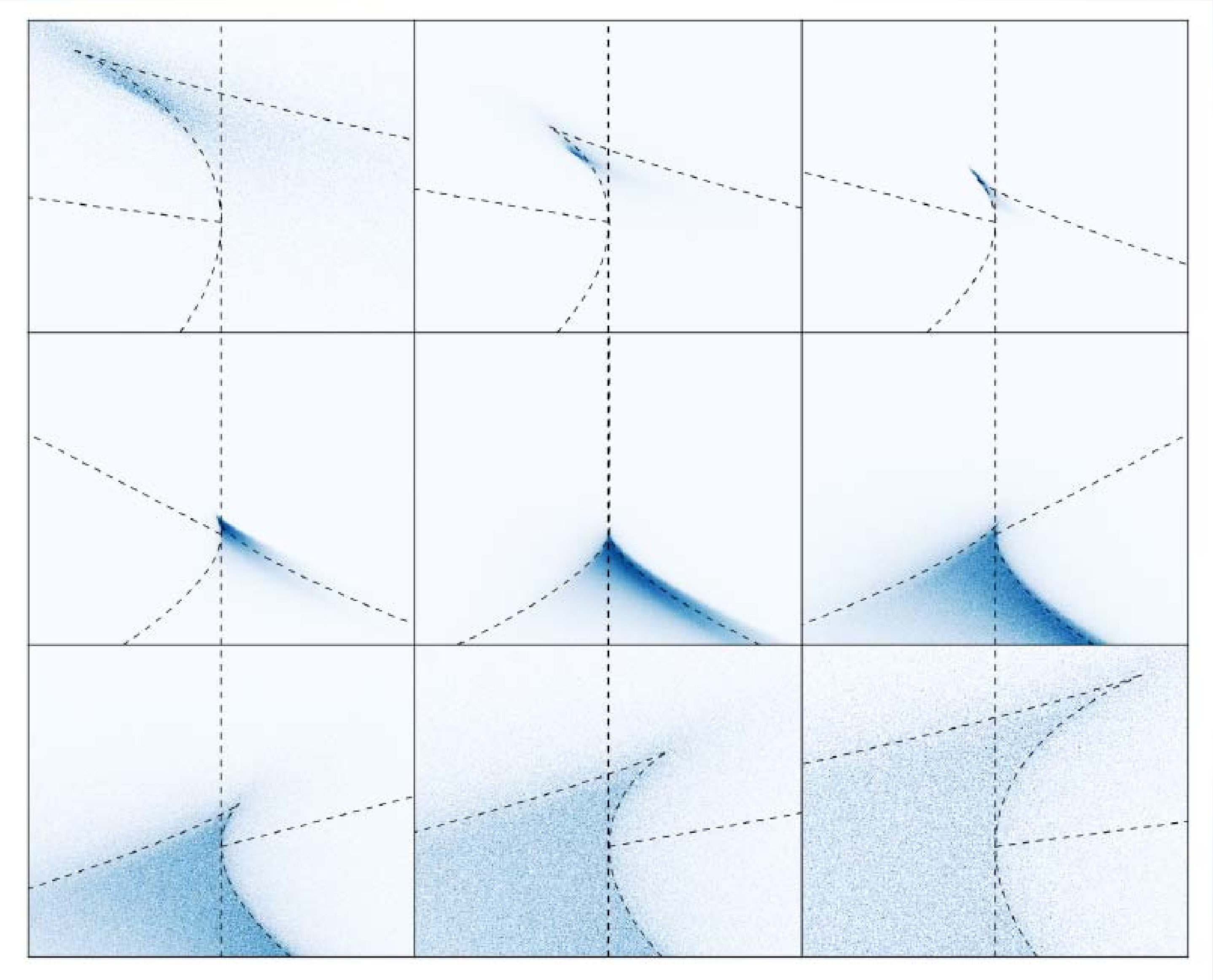
- **Zel'dovich approximation**

$$\tilde{A}_{ij} = \frac{A_{ij}(\tau)}{D^{(v)}(\tau)} = \frac{\partial \Psi_i}{\partial x_j} = \frac{\partial \Psi_i}{\partial q_k}(J^{-1})_{kj},$$

$$\mathcal{P}(\tilde{s}_i) = \mathcal{P}(\lambda_i)\left|\frac{\partial \lambda_i}{\partial \tilde{\eta}_k}\frac{\partial \tilde{\eta}_k}{\partial s_j}\right| = \frac{15^3}{8\pi\sqrt{5}\,\sigma^6\,T^4} Real(\tilde{s}_i)$$
$$\times \exp\left[\frac{-3(\tilde{s}_1 + 2\tilde{s}_2 + 3\tilde{s}_3)^2}{\sigma^2 T^2} + \frac{15(3\tilde{s}_3 + \tilde{s}_2)}{2\sigma^2 T}\right],$$



Beyond Zel'dovich Approximation



Evolution of Cosmic Web Dynamics

--- vortical flow

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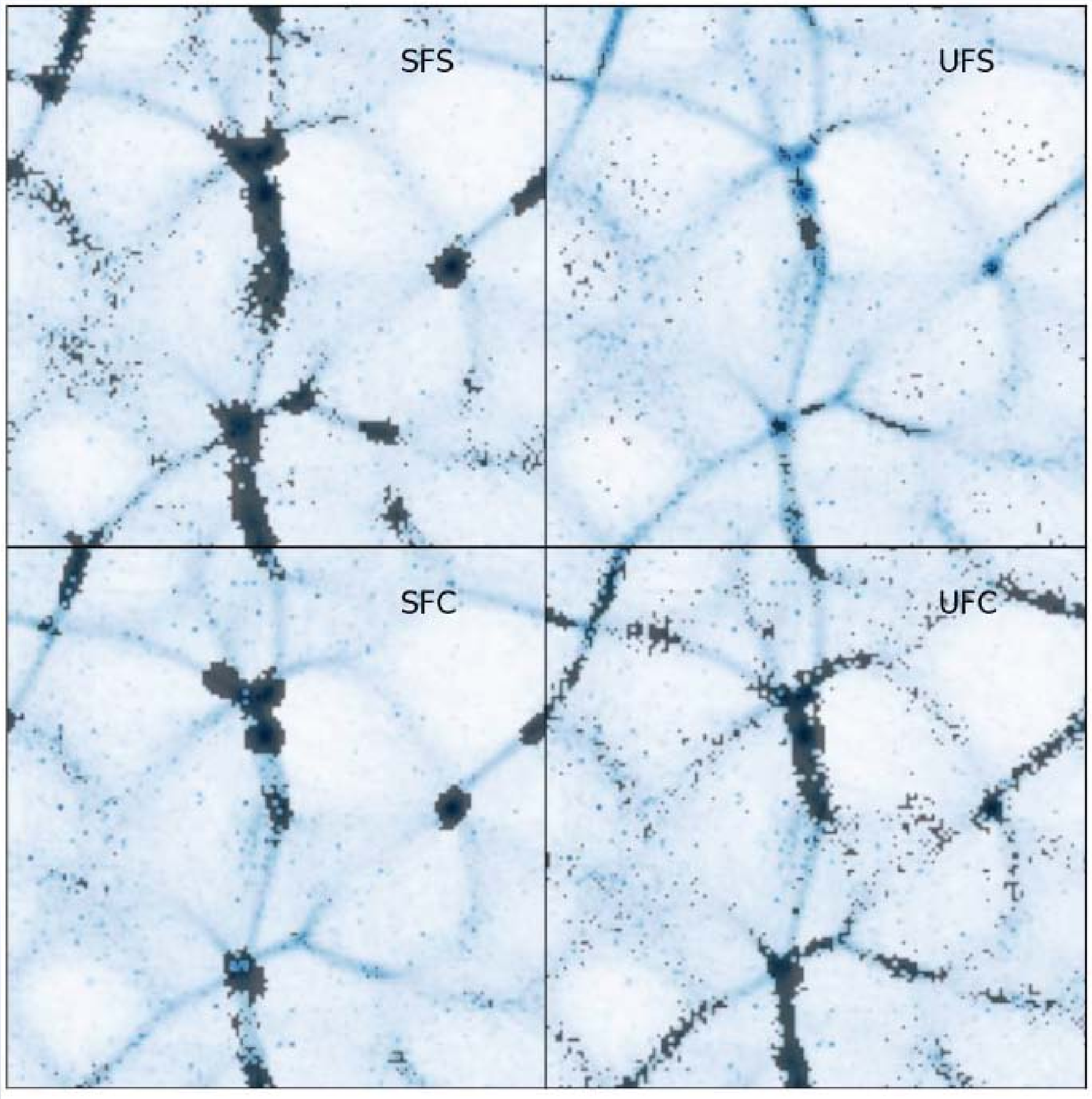
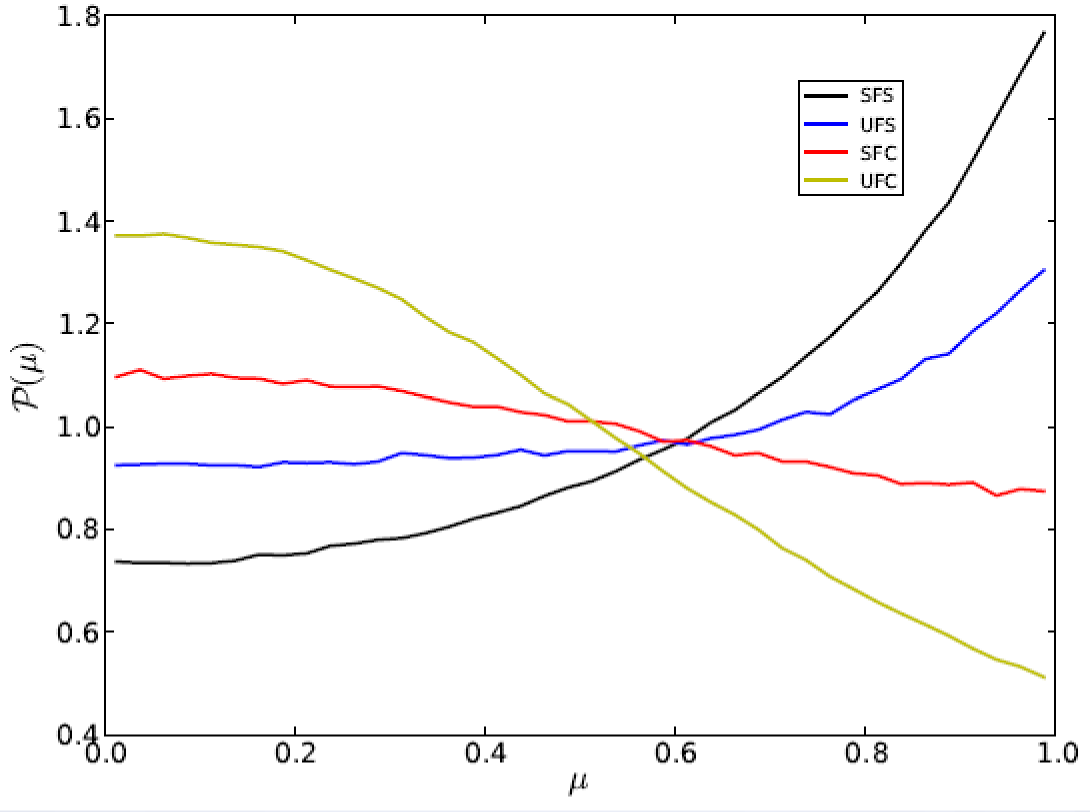
Vortical flow – *spatial distribution*



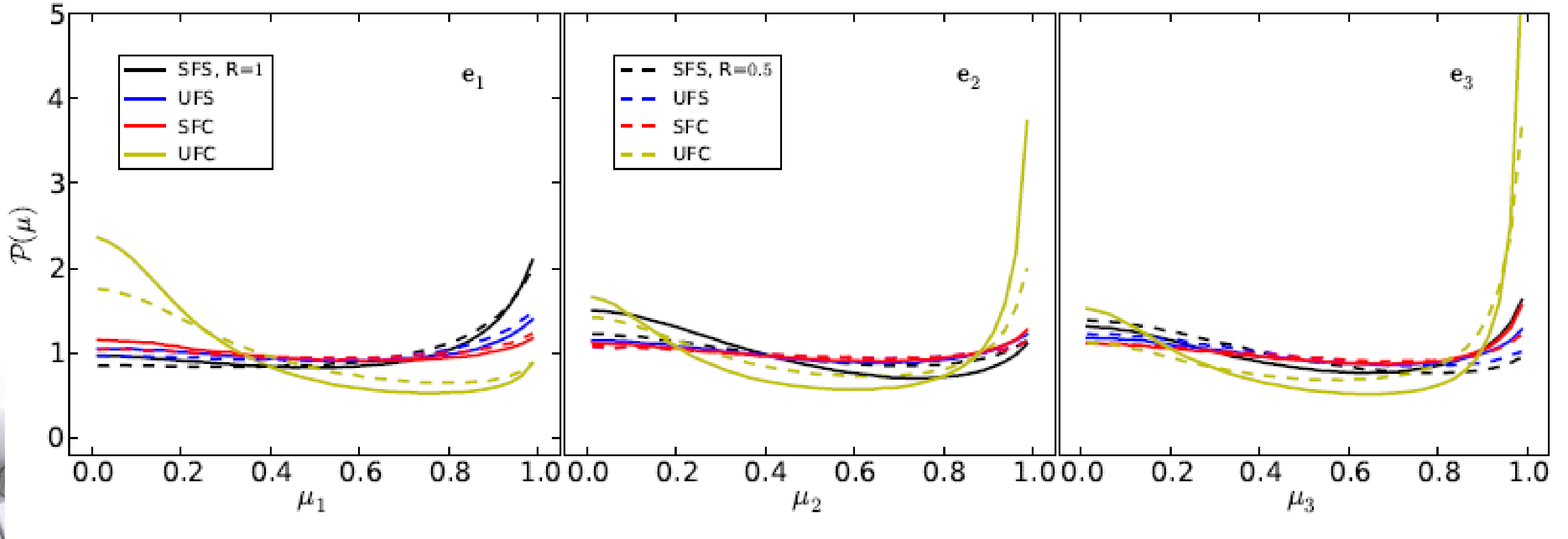
Vortical flow – *vorticity orientation*

- **Flow-type dependence**
  - angle between vorticity and velocity



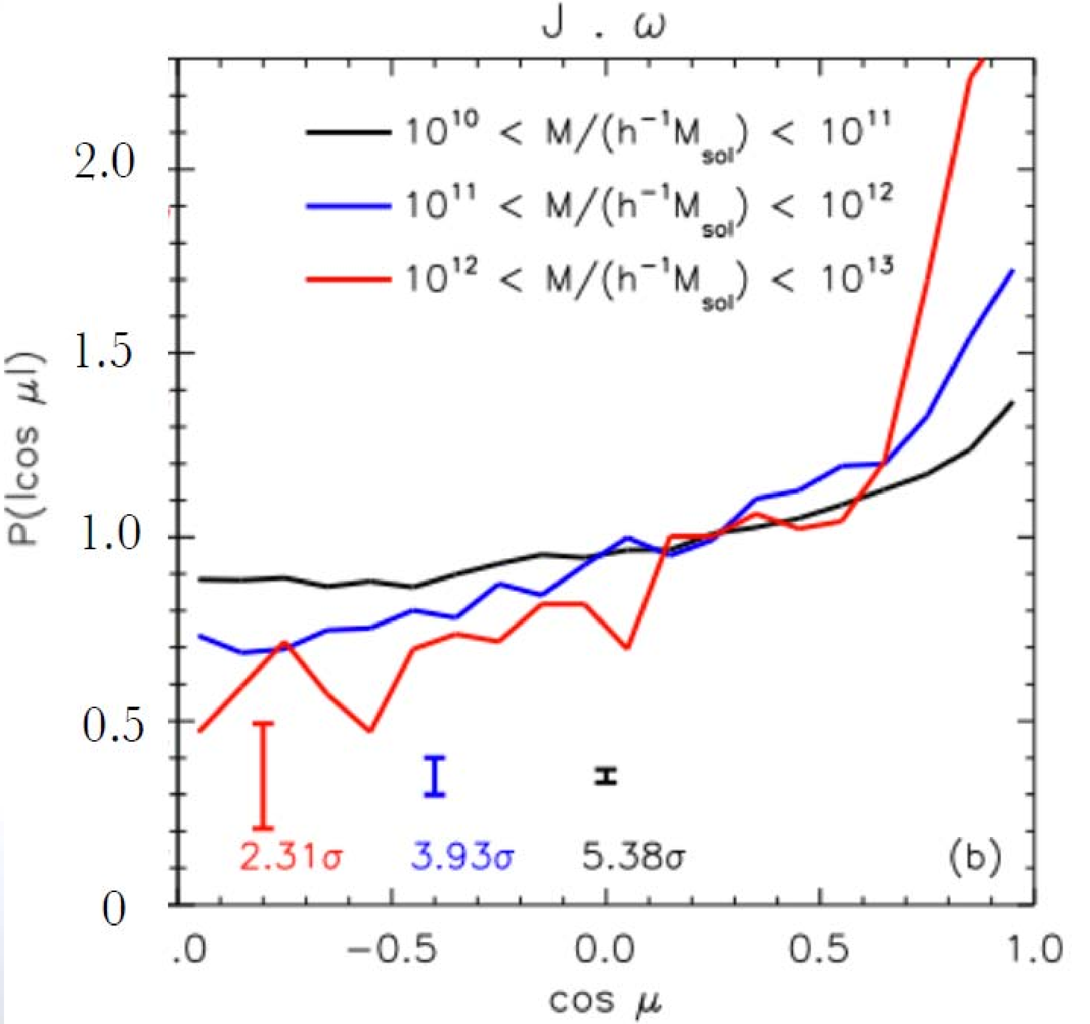
Vortical flow – *vorticity orientation*

- **Flow-type dependence**
  - angle between vorticity and structure direction



Vortical flow – *vorticity orientation*

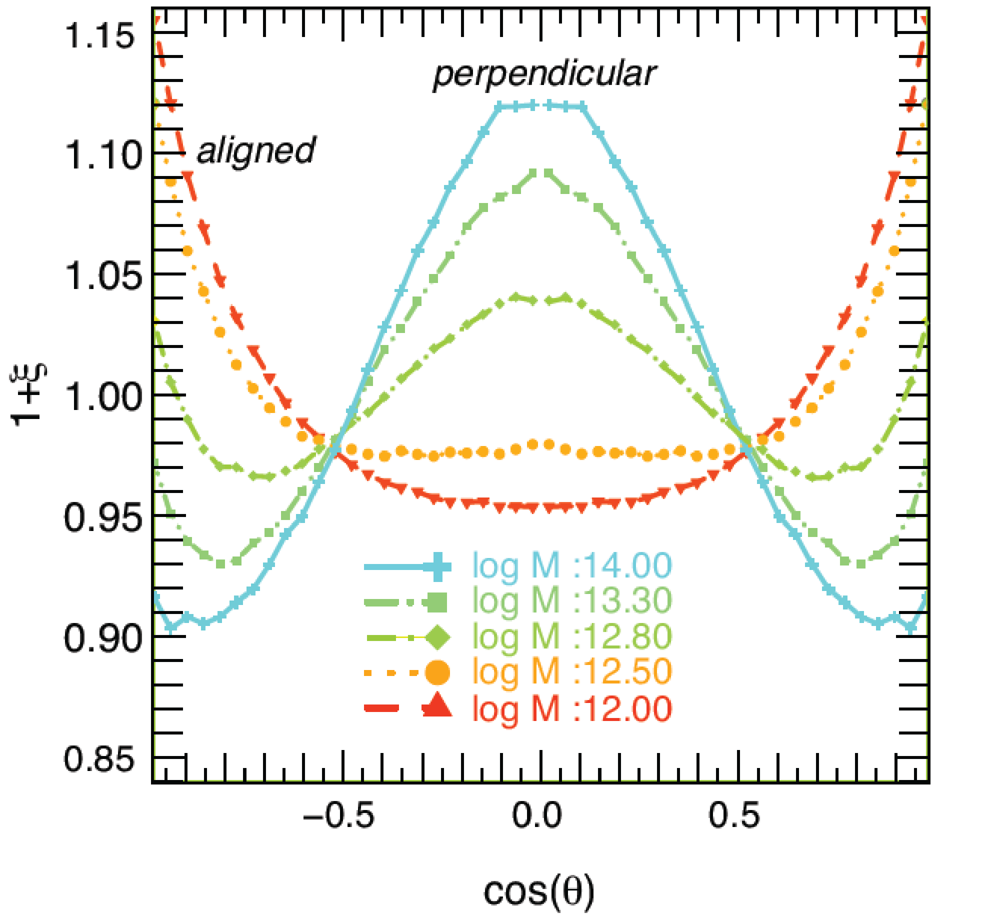
- **Vorticity & halo spin**



- **Libeskind et al. (2013)**

Vortical flow – *vorticity orientation*

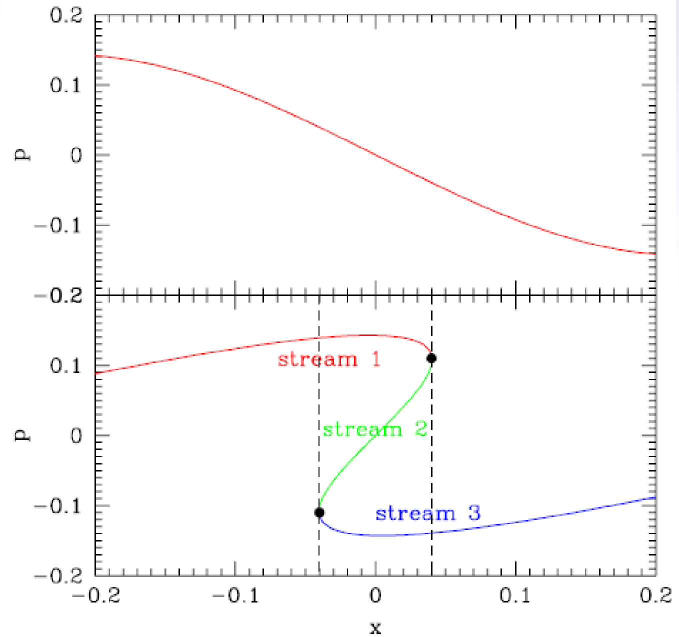
- halo spin & cosmic web



- Pichon et al.

Shell-crossing and Vorticity

- **Multi-streaming**
  - Vlasov-Poisson equation



$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \frac{\partial f}{\partial \mathbf{x}} - am\nabla\Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0,$$

$$\rho = \int d^3\mathbf{p}\ f(\mathbf{x}, \mathbf{p}, \tau), \quad u_i = \int d^3\mathbf{p}\ f(\mathbf{x}, \mathbf{p}, \tau)\ \frac{p_i}{am},$$
$$\varsigma_{ij} = \int d^3\mathbf{p}\ f(\mathbf{x}, \mathbf{p}, \tau)\ \left(\frac{p_i}{am} - u_i\right)\left(\frac{p_j}{am} - u_j\right), \quad (60)$$

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u}(\mathbf{x}, \tau) + \mathbf{u}(\mathbf{x}, \tau) \cdot \nabla \mathbf{u}(\mathbf{x}, \tau) = -\nabla\Phi(\mathbf{x}, \tau) - \frac{1}{\rho}\nabla_j(\rho\varsigma_{ij}),$$

Shell-crossing and Vorticity

$$\frac{dA_{ij}}{d\tau} + \mathcal{H}(\tau)A_{ij} + A_{ik}A_{kj} = \varpi_{ij} + \zeta_{ij},$$

$$\zeta_{ij} = -\nabla_j \left[\frac{1}{\rho}\nabla_k \left(\rho \varsigma_{ik}\right)\right].$$

- **Invariants with velocity dispersion**

$$\begin{aligned}\frac{d}{d\tau}s_1 + \mathcal{H}(\tau)s_1 - s_1^2 + 2s_2 &= -\varpi - \zeta \\ \frac{d}{d\tau}s_2 + 2\mathcal{H}(\tau)s_2 - s_1 s_2 + 3s_3 &= -s_1(\varpi + \zeta) - \varpi_A - \zeta_A \\ \frac{d}{d\tau}s_3 + 3\mathcal{H}(\tau)s_3 - s_1 s_3 &= -s_2(\varpi + \zeta) - s_1(\varpi_A \\ &\quad + \zeta_A) - \varpi_{A^2} - \zeta_{A^2} \quad (64)\end{aligned}$$

Where

$$\begin{aligned}\frac{\partial \varsigma_{ij}}{\partial \tau} + 2\mathcal{H}(\tau)\varsigma_{ij} + (\mathbf{u}\cdot\nabla)\varsigma_{ij} + \varsigma_{jk}\nabla_k u_i + \varsigma_{ik}\nabla_k u_j \\ = -\frac{1}{\rho}\nabla_k(\rho\pi_{ijk}),\end{aligned}$$

Shell-crossing and Vorticity

$$\frac{dA_{ij}}{d\tau} + \mathcal{H}(\tau)A_{ij} + A_{ik}A_{kj} = \varpi_{ij} + \zeta_{ij},$$

$$\zeta_{ij} = -\nabla_j \left[\frac{1}{\rho}\nabla_k \left(\rho\varsigma_{ik}\right)\right].$$

- **Invariants with velocity dispersion**

Highly Non-linear

$$\frac{d}{d\tau}s_1 + \mathcal{H}(\tau)s_1 - s_1^2 + 2s_2 = -\varpi - \zeta$$

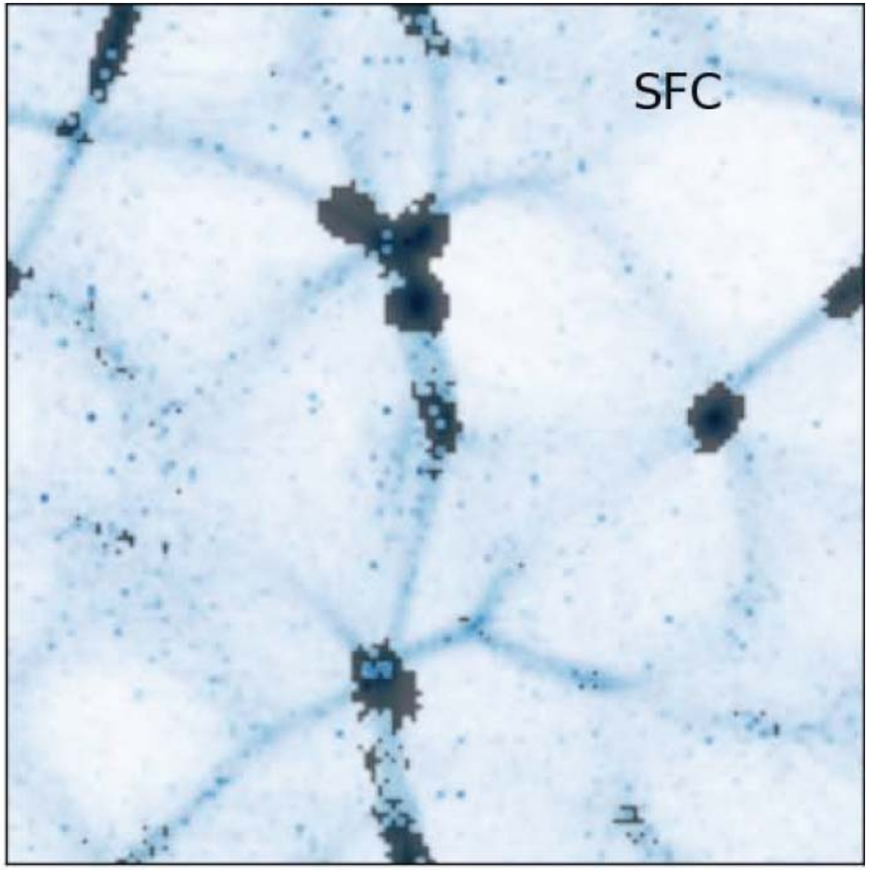
$$\frac{d}{d\tau}s_2 + 2\mathcal{H}(\tau)s_2 - s_1 s_2 + 3s_3 = -s_1(\varpi + \zeta) - \varpi_A - \zeta_A$$

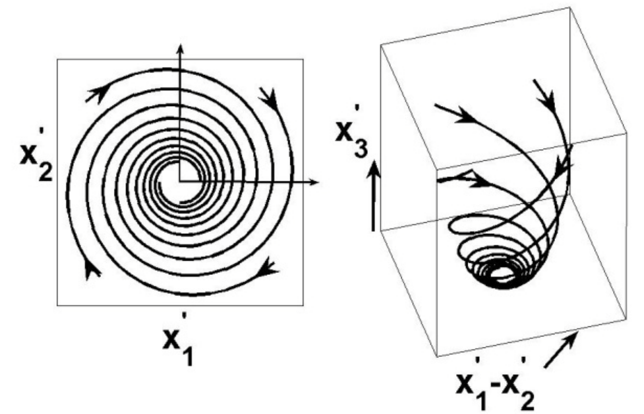
$$\frac{d}{d\tau}s_3 + 3\mathcal{H}(\tau)s_3 - s_1 s_3 = -s_2(\varpi + \zeta) - s_1(\varpi_A + \zeta_A) - \varpi_{A^2} - \zeta_{A^2} \quad (64)$$

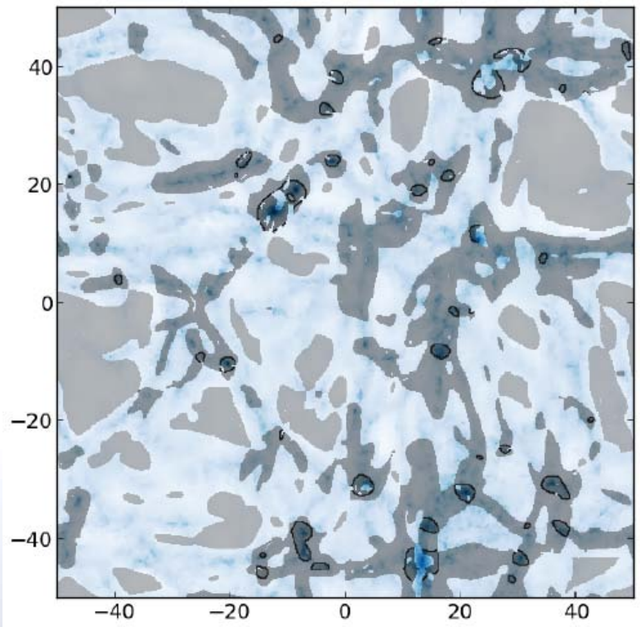
Where

$$\frac{\partial\varsigma_{ij}}{\partial\tau} + 2\mathcal{H}(\tau)\varsigma_{ij} + (\mathbf{u}\cdot\nabla)\varsigma_{ij} + \varsigma_{jk}\nabla_k u_i + \varsigma_{ik}\nabla_k u_j = -\frac{1}{\rho}\nabla_k(\rho\pi_{ijk}),$$

Emergence of Vorticity

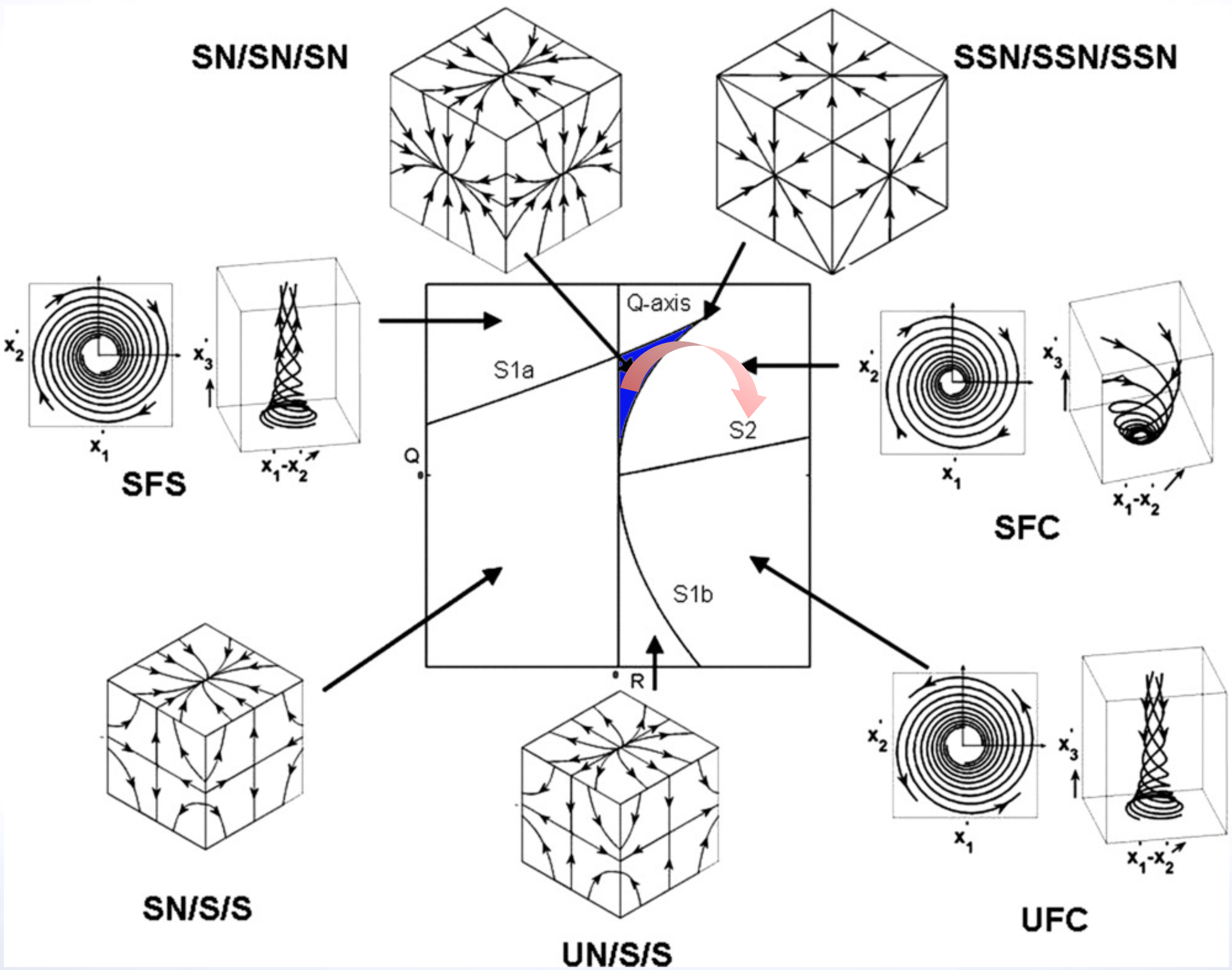






Potential – Vortical transition

$s_1 > 0,$



Potential – Vortical transition

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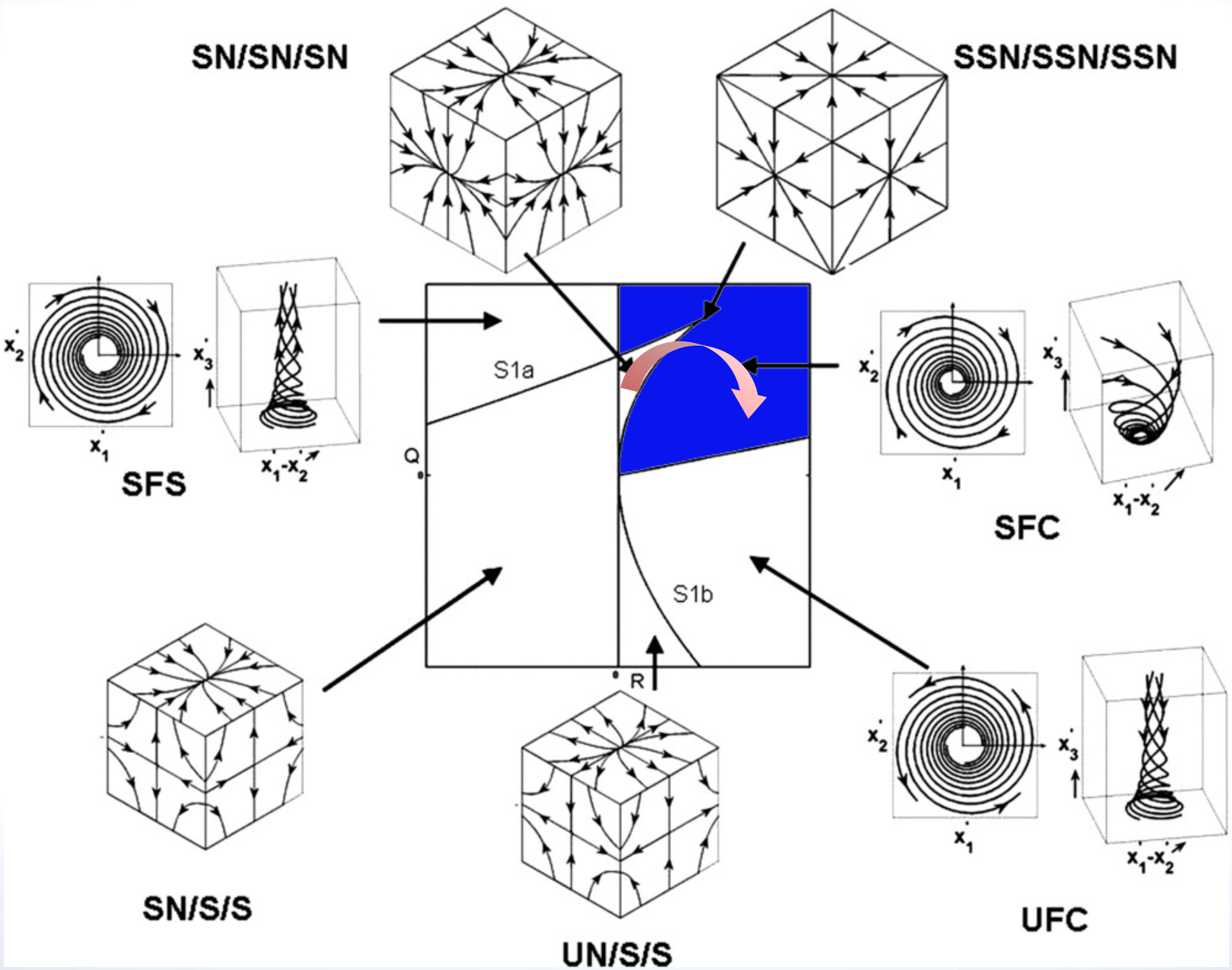

Potential – Vortical transition

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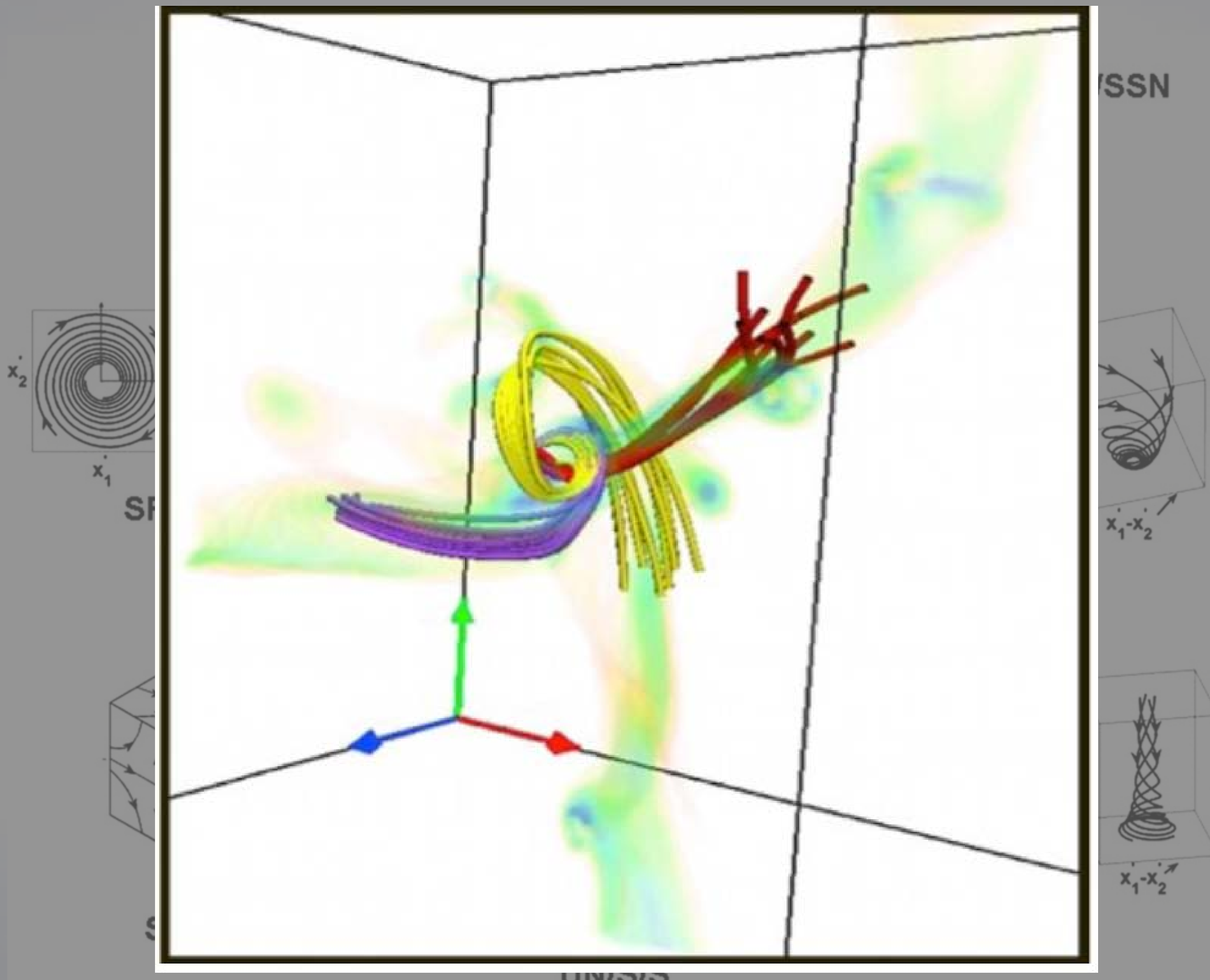
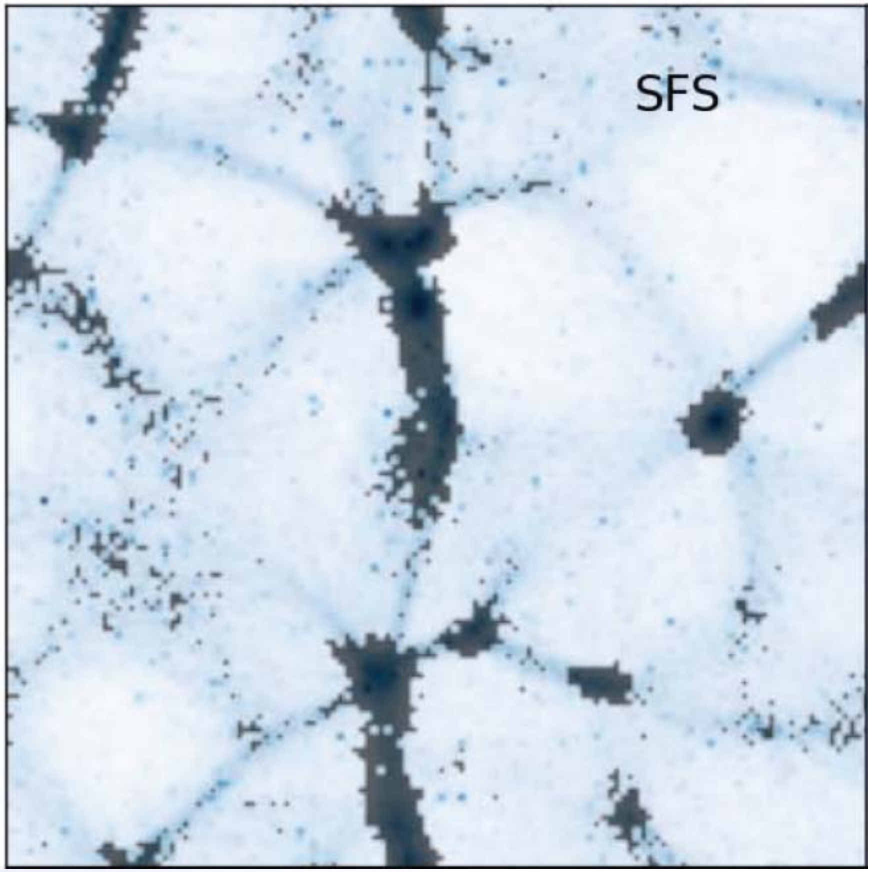
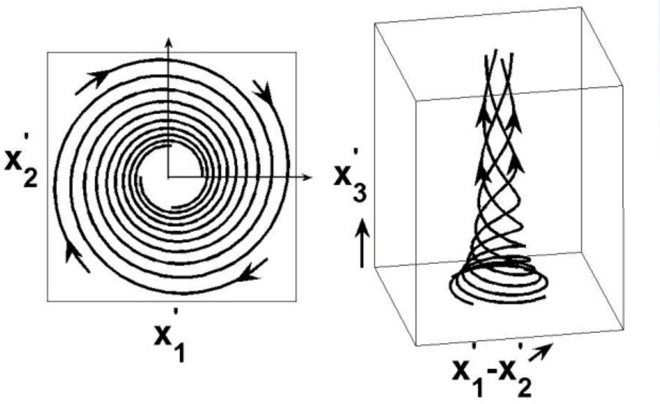
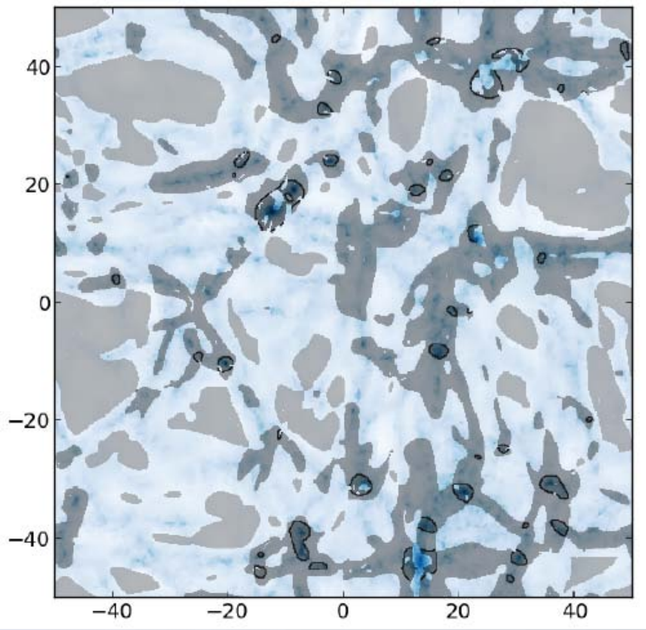


- Pichon, C. et al

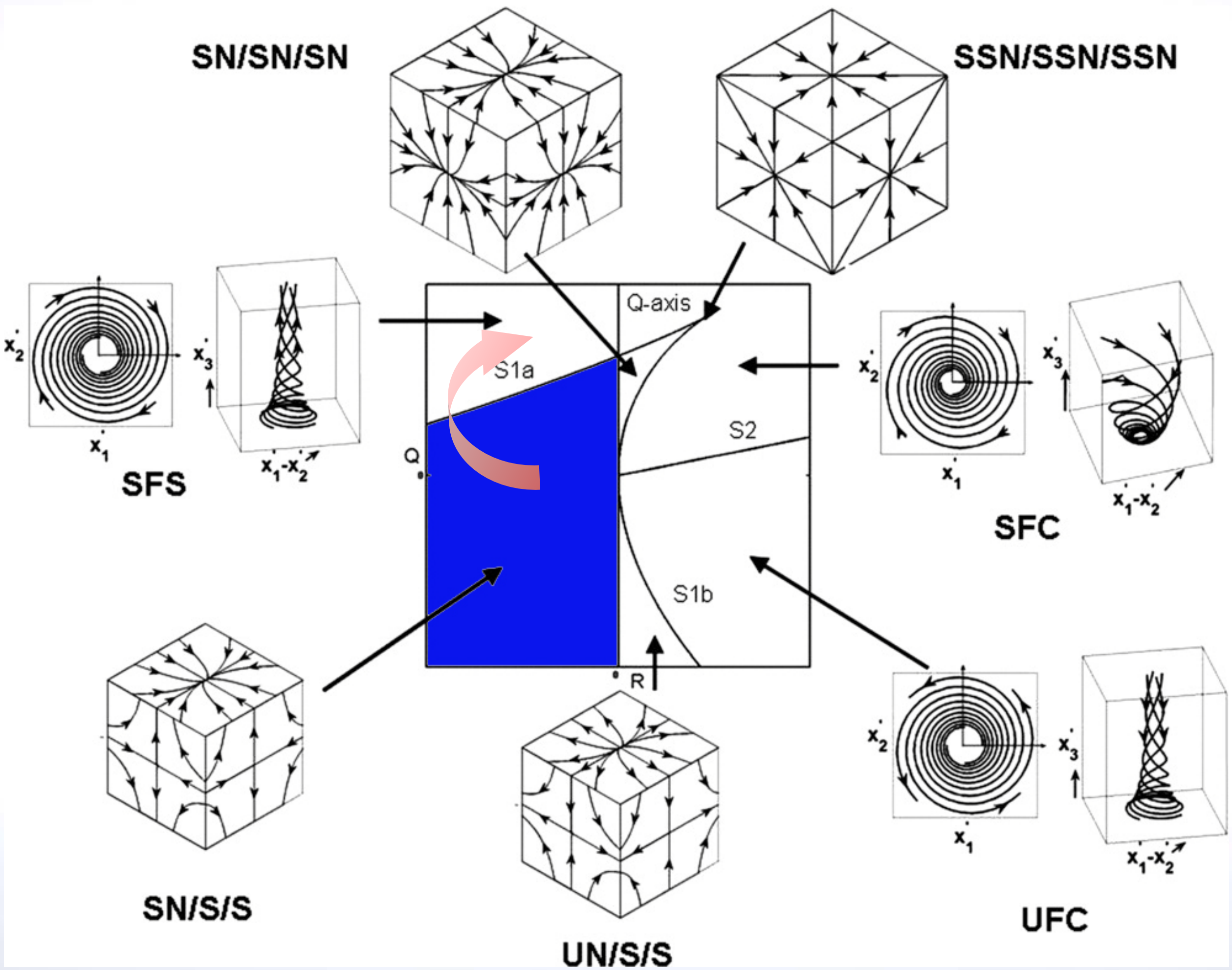
Emergence of Vorticity



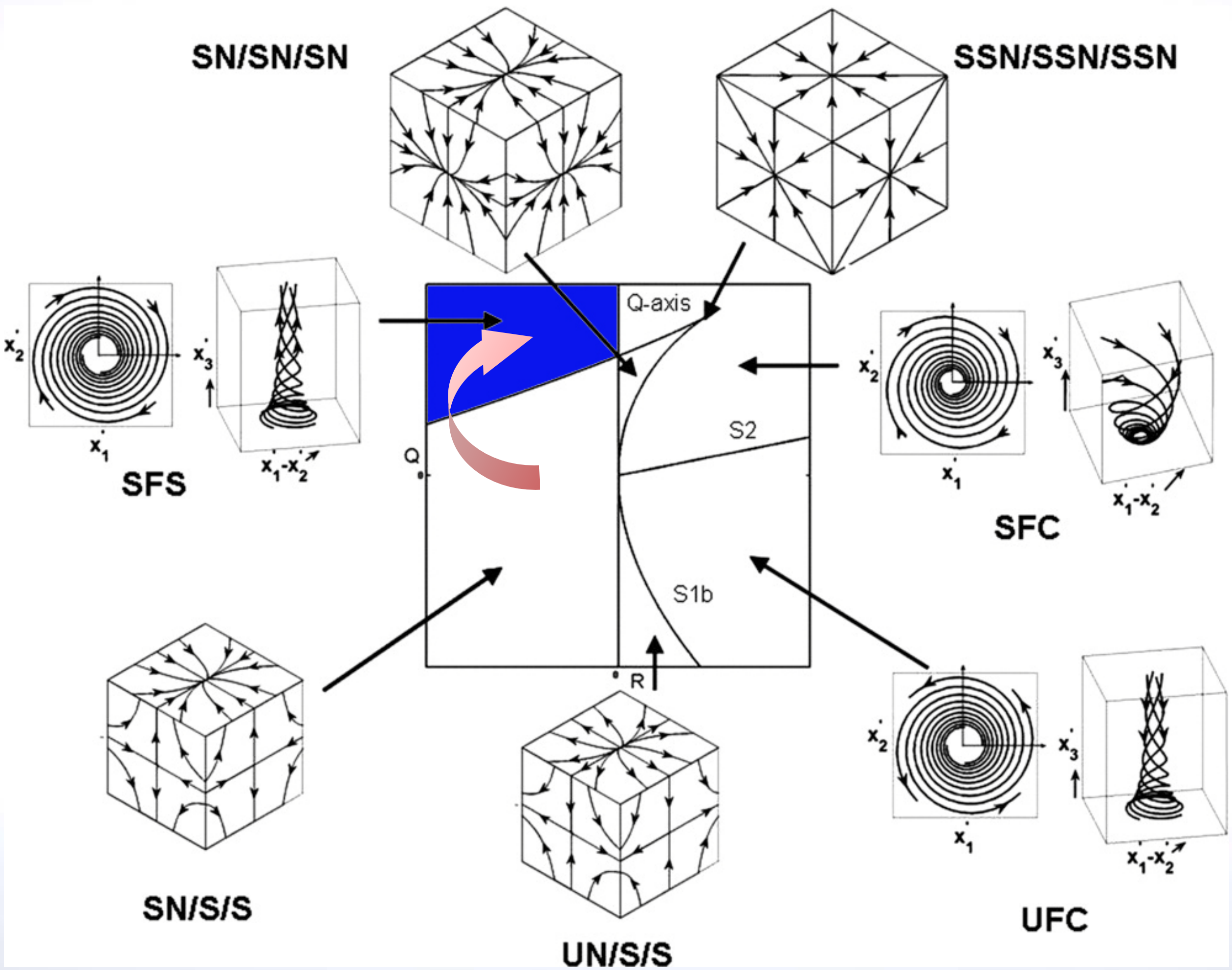




Potential – Vortical transition



Potential – Vortical transition

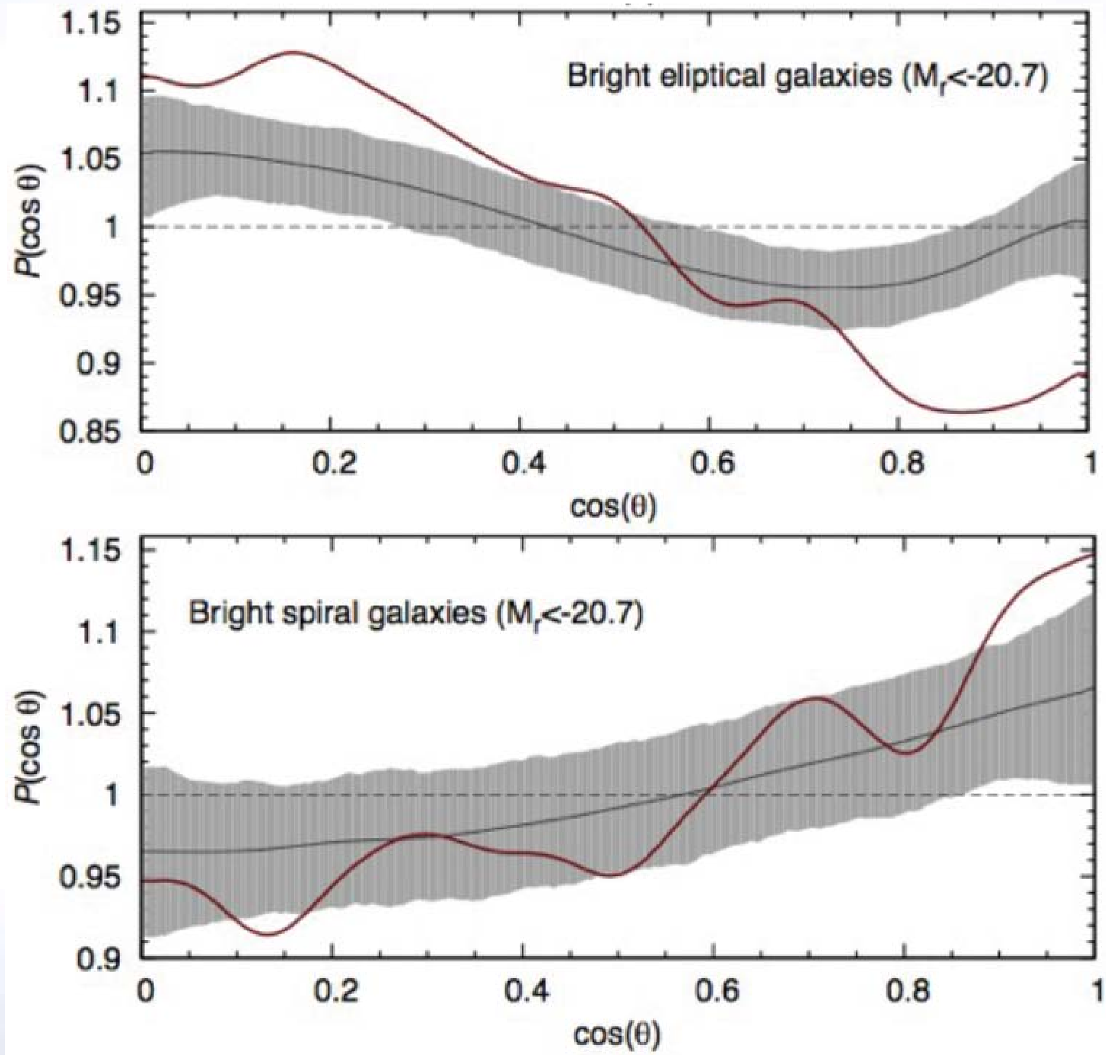


Summary

- Coherent evolution of halo/galaxy spin, vorticity and large-scale structure
- Vorticity is generated in a particular way that related to cosmic web structure
- Environmental dependence is inherited from potential to vortical flow, and eventually to the halo/galaxy spin

Thank you !

Galaxy formation in Cosmic Web



- Tempel et al (2013)